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B L

THE PREFACE.

MATHEMATICKS *receive their Name from* Matheſis, *which ſignifies Diſcipline and Doctrine, and doth not only ſignify to learn and underſtand, but alſo to call to Remembrance the Knowledge of thoſe Things which are imprinted in the Mind, that is, a Remembrance raiſed from Appearance. But now 'tis properly that Science which teaches or contemplates whatever is capable of being Number'd or Meaſured.*

And that Part of Mathematicks which relates to Number only, is called Arithmetic: *That which relates to Meaſure is called* Geometry.

Mathematicks, alſo are divided into Speculative, which propoſes only the ſimple Knowledge of the Thing propoſed, and the bare Contemplation of Truth and Falſhood; And, Practical, which teaches how to demonſtrate ſomething uſeful, or to perform ſomething that ſhall be propoſed for the Benefit and Advantage of Mankind. In all Ages and Countries where Learning hath prevailed, the Mathematical Sciences have been look'd upon as the chief and moſt conſiderable among them.

There is no Science in the World that doth improve the Mind of Man ſo much as this; by giving it a Habit of cloſe and demonſtrative Reaſoning; by freeing it from Credulity, Prejudice and Superſtition; by rendering it exact and capable of ſolving the greateſt Difficulties; and laſtly, by regulating the Imagination, and giving the Mind the greateſt Extention and Capacity that Human Nature is capable to attain.

The Uſefulneſs of this Science is almoſt infinite, there being ſcarce any Knowledge, Art, or Science, in the Univerſe, but may be aſſiſted and advanced by it. And indeed it is to this Science we owe the vaſt Improvements of Natural Knowledge in theſe laſt Ages, and ſome of the moſt Noble Inventions of the World.

Who then would be ignorant in a Science ſo Excellent, ſo Uſeful, and Beneficial to Mankind? Who would not take ſome Pains to attain a Competent Knowledge of an Art ſo truly valuable? I am confident there is no thoughtful and contemplative Perſon but would find unſpeakable Pleaſure and Satisfaction in the Study of it. And to ſuch as are ſo inclin'd, I have here ſet down the beſt Method; in Arithmetic *the Reader will find many uſeful aud pleaſant Queſtions, not to be found in any other Tract.*

The Second Chapter treats of Geometry, *in which I have given every Thing that is Uſeful in Practical Mathematicks, having every where purpoſely omitted the Speculative Part, or Things that are uſeleſs, which to Beginners would rather prove a Stumbling Block than any ways improve the Mind.*

The Third Chapter treats of the Menſuration of all manner of Superficies, *in a plain and familiar Method fitted to the Capacity of Beginners.*

The Fourth Chapter treats of the Menſuration of all manner of Solids, *concluding with two uſeful Propoſitions, the one in* Statuary, *and the other in* Staticks.

The Fifth Chapter is, the Diſcription and Uſe of the Sector, *which Inſtrument being of ſuch Univerſal Uſe, claims a Place in this Work.*

The Sixth Chapter treats of Plain Trigonometry, *in a new and conciſe Method, perform'd by Natural Numbers and by Logarithms, concluding the ſame with ſome uſeful Problems.*

Chapter Seven contains Spheric Trigonometry, *both right and oblique Angled, in a clear Manner, adapted to the Capacity of young Beginners.*

Chapter Eight treats of Aſtronomy *in general.*

The Ninth of Dialling.

And the Tenth of Surveying of Land.

Theſe are the Subjects of the enſuing Work, and herein I have ſhewn the great Uſe and Advantage that may accrue *to Mankind from them, being but ſeriouſly peruſed by a thoughtful Mind, the Reader may come to attain to a competent Knowledge in theſe Uſeful Arts.*

Theſe Arts are called by ſome, The Internal, or Liberal Arts, becauſe they are attained by the Faculties of the Soul, *which is a liberal or free Agent, and not by the Labour or Miniſtry of the Hands.*

How commendable it is to ſee Youth give their Minds to the Study of theſe Sciences! It frees the Mind of ill Habits, and raiſes the Soul above the common Way of thinking; he takes a View of the Univerſe, and ſees at one Glance how the Great Architect has fram'd this wonderful Fabric.

Sophia, *or* Philoſophia, *Wiſdom, or the Love of Wiſdom, is the Knowledge of all Arts and Sciences which any way do conduce to the Knowledge of God: And next to Divinity are the Mathematical Sciences.*

Moſt People now in this Age have an Ambition of giving their Children the beſt Education that they can, and truly, if I might adviſe, when Youth has attain'd Reading and Writing good Engliſh, *let him be put to the Mathematical School; but if the Ability of the Pa-*

rent be in ſo lew Circumſtances that they cannot afford it, then let them buy this Book, and the Boy by his own Genius *may come at what will pleaſe the Mind, and not only ſo, but make him capable of making Satisfaction to the Parent for the Expence they have been at, for his own Improvement. The Uſefulneſs of theſe Arts cannot be denied, and therefore my hopes are; that Youth will bend their Minds to the Study of theſe uſeful Sciences, and by that Means become uſeful Inſtruments in their Generation, which that every one of my Readers may do, is the hearty Prayer of their very*

Humble Servant,

Charles Leadbetter.

THE
Young Mathematician's
COMPANION.

Of ARITHMETIC:

DIFINITION:

ARITHMETIC, is the Art of Numbering well; and in it we make uſe only of theſe Nine Figures, called Digits, *viz.* 1, 2, 3, 4, 5, 6, 7, 8, 9, and the Cypher 0, in which there are divers Rules which ſhall be treated of in the following Order. And Firſt of

NUMERATION.

I. *Numeration*, is the true Diſtinction, Eſtimation, and Pronunciation of Numbers, or the Rule to read any Number tho' never ſo great, and to have a diſtinct Idea of each Place or Figure of it, which may eaſily be done, by getting the following Table by Heart. For the firſt Place to the Right Hand is the Place of Units, that is, every Figure poſſeſſing that Place is no more than the real Value, as if the Figure 1 ſtands there, it is no more than one, if 2 'tis two, &c. but every Figure that ſtands in the Place of Tens, are as many Tens as the Figure is in Value; thoſe in the third Place are Hundreds, the fourth Place are Thouſands, the fifth, Tens of Thouſands, the ſixth Place are Hundreds of Thouſands, the ſeventh, Millions, &c. there are ſeveral Ways of diſpoſing of Figures to form a Table, but this which follows, I conceive the moſt eaſy for young Beginners.

									9	Nine.
								9	8	Eight.
							9	8	7	Seven.
						9	8	7	6	Six.
					9	8	7	6	5	Five.
				9	8	7	6	5	4	Four.
			9	8	7	6	5	4	3	Three.
		9	8	7	6	5	4	3	2	Two.
	9	8	7	6	5	4	3	2	1	One.
9	8	7	6	5	4	3	2	1	0	Cypher.
Thouſands Millions.	C. Millions.	X. Millions.	Millions.	C. Thouſands.	X. Thouſands.	Thouſands.	Hundreds.	Tens.	Units.	

The lowermoſt Line of the Table is thus read, *viz.* Nine Thouſand Eight Hundred Seventy Six Millions, Five Hundred Forty Three Thouſand, Two Hundred and Ten: And ſo of the reſt. Theſe Ten Places are as far as any common Buſineſs will require, but for Curioſity's Sake, I will here ſhew, how to read 66 Places or more at Pleaſure, thus; begin at the Right Hand, and point or prick under the Seventh Place, and over it place the Figure 1, that is the Place of Millions; tell on Seven Places more, and over that Place the Figure 2, the Number 2 ſtands in the Place of Million of Millions, which is better expreſſed by Billions, (*i. e.* Bi-

Millions) tell on again Seven Figures more, and there place the Figure 3; this is the Place of Trillions, and Seven Places more will be Quartrillions, the next Seven Quinquillions, &c. according as the Figures over every Seventh Place doth ſhew. So that the Firſt Place to the Right Hand is Units, the Seventh is Millions, the Thirteenth is Bi-Millions, the Nineteenth Place is Trimillions, the Twenty Fifth Place is Quartrillions, the Thirty Firſt Place Quinquillions, the Thirty Seventh Place is Sexquillions, the Forty Third Place is Septillions, the Forty Ninth Place is Octillions, the Fifty Fifth Place is Nonillions or the Place of a Million multiplied Nine Times by itſelf; as for Inſtance, ſuppoſe I would Number this Train of Figures, *viz.*

$$78987654321\overset{9}{0}12345\overset{8}{6}78987\overset{7}{6}54321\overset{6}{0}12345\overset{5}{6}78987\overset{4}{6}$$

$$54321\overset{3}{0}12345\overset{2}{6}78987\overset{1}{6}543210$$

Here are Sixty Six Places, and are thus read, ſuppoſing them to ſtand all in one Line, begin with the uppermoſt Line at the Left Hand, and by the Directions of the Table above, begin with the Cypher, over which the 9 ſtandeth, and tell the Value of Places to the 7 at the End of the Line, and they are Seven Hundred Eighty Nine Thouſand Eight Hundred Seventy Six Millions, Five Hundred Forty Three Thouſand Two Hundred and Ten Nonillions, One Hundred Twenty Three Thouſand Four Hundred Fifty Six Octillions, Seven Hundred Eighty Nine Thouſand Eight Hundred Seventy Six Septillions, Five Hundred and Forty Three Thouſand Two Hundred and Ten Sexquillions, One Hundred Twenty Three Thouſand Four Hundred Fifty Six Quinquillions, Seven Hundred Eighty Nine Thouſand, Eight Hundred Seventy Six Quartrillions, Five Hundred and Forty Three Thouſand Two Hundred and Ten Trimillions, One Hundred Twenty Three Thouſand Four Hundred Fifty Six Bi-Millions, Seven Hundred Eighty Nine Thouſand Eight Hundred Seventy Six Millions, Five Hundred Forty Three Thouſand Two Hundred and Ten. And thus you ſee 'tis eaſy to read or pronounce any Number at Pleaſure.

II. *Of ADDITION:*

ADDITION, in General, is the putting of two or more Quantities together, and that Quantity which ariſes or reſults from thence, is called the Sum, or Aggregate of thoſe Quantities, as we ſhall make plain by Examples.

The PENCE TABLE:

s.		d.			d.		s.	d.	
1	is	12		And	10	is	0—	10	
2		24			20		1—	8	
3		36			30		2—	6	half a Cro.
4		48			40		3—	4	
5		60			50		4—	2	
6		72			60		5—	0	Crown.
7		84			70		5—	10	
8		96			80		6—	8	Noble.
9		108			90		7—	6	
10		120			100		8—	4	
11		132			110		9—	2	
12		144			120		10—	0	Angel.
13		156			130		10—	10	
14		168			140		11—	8	
15		180			150		12—	6	
16		192			160		13—	4	Mark,
17		204	Piſtole.		170		14—	2	
18		216			180		15—	0	
19		228			190		15—	10	
20		240	Pound.		200		16—	8	
21		252	Guinea.		210		17—	6	
27		324	Moidore.		220		18—	4	
					230		19—	2	
					240		20—	0	Pound.

This Table got by Heart, will be very ready for Tradesmen to caſt up Things by the Head.

And Firſt of one Denomination.

Set your Numbers down one under another, Units under Units, Tens under Tens, Hundreds under Hundreds, &c. Then begin at the Right Hand, and collect each Column ſingly into one Sum, and if the Units in that Row are leſs than Ten, ſet them down under the Line, but if they are more than Ten, ſet down all above Ten, and carry the Tens to the next Row; in which ſet down under the Line the odd Tens, and carry the Hundreds to the next Row, and ſo continuing to ſet down the Units, and carry the Tens to the next Column 'till all be done. Thus, Suppoſe I have 481 Yards of Velvet, 387 Yards of Pluſh, 245 Yards of Calicoe, 129 Yards of Camblet, how many Yards are there in all.

Velvet	481
Pluſh	387
Calicoe	245
Camblet	129

Total 1242 Yards, the Anſwer.

Thus, the Numbers being ſet down as above, begin and ſay 9 and 5 is 14 and 7 is 21 and 1 makes 22, ſet down the 2 Units, under the Line and carry the 2 Tens to the next Row; ſaying 2 that I carry and 2 is 4 and 4 is 8, and 8 is 16, and 8 is 24, ſet down 4 under the Line and carry 2 to the next Row, ſaying 2 that I carry and one is 3, and 2 is 5, and 3 is 8, and 4 makes 12, now becauſe there are no more Figures to carry, I ſet down 12, and the Sum of all the given Number is 1242, as above.

The common Way of Proving Addition (as taught in Schools) is to cut off the Top Line, and caſt the Sum up again, only leaving out the Top Line, then the Sum of theſe added to the Top Line gives the whole Total. But this ſeems to be invented more to hinder, than forward the Work, and is ſo unmaſterly, that I ſhall not recommend it, but rather adviſe the Reader to caſt up his Sum over again, and that's the beſt Way of proving his Work.

More Examples for Practice.

Years.	*Miles.*	*Bar.*	*Lib.*
17064	2706	4617	87069
8517	1042	3570	10952
321	875	2406	7403
298	263	1041	276
106	21	82	95
26306	4907	11716	105795

But if the Number be of different, or of ſeveral Denominations, then they muſt be added by ſumming up each Denomination by itſelf, and ſeeing how many of them will make one of the next Denomination, and carry them to the next Place. As for Example, ſuppoſe the following Sum of *Engliſh* Money were to be caſt up, firſt conſider that four Farthings make one Penny, twelve Pence make one Shilling, and twenty Shillings make one Pound.

Note, Over every Denomination is commonly placed *L. S. D. Q.* ſignifying *Libra*, *Solidi*, *Denarii*, *Quadrantes*, that is Pounds, Shillings, Pence, Farthings.

10.	20.	12.	4.
l.	*s.*	*d.*	*q.*
4862	14	11	1
3721	18	10	2
2951	10	04	3
2563	19	01	2
1085	12	03	1
977	15	09	2
420	04	06	3
16583	15	11	2

Over every Denomination is placed the Denominative Parts, that is, what you are to dot at, or ſhews how many of one Denomination make one of next greater. As in the following Example of *Avoirdupoiſe Weight*, Sixteen Drams make one Ounce, Sixteen Ounces make one Pound, Twenty-eight Pound make one Quarter of a Hundred, four Quarters make one Hundred Weight, and twenty Hundreds make one Ton.

	10.	20.	4.	28.	16.	16.
	T.	*C.*	*Q.*	*lib.*	*oz.*	*dr.*
	7843	14	2	20	12	10.
	6382	18	3	26	14	15.
	5671	11	1	18	10	07.
	4329	19	2	21	11	04.
	3571	17	3	25	13	13.
	2468	10	2	14	02	01.
	1073	16	1	17	06	04.
Sum	31344	09	3	05	07	06

Note, The *Troy* and *Avoirdupoiſe Weight*, bear Proportion one to another according to

Mr. *Boyle* found that $18\frac{1}{8}$ *lib. Troy*, are equal to $15\frac{1}{17}$ Pounds *Avoirdupoiſe*.

Mr. *Wingate* found that one Pound *Avoirdupoiſe*, was equal to 14 *oz.* : 12 *pw. Troy*. Here the *Troy* Ounce is more than the *Avoirdupoiſe* Ounce, but the *Avoirdupoiſe* Pound is greater than the Pound *Troy*.

Mr. *Everard* found in the year 1696, that fifteen Pounds *Avoirdupoiſe* were equal to 18 *lib* : 2 *oz* : 15 *pw. Troy*.

Mr. *John Ward* found by a very nice Experiment, that 14 *oz* : 11 *pw* : $15\frac{1}{2}$ *gr. Troy*, was equal to one Pound *Avoirdupoiſe*.

Mr. *Vincent Wing* ſays, the Proportion of the Pound *Avoirdupoiſe* to the Pound *Troy*, is as 60 to 73. And the Proportion of the Ounce *Avoirdupoiſe*, to the Ounce *Troy*, s as 80 to 73. By which the Pound *Avoirdupoiſe* is greater than the Pound *Troy*; but the Ounce *Avoirdupoiſe* is leſſer than the Ounce *Troy*, as is noted in *Win-* *ate*'s Experiment.

III. Of SUBTRACTION.

DEFINITION.

SUBTRACTION is taking a leſſer Quantity, or Number, from a greater, in order to find the Difference between them, which Difference is called the Remainder.

In order then to take one Number or Sum out of another, if they be Things of one Denomination or Name, ſet Units under Units, Tens under Tens, Hundreds under Hundreds, &c. under which two Numbers draw a Line, and remember that what you carried in *Addition* you muſt here borrow in *Subtraction*, and in Things of one Name, you always carried one for every Ten they amounted to; ſo here when the Under Figure is greater than the Upper, you muſt in this Caſe add Ten to the Upper, and then take the Under out of it, then for this Ten which you muſt call One that you borrowed, add One to the next Figure, and take it out of that which ſtands over it and ſet the Remainder under the Line, proceed thus 'till all the Differences be taken, and then the Work is done, as in theſe Examples.

	C.	*L.*	*lib.*
From	5026	2813	47015
Sub.	3578	1642	38148
Remains	1448	1171	8867

But in Numbers of different Denominations, you muſt always borrow one of the next greater Denomination and ſubtract, remembering always to pay the One you borrowed, and ſet the Remainder under the Line, as in theſe Examples of Money.

	l.	*s.*	*d.*	*q.*
Lent	32051	14	08	0
Paid	9447	16	10	1
Rem.	22603	17	09	3

PROOF.

The Proof of this Rule, is to add together the Remainder, and the Number to be ſubtracted, and that Sum will be equal to the Number from which *Subtraction* was made. See an Example of *Troy Weight*.

	10	12	20	24	
	lib.	*oz.*	*pw.*	*gr.*	
Bought	576	04	11	14	
Sold	427	07	16	19	} add
Rem.	148	08	14	19	
Proof	576	04	11	14	

QUESTION.

In the Year of our Lord 508, *Tarquin* was baniſhed from *Rome*, for the Raviſhing of *Lucretia*, I demand how many Years it is ſince, to this preſent Year 1793? Set the Numbers down thus.

Preſent Year - - - - -	1793
Lucretia Raviſhed in - - -	508
Anſwer - - - - - -	1285

To conclude this Rule, I ſhall here ſubjoin an Example or two uſeful in Buſineſs, which are theſe. 1. Two Men *A* and *B*, enter into Partnerſhip, and after ſome Time *A* had diſburſt 17*l.* 3*s.* 3*d.* *B* had paid 10*l.* 10*s.* they are now indebted 16*l.* 13*s.* and from the Beginning of their Partnerſhip they are to pay equal, *Quere*, What has each now to pay of the Debt?

SOLUTION.

	l.	*s.*	*d.*
From the whole Debt — —	16	13	0
Subtract the X of their Payments —	6	13	3
Remains — — —	9	19	9
Half is what *A* is now to pay —	4	19	10½
A has paid more than *B* — —	6	13	3
Sum is what *B* muſt pay — —	11	13	1½

PROOF.

A	muſt pay	—	—	4	19	10½
B		—	—	11	13	1½
Sum is = to the Debt		—	—	16	13	0

Or thus,

		£.	s.		£.	s.	d.			
A	paid in all	17	3¼	and	4	19	10½	22	3	1½
B		10	10	and	11	13	1½	22	3	1½
Their whole Account was					—			44	6	3

Example 2. *A* hath Diſburſt 40*l.* 16*s.* 8*d.* ½, *B* has paid 26*l.* 13*s.* 4*d.* they are now indebted 27*l.* 18*s.* 2*d.* I demand what is each to pay, and what was their whole Debt at firſt.

			£.	s.	d.			
Anſwer	*A*	muſt pay	6	17	4¾			
	B		21	0	9¼			
And their whole Account at firſt was					—	47	14	1¼

IV. *Of MULTIPLICATION.*

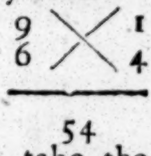
DEFINITION.

MULTIPLICATION is the taking or repeating of one Number or Quality as often as there are ſuppoſed Units in the other. The Number multiplied is called the Multiplicand, the Number by which you multiply, is the Multiplicator, or Multiplier, and that which is found or produced, is called the Product. Alſo for Brevity's Sake the Multiplicand and Multiplier are called Factors, becauſe between them they make up the Product. Multiplication ſerves inſtead of many Additions, and this is the Table.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

The Table above is obvious to the meaneſt Capacity, for to find one of your Factors on the Head, and the other in the firſt Column on the Left Hand, and in the Common Angle, or Place of Meeting is the Product. Example, what is the Product of 6 multiplied by 9? Now it matters not whether you ſeek for the 6 or the 9 on the Head, and the other on the Side, for tis all one, you'll find the Product to be 54, and ſo of any other.

The Ground of *Multiplication Table* may be demonſtrated by this Croſs in the Margin, for place the two *Digits* 6 and 9, (or any other two *Digits)* on the Left Side, and their Difference to Ten on the Right Hand, then take the Difference from the *Digit*, (not from it's own, but croſs-wiſe) as Four out of Nine, or one out of Six, the Remainder will be Five, then the two Differences, *viz.* Four and One multiplied together make Four, which with the Five, place under the Line, as you ſee done, and it makes 54 for the Product.



This may alſo be explained by the Fingers and Thumbs of your Hands, for as there are Nine Figures and the Cypher makes Ten, ſo have we Eight Fingers and Two Thumbs make alſo Ten; then if you would know how many ſix times Nine, or nine times Six is, (for it's all one) Gripe or Clench all your Fingers and Thumbs cloſe, being thus, they are all Units, but every one as you raiſe up is Ten, then raiſe the Thumb on the Right Hand,

and call it Six, to anſwer the Six of the Multiplier, then raiſe the Thumb of the Left Hand, and call it Six, raiſe the Firſt Finger, and call it Seven, raiſe the Second Finger and call it Eight, raiſe the Third Finger and call it Nine for your other Factor; now all the Fingers and Thumbs that are up are Tens, and thoſe down are Units; add the Tens together make Five, and multiply the Units together make Four, ſo I ſee that ſix times Nine make 54 as before.

Here follows Examples for Practice.

Multiplicands, - - -	52		365
Multipliers, - - - -	7		9
Products, - - - -	364		3285
Multiply me - -	1681	And - -	2561
By - - - - - -	12	By - - -	28
	3362		20488
	1681		5122
Products, - -	20172		71708

Note, Ever remember that if your Multiplier conſiſt of ever ſo many Places, that the firſt Figure of it's Product ſtand under the ſame Place that it's Multiplier poſſeſſeth, then add the particular Products together, gives the Product of the Whole.

Notes upon the Nine Digits.

I. The *Digit* One, hath a Property which no other Number hath, for it neither multiplieth nor divideth, but leaveth the Number to be multiplied or divided the ſame.

II. Multiplying any given Number by Two, is the ſame with doubling of it; and that whether you add Two to itſelf, or multiply it in itſelf the Sum and Product are equal. And it is moreover worth taking Notice thereof, that no ſquare Number, how large ſoever, can ever terminate, or end with the *Digit* 2.

III. If you would multiply any given Number by 3, to that Number add the Double thereof, and the Sum ſhall be equal to the Product.

IV. If you would multiply any Number by Four, you muſt double the Dublication, and the Sum is the Product.

V. If you would multiply any Number by Five, add a Cypher to the given Number, and take half that Sum for the Product.

VI. If you would multiply any Number by Six, add a Cypher to the given Number, and take half thereof, to which add the given Number, the Sum ſhall be equal to the Product.

Note, Between theſe two laſt mentioned *Digits* Five and Six, there is a ſecret Property; for if you multiply either of them in themſelves, the Number produced by ſuch Multiplications, ſhall terminate in themſelves.

The Number Six hath another eminent Property, for all it's Aliquot Parts are equal to itſelf, as it's Half, it's Third, and it's Sixth, being all added together make Six. And of Numbers that have this Property, there are but ten to be found, between one, and one Million of Millions, which are theſe exhibited in this Table.

	6
	28
	486
	8128
Numbers of Aliquot Parts. $\frac{1}{2}$ $\frac{1}{3}$ $\frac{1}{6}$	120816
	2096128
	33550336
	536854528
	8589869056
	137438691328

VII. If you would multiply any Number by Seven; add a Cypher to the given Number, and take half thereof, to which half add the double of the Number given, the Sum of them ſhall be the Product of the given Number, multiplied by Seven.

VIII. To multiply any Number by Eight, to the given Number add a Cypher, and from thence ſubtract the double of the Number given, the Remainder is the Product.

IX. To multiply by Nine, add a Cypher to the given Number, and from that Number ſubtract the given Number, the Remainder is the Product.

Obſervation.

This *Digit* Nine, hath a Privilege above all the other *Digits*; for if you take any Number, the Nines taken out of the groſs Sum of that Number, or of all the Parts thereof ſeverally, the remaining *Digit* will be ſtill the ſame.

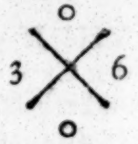
EXAMPLE,

In the Number 45 there are 5 Nines contained therein, ſo if you multiply Nine by Five; the Product is 45. In like manner, if you take the Nines out of this Number 476, it is all one as if you ſhould take the Nines out of the ſingle Figures 4, 7, 6, which do make Seventeen, from which Nine being taken, there will remain the *Digit* Eight, and alſo if you divide 476 by Nine, the Quotient will be Fifty two and Eight remaining.

Proof of Multiplication.

For hence proceeds that falſe Way of proving Multiplication by the Croſs, or by throwing away the Nines out of the Factors and Product.

As for Inſtance, if 272802 be multiplied by 34125, the Product will be 9309432384.



Make a Croſs as in the Margin, throw the Nines out of the Multiplicand 272802, and there reſt 3 which I place on the Left Side the Croſs. Secondly, throw the Nines out of the Multiplier 34125, and the Overplus is Six, which place at the Right Side the Croſs: Thirdly, the Product of Three by Six is Eighteen, out of which throw the Nines, and there remains 0, which place at the Top of the Croſs. Laſtly, the Nines thrown out of the Product, Reſt 0, which place at the Bottom of the Croſs.

Hence, becauſe the Top and Bottom Figures are the ſame, the Work is proved to be right: I ſay this is true, but ſuppoſe any of the Figures in the Product are tranſpoſed (as I have often known) I'll ſuppoſe thus, 9390423384, here the Figures being the ſame, when the Nines are thrown out, there will alſo remain 0, equal with the Figure at the Top of the Croſs, which proves the Sum Right, tho' at the ſame Time it be moſt groſsly falſe.

Compendiums in Multiplication.

Whenever your Multiplier has Cyphers in the Unites, Tens, or Hundreds Places, *&c.* place them to the Right Hand; thus.

```
487        392        726
 10         20        500
----       ----     ------
4870       7840     363000
```

How to multiply by any Number under 20, 30, or 40, ſo as to bring the Whole Work into one Line.

Firſt, Any Number under Twenty.

RULE.

Multiply each Figure in the Multiplicand, by the Unites Figure in the Multiplier, adding to each it's back Figure, that ſtand on the Right Hand of that you multiplied, only mind that the firſt Figure you begin with, is the Thing itſelf, there being none to be added to it.

EXAMPLE.

Let 365 be multiplied by 12, and poſſeſs but one Line.

```
 365
  12
----
4380
```

Firſt ſay 2 times 5 is 10, that is 0, and carry one, then 2 times 6 is 12, and 1 I carried is 13, and the 5 in the Unites Place of the Multiplicand makes 18, that is, ſet down 8 and carry one, then 2 times 3 is 6 and 1 I carried is 7 and the 6 in the Multiplicand makes 13, that is, 3 ſet down and carry 1, which is added to the 3 in the Multiplicand, and it makes 4 to ſet down, ſo the Product of 365 by 12, is = 4380.

More Examples.

$$\begin{array}{r} 764 \\ 15 \\ \hline 11460 \end{array} \qquad \begin{array}{r} 6812 \\ 18 \\ \hline 122616 \end{array} \qquad \begin{array}{r} 81263 \\ 19 \\ \hline 1543997 \end{array}$$

Secondly, Any Number between 20 and 30.

Multiply by the Unites of the Multiplicand, add to each Product the double of the Right Hand Figure, and at laſt add what you carried to the double of the laſt Figure in the Multiplicand, and that will be the true Product. See theſe Examples and mark them well.

$$\begin{array}{r} 7384 \\ 23 \\ \hline 169832 \end{array} \qquad \begin{array}{r} 65981 \\ 25 \\ \hline 1649525 \end{array} \qquad \begin{array}{r} 5486 \\ 28 \\ \hline 153608 \end{array}$$

In the firſt ſay, 3 times 4 is 12, that is, ſet down 2 and carry 1, then 3 times 8 is 24, and 1 I carried in Mind makes 25, and the double of 4 in the Multiplicand is 8 added to 25, makes 33, ſet down 3 and carry 3; then 3 times 3 is 9, and 3 I carried is 12, and 16 (the double of 8) makes 28, ſet down 8 and carry 2; then 3 times 7 is 21, and 2 I carried is 23, and 6 the double of 3 is 29, ſet down 9 and carry 2, which added to the double of 7, makes 16, which ſet down on the Left Hand of the others under the Line, and it makes the Whole Product 169832.

Thirdly, Any Number between 30 and 40.

Multiply as before, and add the Triple of the Right Hand Figure in the Multiplicand to your Product, and at laſt, add the Triple of the laſt Figure in the Multiplicand, to what you carried, and ſet them down under the Line and then 'tis done.

See theſe Examples.

57806	12948	705381
34	36	39
1965404	455328	27509859

Fourthly, By the firſt Rule, to multiply any Number compounded of any Number under 20, the Product is readily found, as by Examples.

46863	79062
112	112
562356	948744
46863	79062
5248646	8854944

Fifthly, To multiply by 1614, or 1517, or by 1716, *&c.*

Firſt multiply by 14 in one Line, and by 16 in the other, and the other Sum by 17 in one Line, and by 15 in the other, *&c.* See the Work.

357948	79062
1614	1517
5011272	486285
5727168	429075
577728072	43393785

Ten Merry Men come to an Alehouſe, and agree with the Landlord for a Guinea, *per* Man, are to have two Pints of

Drink, *per* Piece, *per* Day, for ſo long time as they all being placed together on one Form at every Drinking. There might be a divers Tranſmutation of Place; *Query*, How comes the Landlord off with his Gueſts, both as to Money and Time?

Sixthly, How to multiply by the Component Parts, inſtead of the Whole.

Example, - -	542, by 63? take 7 and 9.
	7
	3794
	9
Product - -	34146

If any odd Numbers are given which are not an even Product of any two of the Nine *Digits*. *&c.* then take two Figures whoſe Product comes neareſt either, more or leſs than the given Multiplier, and add if you took a leſs Number, but ſubtract if you took more, as in theſe Examples.

Multiply	9684 by 35,	and	738 by 29.	
	6		7	
	58104		5166	
	6		4	
Too much	348624		20664	Too little.
Subtract	9684	Add	738	
Products	338940		21402	True.

Seventhly, How to multiply ſeveral Denominations.

	l.	*s.*	*d.*	*q.*
How much is 7 times	58	12	9	2
				7
Anſwer - - -	410	9	6	2

TROY WEIGHT.

	lib.	oz.	pw.	gr.
Multiply - -	476	8	14	17
By - - -				29
Anſwer - -	13825	1	6	13

To multiply by Logarithms.

Logarithms were firſt invented by Lord *Neper*, a *Scotch* Baron, about the Year 1614, and after compleated by our Countryman, Mr *Henry Briggs*, Profeſſor of Geometry at *Oxford*, and at *Greſham College* in *London* : They perform that by *Addition* and *Subtraction*, that Natural Numbers do by *Multiplication* and *Diviſion*. And here

Note, That the Characteriſtick, or Index is always leſs by one, than the Number of Places of the abſolute Number.

The RULE.

To multiply by *Logarithms*, find the *Logarithm* of two given Factors, and the Sum (rejecting *Radius*) is the *Logarithm* of the Product ſought.

EXAMPLE.

Let the Product of 4736, by 28 be ſought ?

OPERATION.

4736 - - -	3.6754117
28 - - -	1.4471580
132608 - -	5.1225697

EXAMPLE II.

Multiply me	375	2.5740313
By	15	1.1760913
Product	5625	3.7501226

VI. *Of DIVISION.*

DIFINITION.

DIVISION, ſignifies the diſtributing or parting of any Whole into Parts, and is only a Compendium of *Subtraction* ; for the Diviſor is ſo many Times contained in the Dividend, as there are Unites in the Quotient.

The Number to be divided is called the Dividend ; that by which you divide the Diviſor, and the Number of Times that the Dividend contains the Diviſor is called the Quotient. And here Note, That *Multiplication* and *Diviſion* interchangeably and infallibly prove each other, as will be ſhewn by and by.

Firſt ask how oft in Quotient, Anſwer make.
Then multiply, ſubtract, a new dividual take.

What's the Quotient of 354, divided by 6 ? Set the Numbers thus.

Diviſor. Dividend. Quotient.

```
6 ) 354 ( 59
     ..
    ---
     54
    ---
Rem:  0
```

More Examples for Practice.

12) 4170 (347
57
90
Rem. 6

19) 17501 (921
40
21
Rem. 2

Compendium.

If the Diviſor hath Cyphers in the Unites, Tens, Hundreds Place, &c. cut them off with a Line, and alſo cut as many Places off in the Dividend and divide with the remaining Figures, as you ſee in theſe Examples.

2|0) 17|6 (8
Rem. 16

46|00) 1357|00 (29
92
437
414
Rem. 2300

PROOF of Diviſion.

The ſureſt Way of proving Diviſion is by Multiplication, and of proving Multiplication is by Diviſion.

For if the Product be divided by either Factor, the Quotient will be the other, and nothing will remain if the Sum is right. And in Diviſion if you multiply the Diviſor by the Quotient (and add the Remainder thereto, if any) the Product will be the Dividend. Example in Page 17, we have multiplied 357948, by 1614, and the Product is 577728072: Now for Proof, if we divide 577728072 by 1614, the Quotient will be 357948, and nothing will remain, which proves the Work to be right.

Or, if we divide 577728072, by 357948, the Quotient will be 1614, and nothing will remain.

Short Diviſion.

Short Diviſion is done by ſubtracting as you go on, and only ſet down the Remainder, and not the Products, as you may perceive by this Work.

```
1614) 577728072 (357948
         ......:
      ---------
      9382
      -------
       12828
       -------
        15300
        ------
         7747
         ------
          12912
          -------
Rem.          0
```

If you have any Thing to do by Diviſion, and you would perform the ſame by Multiplication, you may find a Factor that will do it, that is, if you divide Unity by the Diviſor, that Quotient will be a Multiplicator, by which if you multiply your Dividend, this Product will be equal to the Quotient.

As for Example.

Suppoſe I had 1740 to be divided by 4, what Number is that by which if I multiply 1740, the Product ſhall be the ſame as if I had divided it by

```
4) 1740 (435
     ..
   ------
    14
    ----
     20
     ----
      0
```

4) 100 (.25 Factor.

1740
.25

8700
3480

Product 435 The ſame with the Quotient above.

Note, When 1 is divided by 4, there are two Cyphers added, for which Reaſon I omit two Cyphers in the Product.

Diviſion by Logarithms.

Diviſion by *Logarithms* is performed by Subtraction, for ſubtract the *Logarithm* of the Diviſor, from the *Logarithm* of the Dividend, and the Remainder is equal to the *Logarithm* of the Quotient.

EXAMPLE I.

Let 132608 be divided by 28, what's the Quotient?

See the Work.

Dividend - -	132608 - -	5.1225697
Diviſor - -	- 28 - -	1.4471580
Quotient - - -	4736 - -	3.6754117

EXAMPLE II.

Dividend - -	5625 - - - -	3.7501226
Diviſor - -	15 - - - -	1.1760913
Quotient - -	375 - - - -	2.5740313

CHAP.

CHAP. II.

Of REDUCTION.

DIFINITION.

REDUCTION is no Rule properly of itſelf, but depends wholly on *Multiplication* and *Diviſion.* It is either Aſcending or Deſcending. Aſcending is performed by *Diviſion,* and is when you would Reduce, or bring Things of a leſs Name, to Things of a greater, as Farthings into Pounds, *&c.* Deſcending is when you would Reduce, or bring Things of a greater, to Things of a leſſer Name, as Pounds into Shillings, Pence, or Farthings, and this is always done by *Multiplication.*

QUESTION.

A Gentleman dying, left 410625 Pounds, to be equally divided amongſt Twenty Nine Perſons, I demand, what each muſt have to his Share?

See the Work.

29) 410625 (14159 *l.*
....
29
120
116
46
29
172
145
275
261
Rem. 14 Pounds.
20
29) 280 (9 Shillings.
261
19
Pence in a Shilling 12
29) 228 (7 Pence.
203
Rem. 25 Pence.
Farthings in a Penny 4
29) 100 ($3\frac{13}{29}$ Farthings.
87
Rem. 13 Farthings.

Anſwer 14159 *l.* 9 *s.* 7 *d.* $3\frac{13}{29}$ *q.*

In reducing of great Things into leſſer, multiply by the Denominate Parts, and add in the odd Parts if there be any, and the laſt Product is the Anſwer.

EXAMPLE.

In 95*l.* 17*s.* 2*d.* 1*q.* how many Farthings?
Anſwer 92025 Farthings.

In 92025 Farthings, how many Pence, Shillings and Pounds?

$$\begin{array}{llll} & 12 & 2|0 & l.\ \ s.\ \ d.\ \ q. \\ 4)\ 92025 & (\ 23006 & (\ 191|7 & (\ 95\ \ 17\ \ 2\ \ 1 \\ \hline \quad 1 & \quad 2 & \quad 17 & \end{array}$$

How many Pounds are there in 527 Moidores? Multiply by 27, and divide by 20, and the Anſwer will be 711 *l.* 9 *s.*

In 711 *l.* 9 *s.* how many Moidores? Multiply by 20, and divide by 27, and the Anſwer you will find to be 527 Moidores.

Queſt. 3. In 627 *l.* 15 *s.* 6 *d.* how many Pieces of 16 *d.* of $14\frac{1}{2}$, of $12\frac{1}{4}$, of $10\frac{3}{4}$, of $8\frac{1}{2}$, of $5\frac{3}{4}$, and of $1\frac{1}{4}$, and of each an equal Number? Add all the Particulars together, and they make 69 Pence, and Reduce the Money into Pence, are 150666 for a Dividend, the Quotient is the Anſwer $2183\frac{39}{69}$ Pieces.

Note, In Things of this Nature, the Diviſor and Dividend muſt always be of one name.

Queſt. 4. In 1580 Ells *Flemiſh*, how many Ells *Engliſh*?

Operation.

$$\begin{array}{rrl} & 1580 & \\ & 3 & \\ \hline 5) & 4740 & (948\ \text{Ells } \textit{Engliſh} \end{array}$$

Queſt. 5. How many Miles, Furlongs, Poles, Yards, Feet, Inches, and Barley Corns will reach round the World, allowing *Norwood*'s Meaſure of $69\frac{1}{2}$ Miles, to one Degree of a great Circle.

Anſwer, Miles 25020, Furlongs 200160, Poles 8006400, Yards, 44035200, Feet 132105600, Inches 1585267200, Barley Corns 4755801600.

Queſt 6. How many ſquare Yards are there in four ſquare Miles.

Anſwer, 49561600 ſquare Yards.

Of TIME.

From the Creation of the World, to this preſent Year 1739, by the beſt of Prophane Hiſtory it is reckoned 5688 Years, how many Minutes are there in the ſame?

See the Work.

	d.	*h.*	
One Year is	365	6	
	24		
	1466		
	730		
Hours	8766		in one Year.
	60		
Minutes	525960		in one Year
Years	5688		
	4207680		
	4207680		
	3155760		
	2629800		
Anſwer	2991660480		Minutes.

Queſt. 7. How many times doth a Clock that ſtrikes Hours, Halves and Quarters, ſtrike in a Year?

Hence, becauſe it ſtrikes Quarters, that is ten times in one Hour, excluſive of the Hours, amounts to 240 times in 24 Hours, and the times it ſtrikes in 24 Hours excluſive of the Quarters, are 156 added to 240, makes 396 times the Clock ſtrikes in 24 Hours one Day, which multiply'd by $365\frac{1}{4}$, will produce 144639 times that the Clock ſtrikes in a Year, But $396 \times 365 = 144540 + 81 = 144621$ the true Anſwer.

Of MOTION.

Queſt. 8, If the Difference of the Times of the Lunations in 19 Years be 1 *h.* 28' 15'', how long will it be before it comes to make one Day?

Anſwer 310 $\frac{150}{5295}$ Years.

Queſt. 9. I would put 100 Hoſheads of *London* Beer into 50 Wine Pipes, what muſt the Caſk contain which receives the Difference?

Anſwer 4 $\frac{18}{54}$ Beer Meaſure.

CHAP. III.

Of PROGRESSION.

DIFINITION.

PROGRESSION is two fold, *viz.* Arithmetical, and Geometrical.

Firſt, Arithmetical *Progreſſion,* is when Numbers do proceed by equal Difference, either increaſing or decreaſing.

As { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12.
2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, *&c.*
40, 38, 36, 34, 32, 30, 28, 26, 24, 22, 20.

In the Firſt of theſe, is a continual increaſe of 1; in he Second of 3; in the Third a decreaſe of 2.

If never ſo many Quantities are ſo proportional (as above) the double of the Mean is equal to the Sum of the two Extreams, as 1, 2, 3, the double of the 2, the mean, or middle Term, is equal to the Sum of 1 and 3 the two Extreams, *viz.* 4.

To find the Sum.

The Sum of any Arithmetical Proportion, the Sum of all the Terms may be found if you multiply the Sum of the firſt and laſt by half the Number of Places, or the Number Places, by half the Sum of the firſt and laſt

Terms, the Product will be equal to the Sum of all the Terms.

So in the Firſt Rank of Numbers, above the Sum of 1 and 12 is 13, multiply'd by 6 half Number of Places, the Product 78 is equal to the Sum of all the Terms. And ſo often doth a Clock ſtrike in 12 Hours.

Four Numbers in an Arithmetical Proportion, the Sum of the two Means, is equal to the Sum of the two Extreams. So 2, 5, 8, 11, the Sum of 5 and 8 is 13, and the Sum of 2 and 11 is 13.

A QUESTION.

There was a Wager laid between a Taylor and a Gentleman's Servant, the Taylor was to gather 100 Stones laid a Yard aſſunder, and the other was to run two Miles and a half reckoned, ſrom the Place where the Baſket was to ſtand, and back again. (I was an Eye Witneſs of the Performance). The Taylor won the Wager. Now let us ſee how many Miles the Taylor Run, in gathering up the Stones.

An Hundred Stones right in a Line,
Exact a Yard aſunder,
How many Miles then doth he go,
That gathers up that Number.

See the Work.

Sum of the firſt and laſt Terms = 101
Half Number of Places - - - - - 50

Sum of all the Terms - - - - - 5050
2

Doubled are the Number of Yards = 10100

The Place where the Race was, they have 8 Yards to the Pole, ſo that it amounted to of their Miles 3, Furlongs 7, Poles 22, and 4 Yards. But $5\frac{1}{2}$ Yards to the Pole, it amounted to 5 *M.* 5 *F.* 36 *P.* 2 *Y.*

Queſt. 2. By having the firſt Term and Common Exceſs, to find the laſt Term of any Progreſſion, tho' never ſo large.

RULE.

Multiply the Number of Places leſs by one, by the Common Exceſs, and to the Product add the firſt Term, this Sum is equal to the laſt Term ſought.

EXAMPLE.

In a Series of Numbers, where the firſt Term is 2, the Common Exceſs 3, and the Number of Places 11, what's the laſt Term ?

Operation.

Number of Places leſs 1 = 10
Common Exceſs - - - - - - 3

Product - - - - - - - 30
Firſt Term add - - - - - - 2

Laſt Term - - - - - - - 32

Queſt. 3. The firſt and laſt Terms and Total given, to find the Number of Places.

RULE.

Divide the Sum of all the Terms by the Sum of the firſt and laſt Terms, the Quotient is equal to half the Number of Places.

EXAMPLE.

Let the Sum of all the Terms be 187, and the Sum of the firſt and laſt Terms 34, what's the Number of Places.
Anſwer 11.

Queſt. 4. The firſt and laſt Terms, and Number of Places given, to find the common Exceſs.

RULE.

From the laſt Number take the firſt, the Remainder is a Dividend, which divide by the Number of Places leſs by 1, the Quotient is the Common ?

EXAMPLE.

Let the laſt Term be 32, and the firſt 2, the Number of Places 11. What's the common Exceſs ?

Anſwer 3.

Queſt. 2. Progreſſion Geometrical, or Geometrical Proportion continued, is when Numbers proceed by equal Proportion, or Ratio's, that is, according to one common Multiplier, or Exponent of the common Ratio, whether increaſing or decreaſing.

As, 2, 6, 18, 54, 162, 486, *&c*.

Here 3 is the common Exceſs or Multiplier.

Three Numbers in a Geometrical Proportion, the Square of the Mean, is equal to the Rect-angle of the two Extreams. So 2, 4, 8, being given, the Square 4 is equal to 16, equal to the Product of 2 by 8.

Four Numbers in a Geometrical Proportion, the Rect-angle, or Product of the two Means, is equl to the Rect-angle, or Product of the two Extreams.

EXAMPLE.

Let 6, 24, 96, 384 be given, the Product of 24 by 96, is equal to the Product of 6 by 384, *viz*. 2304.

If over any Rank of Geometrical Numbers, you place a Series of Arithmetical ones, beginning with 0, the *Addition* and *Subtraction* of the Indexes, anſwer to the *Multiplication* and *Diviſion* of the Numbers they ſtand over.

EXAMPLE.

EXAMPLE.

Indexes 0 1 2 3 4 5 6 7.
Numbers 1, 2, 4, 8, 16, 32, 64, 128.

For 2 added to 3 make 5, which is the Index of 32, equal to the Product of 4 by 8, alſo the Index of 2 and 4 is 6, and the Product of 4 by 16 is 64.

In this are two Caſes, which we ſhall briefly ſet down for the Learner's Practice,

CASE I.

The common Exceſs and the Number of Places given, to find the laſt Number.

RULE.

Multiply the Number belonging to any one Place by itſelf, and that Product ſhall be the Number or Term belonging to twice ſo many Places wanting one Place, which Term multiply by the common Exceſs, gives the Term of the Progreſſion that you ſeek.

EXAMPLE.

Let the Term or Number anſwering to the 33d Place, or Sum of all the Terms be ſought, the common Exceſs being 2?

Here by obſerving well the Premiſes the following Work will be evident.

For Firſt, You ſee the Farthings anſwering the 5th Nail are 16, which being ſquared are 296, the Farthings anſwering the 9th Nail, or Place, and ſo on, as you ſee here ſet down.

Farthings.

	Farthings.	Nails.
	1	1
	2	2
	4	3
	16	5
16 ſquared is equal to - - -	256	9
	256	
	1536	
	1280	
	512	
256 ſquared is equal to - -	65536	17
	65536	
	393216	
	196608	
	237680	
	237680	
	393216	
65536 ſquared is =	4195967296	33

Here you ſee that 65536, the Farthings anſwering the 17th Place, or Nail, being ſquared, gives the Farthings anſwering the 33d Place, and ſo on *ad infinitum*.

CASE II.

Given, the common Exceſs and the laſt Term, to find the Sum of all the Terms.

RULE.

Multiply the laſt Term by the common Exceſs, and rom the Product take the firſt Term, divide the Remainder y the common Exceſs leſs by one, the Quotient is the Sum f all the Terms.

EXAMPLE.

What's the Sum of 1, 2, 4, 8, 16, 32.

Operation.

Laſt Term - - - - 32
Common Exceſs - - - 2

64
Firſt Term Subt. 1

1) 63 (63

Anſwer 63.

By this Rule is ſolved the Queſtion of buying a Horſe having four Shoes, and in every Shoe eight Nails, at a Farthing the firſt Nail, and ſo doubling to the End of 32 Nails, by working according as has been taught above, you will find the Sum of all the Terms, or Farthings to be 4294967295, which reduced are 4473924 *l.* 5 *s.* 3 *d.* 3 *q.* and ſo much doth the Horſe coſt at that Price.

CHAP. IV.

The Single RULE *of* THREE *Direct.*

DIFINITION.

THE Rule of Three, or the Rule of Proportion, is commonly called the *Golden Rule*, for it's excellent Uſe, it teaches to find a Fourth Number, which ſhall have the ſame Proportion to the Third, as the Second has to the Firſt, or as the Firſt is to the Second, ſo is the Third to the Fourth.

This Rule is known thus; If more require more, or leſs require leſs, the Queſtion belongs to this Rule:

How to State the Queſtion.

The Demand muſt be always in the Third Place, the Firſt muſt be of the ſame Name.

And the Anſwer to the Queſtion will be of the ſame Name with th the Middle Term: The Queſtion being thus Stated, multiply the Second and Third Terms together, and divide the Product by the Firſt Term, the Quotient is the Anſwer.

EXAMPLE.

Queſt. 1ſt. If 8 Yards of Cloth coſt 16 *l.* 13 *s.* 4 *d.* what will 64 Yards coſt at that Rate?

Note, Here the Demand lyeth in the 64, therefore it muſt poſſeſs the Third Place.

```
      Yds.   l.   s.  d.  Yds.
   If  8 — 16   13   4 — 64
           20              4000
          ———             ————————
          333           8)256000(32000
           12               .....
          ———              ———————
         4000                16
                           ———————
                              0
                   2|0      l.
  12)   32000  (266|6  (133
         ....       |
        ———————   ——————
         80   Rem.  6 s.
        ——————
         80
        ——————
          80
        ——————
  Rem.    8 d.
```

Anſwer 133 *l.* 6 *s.* 8 *d.*

Queſt. 2. A Fruiterer bought 2001 Apples at three for a Penny, and he alſo buys 2001 more, at 2 for a Penny, which he mingles together, and ſells them out at 5 for Two Pence; I demand, whether he gain'd or loſt by the Bargain, and how much?

Anſwer he loſt in the Whole 5 *s.* 6 *d.* $2\frac{4}{5}$ *q.*

Queſt. 3. A Poulterer had 60 *l.* left him for a Legacy, which he laid out in Eggs on this wiſe, *viz.* he bought

30 *l.* worth in at two for a Penny and ſold them out at four for a Penny, he alſo laid out the other 30 *l.* in Eggs, and bought them in at four a Penny, and then out at two a a Penny; now do I demand whether he gained or loſt in the Whole, and how much?

Anſwer he gained 15 *l.*

Queſt. 4. How many Ten Inch Tiles will ſerve to pave a Floor that is 18 Yards long, and 18 Yards broad?

Anſwer 4199 $\frac{4}{100}$.

Queſt. 5. If one Pound of Nutmegs be in value to 50 Oranges, and 70 Oranges to 40 Lemons, and one Lemon worth three-half Pence, what's the Price of one Pound of Nutmegs at that Rate?

Anſwer 3 *s.* 6 *d.* 1$\frac{3}{7}$*q.*

Queſt. 6. If 6 Pounds of Pepper be worth 13 Pounds of Ginger, and 19 Pounds of Ginger be worth 4$\frac{1}{4}$ of Cloves, and 10 Pounds of Cloves be worth 63 Pounds of Sugar, at 5 *d.* per Pound, what's the Value of 100 Weight of Pepper?

Operation.

Pepper 6 = 13 Ginger,
Ginger 19 = 4 25 Cloves.
Cloves 10 = 63 Sugar.
Sugar 1 = 5 Pence.
What Pence = 112 Pepper.
Anſwer 7 *l.* 2 *s.* 5 *d.* 3 $\frac{420}{1140}$ *q.*

CHAP. V.

The Single RULE *of* THREE *Indirect, or the backward* RULE, *commonly called Inverſe.*

DIFINITION.

IN this Rule there are always three Things given to find a fourth, that bears ſuch proportion to the firſt, that the Second doth to the Third.

How

How to State the Queſtion.

Here, the Demand muſt be in the third Place, the firſt and third of one Name, every Term brought into it's loweſt Name, then the Product of the Firſt and Second divided by the Third, gives the Anſwer, which is always of the ſame Name with the middle Term. And the Queſtions that belong to this Rule are always known thus; if leſs require more, or more require leſs, the Queſtion is indirect.

EXAMPLE.

Queſt. 1. If for 20*s.* I have 7 *C.* 3 *Q.* 4 *lib.* carried 40 Miles, how much Weight may I have carried 30 Miles for the ſame Money?

Here leſs requires more, that is, leſs Miles, requires more Weight.

Operation.

```
     M.    C.  Q.  lib.    M.
If   40 :  7   3   4   ::  30
           4
         ----
          31
          28
         ----
         252
          62
         ----
         872
          40     lib.     C.  Q.  lib.
        ------
  30) 34880 (1162 20/30 = 10  1   14 2/3.
```

Queſt. 2. How many Yards of half Yard-wide will line a Cloak, which hath in it 5 Yards of 7 Quarters wide? Anſwer 17 *y.* 2 *q.*

Queſt. 3. A Meſſenger makes a Journey in 8 Days when the Day is 10 Hours long, I would know in how many Days he will go the ſame when the Day is 15 Hours long? More requires Leſs.

Anſwer 5 *D.* 8 *H.*

CHAP. VI.

Of PLURAL PROPORTION, Or the Double Rule of Three Direct.

DEFINITION.

HERE are always five Numbers given to find a Sixth in proportion; in three of which there always lyeth a Suppoſition, and a Demand in the other two, and this may be done either by two ſingle Rules of three, or elſe at one Operation by ſtating the whole five Numbers, as I ſhall ſhew in an Example.

Queſt. 1. A Carrier receives 14 *s.* for the Carriage of one Hundred Weight 50 Miles, I demand what he ought to receive for the Carriage of 3 *C.* 1 *Q.* 18 *lib.* 20 Miles.

Firſt, I ſhall work it at two Operations by the Single Rule of Three Direct: And here Note, it matters not whether I take the Weight or the Miles in the firſt Operation, for the Anſwer will be the ſame which ever be taken. Firſt I ſhall take the Weight and work it thus,

```
      lib.   s.    C.   Q.   lib.
If    112 : 14 ::  3    1    18
                   4
                 ----
                  13
                 ----
                  28
                 ----
                 112
                  27
                 ----
                 382
                  14
                 ----
                1528
                382
               -----
    11:)       5348 (47 s. 9 d.
```

Secondly,

2dly, If 50 *M.* : 47 *s.* 9 *d.* : : 20 *M.*

$$\begin{array}{r} 12 \\ \hline 573 \\ 20 \\ \hline 50)\overline{11460}(229\,d. = 19\,s.\ 1\,d. \end{array}$$

Anſwer, 19 *s.* 1 *d.* $0\frac{4}{5}$ *q.*

Secondly, If we take the Miles firſt, then the fourth proportional Term will be $5\frac{3}{5}$ *s.* which muſt be the ſecond Term in the ſecond Work. Thus

	lib.	*s.*	*lib.*	*s.*	*d.*	*q.*
If	112 :	$5\frac{3}{5}$: :	382 :	19	1	0

Anſwer 19 *s.* 1 *d.* $0\frac{448}{560}$ *q.*

The laſt Queſtion varied, which is a Proof of the Rule.

Every Queſtion in this Rule may be varied, that is, it may be ſtated by putting what was before given, now in the Demand's Place. Thus,

If a Carrier receive 19 *s.* 1 *d.* $0\frac{4}{5}$ *q.* for the Carriage of 3 *C.* 1 *Q.* 18 *lib.* 20 Miles, what muſt he receive for the Carriage of one Hundred Weight 50 Miles?

Anſwer 14 *s.*

Both Queſtions are anſwered at one Operation as you ſee below.

	lib.	*M.*	*s.*	*C.*	*Q.*	*lib.*	*M.*
Firſt, If	112 :	50 :	14 : :	3	1 :	18 :	20

Anſwer 19 *s.* 1 *d.* $0\frac{4}{5}$ *q.*

	C.	*Q.*	*lib.*	*M.*	*s.*	*d.*	*q.*	*lib.*	*M.*
Secondly, If	3	1	18 :	20 :	19	1	$0\frac{4}{5}$:	112 :	50

Anſwer 14 *s.*

Queſt. 3. What's the Simple Intereſt of 2 Guineas for 76 Days, at 5 *per Cent. per Annum?*

Anſwer 5 *d.* 0 $\frac{7216|00}{7300|00}$ *q.*

CHAP. VII.

Of PLURAL PROPORTION, or the Double RULE of THREE Idnireƈt.

DEFINITION.

HERE are always five Numbers given to find a Sixth in an inverted Proportion. And as in all the foregoing Rules, ſo here, all the Difficulty lyeth in ſtating the Queſtion, and to know truly when a Queſtion falls under this Rule, that they are ſuch as require to find Men, Principal, Time, Weight, *&c.*

To ſtate the Queſtion.

This being known, you muſt obſerve that the three firſt Terms muſt be the Suppoſition, and the Demand in the two laſt Places, and that you put in the firſt Place the Work, the Price, the Intereſt, the Produce, *&c.* and the ſecond Term muſt be Time, Principal, Weight, Miles, Men; the third Term muſt be of that Name you are ſeeking; the ſecond and fourth of one, and the ſecond and fifth of the ſame Name. Theſe things being known, the Produƈt of the ſecond, third and fourth Terms, divided by the Produƈt of the firſt and fifth Terms gives the Anſwer to the Queſtion.

EXAMPLE,

Queſt. 1. A Carrier receives 19 *s.* 1 *d.* $0\frac{4}{5}$ *q.* for the Carriage of 3 *C.* 1 *Q.* 18 *lib.* 20 Miles. I demand how much he muſt carry 50 Miles for 14 *s.*

State it thus,

	s.	*d.*	*q.*		*M.*		*C.*	*Q.*	*lib.*		*s.*		*M.*
If	19	1	$0\frac{4}{5}$	:	20	:	3	1	18	:	14	::	50

Work according to the Directions, and the Anſwer will be 112 *lib.* = one Hundred Weight.

Note, Every Queſtion in this Rule may be varied four ſeveral Ways, as you may perceive by what follows.

Queſt. 2. A Carrier receives 14 *s.* for the Carriage of 112 *lib.* 50 Miles, I demand how much he muſt carry 20 Miles for 19 *s.* 1 *d.* $0\frac{4}{5}$ *q.*

State it thus,

s. *M.* *lib.* *s.* *d.* *q.* *M.*
If 14 : 50 : 112 : : 19 1 $0\frac{4}{5}$: 20
Anſwer 3 *C.* 1 *Q.* 18 *lib.*

Queſt. 3. A Carrier receives 19 *s.* 1 *d.* $0\frac{4}{5}$*q.* for the Carriage of 3 *C.* 1 *Q.* 18 *lib.* 20 Miles, I demand how many Miles he may carry one Hundred Weight for 14 *s.*

State it thus,

s. *d.* *q.* *C.* *Q.* *lib.* *M.* *s.* *lib.*
If 19 1 $0\frac{4}{5}$: 3 1 13 : 20 : : 14 : 112.
Anſwer 50 Miles.

Queſt. 4. A Carrier receives 14 *s.* for the Carriage of one Hundred Weight 50 Miles, how many Miles muſt he carry 3 *C.* 1 *Q.* 18 *lib.* 20 Miles for 19 *s.* 1 *d.* $0\frac{4}{5}$ *q.*

State it thus,

s. *lib.* *M.* *s.* *d.* *q.* *C.* *Q.* *lib.*
If 14 : 112 : 50 : : 19 1 $0\frac{4}{5}$: 3 1 18
Anſwer 20 Miles.

Queſt. 5. Put out for 7 Months a Sum of Money, at 5 *per Cent per Annum*, and received for Intereſt 2 *l.* 17 *s.* $4\frac{1}{4}$ *d.* I demand what was the Sum lent?

Anſwer 98 *l.* 6 *s.* 5 *d.* $0\frac{19200}{33600}$ *q.*

Queſt. 6. An Uſurer receives 2 *l.* 17 *s.* $4\frac{1}{4}$ *d.* for the Intereſt of 98 *l.* 6 *s.* 5 *d.* $0\frac{19200}{33600}$*q.* for 7 Months, I demand in what time 5*l.* will gain 1 C. Principal ?

Anſwer 12 Months.

I have given now the Ground and Foundation of Arithmetic through all Parts of the Golden Rule. I ſhall only touch upon the reſt of the Rules in a few Queſtions, and be as brief as poſſible.

And firſt of Single Fellowſhip, or Fellowſhip without Time.

I. *Single Fellowſhip.*

THIS is a Rule amongſt Merchants for Balancing Accompts.

The RULE,

As the whole Stock of all the Partners, is in Proportion to the whole Gain or Loſs; ſo is each Man's particular Share in the Stock, to each Man's particular Share in the Gain or Loſs.

Queſt. 1. *A*, *B* and *C* bought a Houſe, for 329 *l.* 15 *s.* 7 *d.* *A* paid one half ſo much as *B*, *B* paid one half ſo much as *C*, what did each Man pay ?

Note, You may ſuppoſe paid any Sum at Pleaſure. As ſuppoſe 12*l.*

	l.	*s.*	*d.*	*q.*
Anſwer	188	8	10	$3\frac{9}{21}$
	94	4	5	$1\frac{15}{21}$
	47	2	2	$2\frac{18}{21}$
Proof	329	15	7	0

Queſt. 2. Suppoſe the Grandfather *A*, the Father *B*, the Son *C*, met together and ſpent 20 *s.* when the Reckoning came to be paid, the Grandfather would pay one Half, the Father one Third, the Son one Fourth, now I would know how much each muſt pay of the Reckoning ?

		s.	*d.*	*q.*
Anſwer	*A*	9	2	$3\frac{2}{26}$
	B	6	1	$3\frac{10}{26}$
	C	4	7	$1\frac{14}{26}$
Proof		20	0	0

Queſt. 3. Four Gentlemen at the Tavern drinking till the Reckoning came to 20 *s.* the Duke would pay one Third, the Earl one Fourth, the Knight one Fifth, and the 'Squire one Sixth Part, which Parts all together makes only 19 *s.* now I would know what each muſt pay?

		s.	*d.*	*q.*
Anſwer	Duke	7	0	$0\frac{192}{228}$
	Earl	5	3	$0\frac{12}{19}$
	Knight	4	2	$2\frac{2}{19}$
	'Squire	3	6	$0\frac{96}{228}$
Proof		20	0	0

II. *Fellowſhip with Time, or the Double Rule.*

RULE.

MULTIPLY the particular Stocks of each Perſon by the Time of Continuance, and the Sum of the ſeveral Products, make the firſt Term in the Single Rule of Three Direct, the whole Gain or Loſs the Second, and every Man's particular Stock, Multiplied by it's Time, the Third.

Queſt. 1. Three Merchants, *A*, *B* and *C*, make a Stock of 2840 *l.* of which *A* lays in 899 *l.* for 4 Months, *B* 1681 *l.* for 7 Months, and *C* 260 for 12 Months, with this they gain 1420 *l.* what is each Man's Share?

		l.	*s.*	*d.*	*q.*
Anſwer	*A*	276	5	5	$0\frac{5940}{}$
	B	904	0	6	$2\frac{7122}{}$
	C	239	14	0	$1\frac{5421}{18483}$

Queſt. 2. A Ship's Company take a Prize of 718 *l.* 15 *s.* which is to be divided amongſt them according to their Pay and Time they have been on Board; the Officers and Midſhipmen 6 Months, and the Sailors 4 Months, the Officers one with another 40 *s.* *per* Month, the Midſhipmen

Men 30 Shillings per Month, and the Sailors 24 Shillings per Month, there were 6 Officers, 13 Midſhipmen, and 93 Sailors, what muſt each have to his Share?

		l.	*s.*	*d.*	*q.*
Anſwer	Officers	104	12	9	$2\frac{12648}{}$
	Midſhipm.	127	10	7	$1\frac{6348}{}$
	Sailors	486	11	6	$3\frac{7380}{13188}$
	Proof =	718	15	0	0 (2

III. *Of Alligation Medial.*

DEFINITION.

TEACHES to find a mean Price of ſeveral Simples propounded.

Queſt. 1. Admit there were melted and mixed together two kinds of Silver, one worth 5 *s.* 3 *d.* and the other 4 *s.* 9 *d.* *per* Ounce, and there were 8 Ounces of the Former and 17 of the Latter, what's the Value of one Ounce of that Mixture?

Anſwer 4 *s.* 10 *d.* $3\frac{17}{25}$ *q.*

Queſt. 2. An Hoſtler mix'd Provender for his Horſes, *viz.* 18 Buſhels of Oates, at 2 *s.* 1 *d.* *per* Buſhel, with 16 Buſhels of Beans, at 4 *s.* 9 *d.* *per* Buſhel, and 13 Buſhels of Malt, at 3 *s.* 10 *d.* *per* Buſhel, now do I demand what a Peck of this Mixture is worth?

Anſwer 10 *d.*

IV. *Of Alligation Alternate.*

DIFINITION.

THIS Rule hath it's Name from binding, tying, or uniting many Particulars into one Maſs or Sum; for here are always given the particular Prices of ſeveral

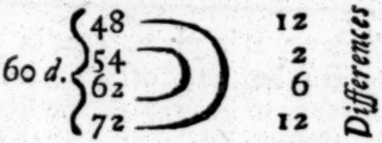
Simples, and thereby we diſcover ſuch Quantities of thoſe Simples, as being mingled together, ſhall bear a certain Rate propounded.

Queſt. A Vintner hath four Sorts of Wines of ſeveral Prices, *viz.* one of 4 *s.* *per* Gallon, another of 4 *s.* 6 *d.* another at 5 *s.* 2 *d.* and another of 6 *s.* *per* Gallon, of which Wines he would have a Mixture made of 60 Gallons, and to bear a Price of 5 *s.* *per* Gallon, I would know how much muſt be taken of each Sort?

Note, You muſt always be careful to link two together, whereof one is greater, and the other leſſer than the common Price.

RULE:

Then as the Sum of the Differences, is to the Sum propoſed, ſo is each particular Difference, to each particular Quantity. And every Queſtion may be linked ſeveral Ways as appears by theſe Examples.

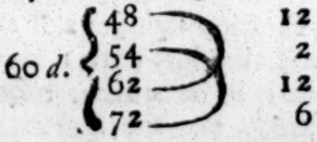


Sum 32

As 32 : 60 :: 12 : $22\frac{16}{32}$
2 : $3\frac{24}{32}$
6 : $11\frac{8}{32}$
12 : $22\frac{16}{32}$

Proof = 60

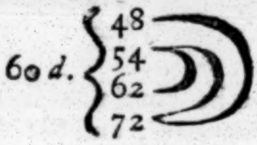
A Second Way of Linking.



Sum 32

$$\text{As } 32 \; : \; 60 :: \begin{cases} 12 : 22\frac{16}{32} \\ 2 : 3\frac{24}{32} \\ 12 : 22\frac{16}{32} \\ 6 : 11\frac{8}{32} \end{cases}$$

A Third Way of Linking.



12	12
2, 12	14
6	6
6, 12	18
Sum	50

$$\text{As } 50 \; : \; 60 :: \begin{cases} 12 : 14\frac{20}{50} \\ 14 : 16\frac{40}{50} \\ 6 : 7\frac{10}{50} \\ 18 : 21\frac{30}{50} \end{cases}$$

Sum 60 Proof.

By the Work above it is plain, that in the firſt Way of Linking, there muſt be $22\frac{1}{2}$ of 4*s.* *per* Gallon, in the Second Way there muſt be $22\frac{1}{2}$ of the ſame Sort; but in the Third Way, there muſt be of that Sort $14\frac{2}{5}$ Gallons, and ſo of the reſt.

V. *Of Vulgar* FRACTIONS.

DEFINITION.

THE Word Fraction ſignifies a Breaking, or Breach of any entire Thing into Parts; now the Unite, or entire Number which is to be broken, may be any thing, as one Pound, one Yard, one Year, *&c.*

This Table will explain the ſeveral Names of Fractions, better than many Words.

Numerator

Numerator	1	and	3
Denominator	4		5

Proper Fractions are, $\frac{2}{3}$ or $\frac{5}{6}$ or $\frac{1}{2}$
Improper Fractions are, $\frac{3}{3}$ or $\frac{7}{5}$ or $\frac{14}{11}$
Single Fractions are $\frac{4}{5}$ or $\frac{11}{13}$ or $\frac{6}{11}$ or $\frac{3}{7}$
Compound Fractions are, $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{5}{8}$ of $\frac{11}{13}$
Mixt Numbers are, $8\frac{2}{3}$ or $12\frac{1}{5}$ or $1\frac{1}{3}$ or $11\frac{1}{6}$

Now before we can proceed in the known Rules, we muſt firſt know how to reduce a Fraction into it's known Quantity, according to the Nature of the Queſtion.

1. And if the Fraction is mixt, multiply the whole Number by the Denominator, and to the Product add the Numerator, that Product is a new Numerator, which place over the Denominator and 'tis done.

2. If you would reduce a whole Number into an Improper Fraction, multiply the given Number by the intended Denominator, and place the Product for a Numerator over the intended Denominator.

3. When you have an Improper Fraction to reduce into a Whole or mixt Number, divide the Numerator by the Denominator, the Quotient is the whole Number, and if any thing remain, place it for a Numerator over the Diviſor, or Denominator.

4. If you would reduce Fractions into their loweſt Term.

Firſt, If they end in Cyphers cut them off, and take the half of the ſignificant Figures as long as you can.

Secondly, If they both end in 5, or one 5 and the other in 0, then divide them ſeverally by 5, and the two Quotients ſhall be equal to the Fraction given.

Thirdly, But the beſt Way is to find a Common Meaſure, by dividing the Denominator by the Numerator, continually till nothing remain, and this laſt Quotient will be a Common Meaſure that will divide both Numerator and Denominator without any Remainder, and reduce the Fraction in it's loweſt Term.

So if $\frac{246}{864}$ were the Fraction given to be brought into it's loweſt Term, the Work will ſtand thus,

```
246) 864 (3
     738
     ---
     126) 246 (1
          126
          ---
          120) 126 (1
               120
               ---
                 6) 120 (20
                     ..
                    ---
                      0
```

6 is the common Meaſure.

```
6) 246 (41        6) 864 (146
    ..                ..
```

Anſwer $\frac{41}{144}$ in it's loweſt Term.

5. When you have a compound Fraction to bring into a ſingle Fraction, multiply all the Numerators together for a new Numerator, and all the Denominators together for a new Denominator. So $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{5}{6}$ will be $\frac{10}{144}$ ſingle.

Queſt. 6. When you would find the Value of a Fraction in any known Meaſure, *&c.* multiply the Numerator of the given Fraction, by the Parts of the next inferiour Denomination, and divide the Product by the Denominator, the Quotient is it's Value in the ſame Parts you multiplied by. So if $\frac{5}{7}$ of a Pound Sterling were given to know it's Value, you will find it to be 14 *s.* 3 *d.* 1$\frac{5}{7}$ *q.*

Alſo $\frac{2}{3}$ of $\frac{1}{2}$ of $\frac{3}{4}$ of 4 Shillings is 1 *s.*

And $\frac{3}{4}$ of $\frac{5}{8}$ of a Mile, you will find to be 3 Furlongs, 30 Poles.

And $\frac{2}{3}$ of $\frac{4}{5}$ of a Yard, is 2 *q.* 0$\frac{8}{15}$ *n.*

And $\frac{13}{14}$ of an Ell *Engliſh*, is 4 *q.* 2$\frac{4}{7}$ *n.*

And $\frac{11}{15}$ of a Week, is 5 *d.* 3 *h.* 12′.

7. When you have Fractions of unequal Denominations, firſt bring them into their loweſt Terms, then multiply all the Denominators together, for a common Denominator, then multiply continually the Numerator of each Fraction by all the Denominators except it's own;

and that will give new Numerators to be ſet over the common Denominator.

Reduce $\frac{2}{3}$ and $\frac{4}{7}$ to a common Denominator.

Anſwer $\frac{14}{21}$ and $\frac{12}{21}$.

8. When you have Fractions to be reduced from one Denomination to another, either aſcending, or deſcending.

1. When a Fraction is brought from a leſſer to a greater Denomination, make of it a Compound Fraction, by comparing it with the intermediate Denominations between it, and that you would have it reduced to, then by the 5 hereof, bring it into a ſingle Fraction, and 'tis done.

EXAMPLE.

What Part of a Pound Sterling is $\frac{1}{4}$ of a Penny ?

Firſt make of it a Compound Fraction, thus $\frac{1}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$, and by the 5 hereof it is, $\frac{1}{960}$ of a Pound Sterling.

So $\frac{1}{2}$ of a Penny is $\frac{1}{24}$ of a Shilling.

What part of a Pound is 8 *d* ?

Anſwer, $\frac{1}{30}$.

What part of a Crown is $\frac{3}{4}$ of a Penny ?

Anſwer, $\frac{3}{240}$.

2. But if you would reduce a Fraction of a greater, to a Fraction of a leſſer Name, multiply the Numerator by the Parts contained in the ſeveral Denominations betwixt it; and that you would reduce it to, and place the laſt Product over the Denominator of the given Fraction.

EXAMPLE.

Reduce the $\frac{1}{2}$ of a Pound to the Fraction of a Penny ?

Anſwer, $\frac{240}{2}$.

VI. *Of ADDITION.*

BEFORE Fractions can be added they muſt be reduced to a common Denominator, if they have not ſuch.

Queſt. 1. What's the Sum of $\frac{1}{2}$ and $\frac{2}{3}$ and $\frac{3}{4}$ and $\frac{1}{4}$ of a Shilling?

Anſwer, 2 *s.* 2 *d.*

Queſt. 2. What's the Sum of $\frac{1}{4}$ of a Farthing, $\frac{3}{4}$ of a Penny, $\frac{1}{4}$ of a Shilling, and $\frac{3}{4}$ of a Pound?

Firſt by reducing them to the Fraction of one Farthing. $\frac{1}{4}$*q.* $\frac{3}{4}$*d.* $\frac{1}{4}$*s.* $\frac{3}{4}$*l.*

Firſt, The Numerator 3 of the Pound is multiplied by the Farthings in a Pound, 960 = $\frac{2880}{4}$.

Secondly, The Numerator of the Shillings is multiplied by the Farthings in a Shilling 48 = $\frac{48}{4}$ *qrs.*

Laſtly, The Numerator 3 of the Farthings, is mutiplied by 4, the Farthings in a Penny, and it is $\frac{12}{4}$ *qrs.* $\frac{1}{4}$; and now becauſe they have all one Denominator, the Work ſtands thus.

$$\text{Numerator.}\ \left\{\begin{array}{r}2880\\48\\12\\1\end{array}\right.$$

$$\begin{array}{rrrr}
\underline{} & 4 & 12 & s.\\
4)\ \underset{\cdots}{2941}\ (& \underset{\cdots}{735}\ (& \underset{\cdot\cdot}{183}\ (& 15\\
\overline{1} & \overline{3} & \overline{3} &
\end{array}$$

Anſwer 15 *s.* 3 *d.* 3$\frac{1}{4}$ *q.*

Secondly, I ſhall give the Solution by reducing them to the Fraction of a Pound, by making of them compound Fractions; thus,

$$\begin{array}{ll}
\frac{3}{4}\text{ of }\frac{1}{12}\text{ of }\frac{1}{20}\text{ Single} & \frac{3}{960}\\
\frac{1}{4}\text{ of }\frac{1}{20}\text{ Single} & \frac{1}{80}\\
\frac{3}{4}\text{ is } = \text{ - - -} & \frac{3}{4}
\end{array}\left.\vphantom{\begin{array}{l}\frac{3}{960}\\ \frac{1}{80}\\ \frac{3}{4}\end{array}}\right\}\text{ Pound.}$$

Being reduced to a Common Denominator will be $\frac{960}{307200}$ $\frac{3840}{307200}$ $\frac{230400}{307200}$.

Anſwer $\frac{235200}{307200}$ = 15 *s.* 2 *d.* 3$\frac{1}{4}$ *q.*

Queſt. 3. What is the Sum of $\frac{2}{3}$ of a Minute, $\frac{4}{5}$ of an Hour, $\frac{5}{6}$ of a Day, $\frac{7}{8}$ of a Week, $\frac{3}{7}$ of a Month, and $\frac{8}{9}$ of a Year; ſuppoſing the Year to contain only 13 Months?

Anſwer 12*m.* 0*w.* 6*d.* 13*h.* 8′ 40″.

VII. *Of SUBTRACTION.*

IF the Fractions have unequal Denominators, they muſt be reduced to equal Denominators as has been taught above.

What's the Difference between $\frac{9}{16}$ and $\frac{5}{16}$?

Anſwer $\frac{4}{16} = \frac{1}{4}$.

What's the Difference between $\frac{3}{4}$ and $\frac{5}{7}$ of any thing?

Anſwer $\frac{1}{28}$.

What's the Difference between $\frac{2}{3}$ of $\frac{3}{4}$, and $\frac{1}{2}$ of $\frac{5}{8}$ of a Pound *Sterling*?

Anſwer $\frac{3}{16}$ = 3 *s.* 9 *d.*

What's the Difference betwen 3 and 5 $\frac{1}{2}$?

Anſwer 2 $\frac{1}{2}$

What's the Difference between 1 and $\frac{3}{4}$?

Anſwer $\frac{1}{4}$.

What's the Difference between 14 $\frac{3}{4}$ and 29 $\frac{4}{7}$?

Anſwer 14 $\frac{3}{4}$.

VIII. *Of MULTIPLICATION.*

1 IF the Fractions be ſingle, multiply the Numerators together for a new Numerator, and the Denominators together for a new Denominator.

2 If the Fractions are mixt Numbers, reduce them to Improper Fractions.

3 If they are Compound, reduce 'em into ſingle Fractions.

Queſt. 1. What's the Product of $\frac{5}{7}$ by $\frac{9}{11}$?

Anſwer 1 $\frac{1}{4}$.

Queſt 2. What's the Product of $\frac{1}{2}$ by 2 $\frac{1}{2}$?

Anſwer $\frac{45}{77}$.

Queſt. 3. What's the Product of $\frac{2}{7}$ by 12?

Anſwer 3 $\frac{3}{7}$.

Queſt. 4. What's the Product of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ by $\frac{2}{3}$?

Anſwer $\frac{1}{16}$.

Queſt. 5. What's the Product of $\frac{1}{4}$ of $\frac{2}{3}$ of $\frac{1}{2}$ by $7\frac{3}{4}$?
Anſwer $\frac{31}{48}$.

Queſt. 6. What's the Product of $5\frac{1}{2}$ by 3?
Anſwer $16\frac{1}{2}$.

Queſt. 7. What's the Product of $\frac{2}{3}$ of a Pound multiplied by $\frac{3}{4}$ of a Pound?
Anſwer $\frac{1}{2} = 10$ *s*.

Queſt. 8. What's the Square of 3*s*. 11*d*. a Pound the Integer?
Anſwer 9*d*. $0\frac{18816000}{230400000}$ *q*.

But if a Shilling be put for the Integer, the Anſwer will be 15*s*. 4*d*. $0\frac{48}{144}$ *q*.

IX. *Of Diviſion of Vulgar Fractions.*

WHAT has been ſaid in the laſt Article of reducing of Fractions, muſt be obſerved here.

Queſt. 1. What's the Quotient of $\frac{4}{5}$ divided by $\frac{3}{4}$?

RULE.

Multiply the Numerator of the Diviſor by the Denominator of the Dividend, and the Product is a new Denominator; multiply the Denominator of the Diviſor by the Numerator of the Dividend, the Product is a new Numerator. See the Work.

$$\tfrac{3}{4})\tfrac{4}{5}(\tfrac{16}{15} = 1\tfrac{1}{15}$$

Queſt. 2. What's the Quotient of $\frac{5}{8}$ divided by $2\frac{2}{5}$?

$$\tfrac{12}{5})\tfrac{5}{8}(\tfrac{25}{96}$$

Queſt. 3. What's the Quotient of $\frac{3}{4}$ divided by 1?

$$\tfrac{1}{1})\tfrac{3}{4}(\tfrac{3}{4}$$

Note, As in whole Numbers, ſo here you ſee Unity neither multiplies nor divides.

Queſt. 4. What's the Quotient of $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ divided by $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$?

$$\tfrac{30}{120})\tfrac{3}{32}(\tfrac{360}{960} = \tfrac{3}{8}$$

Queſt. 5. What's the Quotient of $\frac{4}{5}$ of $\frac{5}{7}$ divided by 7 ?

$$\tfrac{7}{1})\tfrac{20}{35}(\tfrac{20}{245}$$

Queſt. 6. What's the Quotient of $\frac{3}{5}$ of $\frac{3}{7}$ of $\frac{5}{6}$ divided by $\frac{5}{9}$?

$$\tfrac{5}{9})\tfrac{45}{210}(\tfrac{405}{1050}=\tfrac{27}{70}$$

Queſt. 7. What's the Quotient of $4\frac{3}{4}$ divided by $3\frac{2}{3}$?

$$\tfrac{11}{3})\tfrac{19}{4}(\tfrac{57}{44}=1\tfrac{13}{44}$$

Queſt. 8. What's the Quotient of $8\frac{1}{2}$ divided by $\frac{2}{3}$ of $\frac{3}{5}$?

$$\tfrac{6}{35})\tfrac{17}{2}(\tfrac{595}{12}=49\tfrac{7}{12}$$

Queſt. 9. What's the Quotient of $9\frac{5}{11}$ divided by 15 ?

$$\tfrac{15}{1})\tfrac{104}{11}(\tfrac{104}{165}$$

Queſt. 10. What's the Quotient of 16 divided by $\frac{3}{4}$?

$$\tfrac{3}{4})\tfrac{16}{1}(\tfrac{64}{3}=21\tfrac{1}{3}$$

Queſt. 11. What's the Quotient of 20 divided by $\frac{2}{3}$ of $\frac{1}{6}$?

$$\tfrac{2}{18})\tfrac{20}{1}(\tfrac{360}{2}=180$$

Queſt. 12. What's the Quotient of 30 divided by $5\frac{1}{2}$?

$$\tfrac{11}{2})\tfrac{30}{1}(\tfrac{60}{11}=5\tfrac{5}{11}.$$

X. *The Single Rule of Three Direct in Vulgar Fractions.*

QUESTION I.

IF $12\frac{1}{2}$ Yards of Cloth coſt 15 *s.* 9 *d.* what will 48^{1} Yards coſt at that Rate ? A Shilling the Integer.

State it thus,

$$\begin{array}{lcccccccc} & y. & & s. & & y. & & s. \\ \text{If} & \frac{25}{2} & : & \frac{63}{4} & :: & \frac{193}{4} & : & \frac{24318}{400} \end{array}$$

Anſwer 3 *l.* 0 *s.* 9 *d.* $2\frac{64}{400}$ *q.*

The ſame Queſtion anſwered by putting a Pound for the Integer.

State it thus,

$$\begin{array}{lcccc} & y. & l. & y. & l. \\ \text{If} & \frac{25}{2} : & \frac{252}{320} :: & \frac{193}{4} : & \frac{97272}{32000} \end{array}$$

Anſwer 3 *l.* 0 *s.* 9 *d.* $2\frac{5120}{32000}$ *q:*

Queſt. 2. If $\frac{1}{4}$ of a Yard of Cloth coſt $\frac{2}{3}$ of a Pound, what will $\frac{3}{5}$ of an *Engliſh* Ell coſt?

Note, The Yard muſt be reduced to the Fraction of an Ell, or elſe the Ell to the Fraction of a Yard; by the firſt it will ſtand thus,

$\frac{1}{4}$ *y.* is $\frac{1}{4}$ of $\frac{4}{5}$ of an Ell. Single $\frac{4}{20}$ Ell.

$$\begin{array}{lccccc} & ei & l. & e. & l. & l. \\ \text{If} & \frac{4}{20} : & \frac{2}{3} : : & \frac{3}{5} : & \frac{120}{60} & = 2 \end{array}$$

Anſwer 2 *l.*

Queſt. 3. If an Ounce of Gold be worth 3 *l.* 18 *s.* 1 *d.* how much may I buy for $3\frac{12}{20}$ *l.*

Anſwer 18 *pw.* $10\frac{2264}{2748}$ *gr.*

Queſt. 4. If $1\frac{1}{2}$ Herring coſt $1\frac{1}{2}$ *d,* how many may I buy for 11 *d.*

Anſwer 11.

For as the ſecond Term is equal to the firſt, ſo will the fourth be to the third.

XI. *Rules of* PRACTICE.

DEFINITION.

PRACTICE is a Rule, which expeditiouſly anſwers Queſtions in the Rule of Three, when the firſt Term is Unity or 1: And for the Reader's better Improvement, he muſt be ſure to get the following Table by Heart.

A Table of Aliquot Parts:

Even Parts of a Pound.		Fractional Parts.	Uneven Parts of a Pound.		Even Parts.			Fractional Parts.		
s.	d.									
10	0	$\frac{1}{2}$	19	is	10	5	4	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{5}$
6	8	$\frac{1}{3}$	18		10	4	4	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{5}$
5	0	$\frac{1}{4}$	17		10	5	2	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{10}$
4	0	$\frac{1}{5}$	16		10	4	2	$\frac{1}{2}$	$\frac{1}{5}$	$\frac{1}{10}$
3	4	$\frac{1}{6}$	15		10	5	0	$\frac{1}{2}$	$\frac{1}{4}$	
2	6	$\frac{1}{8}$	14		10	4	0	$\frac{1}{2}$	$\frac{1}{5}$	
2	0	$\frac{1}{10}$	13		4	4	5	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{4}$
1	8	$\frac{1}{12}$	12		10	2	0	$\frac{1}{2}$	$\frac{1}{10}$	
1	3	$\frac{1}{16}$	11		5	4	2	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{10}$
1	0	$\frac{1}{20}$	9		5	4	0	$\frac{1}{4}$	$\frac{1}{5}$	
0	10	$\frac{1}{24}$	8		4	4	0	$\frac{1}{5}$	$\frac{1}{5}$	
0	8	$\frac{1}{30}$	7		5	2	0	$\frac{1}{4}$	$\frac{1}{10}$	
0	$7\frac{1}{2}$	$\frac{1}{32}$	6		4	2	0	$\frac{1}{5}$	$\frac{1}{10}$	
0	6	$\frac{1}{40}$	3		2	1	0	$\frac{1}{10}$	$\frac{1}{20}$	

Even Parts of a Shilling.		Even Parts of a Pound.		Even Parts of a Pound.		Even Parts of a Shilling.		Even Parts of a Penny	
d.				q.		q.		q.	
6 is	$\frac{1}{2}$	6	$\frac{1}{40}$	1	$\frac{1}{960}$	1	$\frac{1}{48}$	1	$\frac{1}{4}$
4	$\frac{1}{3}$	4	$\frac{1}{60}$	2	$\frac{1}{480}$	2	$\frac{1}{24}$	2	$\frac{1}{2}$
3	$\frac{1}{4}$	3	$\frac{1}{80}$	3	$\frac{1}{320}$	3	$\frac{1}{16}$	3	$\frac{3}{4}$
2	$\frac{1}{6}$	2	$\frac{1}{120}$						
$1\frac{1}{2}$	$\frac{1}{8}$	$1\frac{1}{2}$	$\frac{1}{160}$						
1	$\frac{1}{12}$	1	$\frac{1}{240}$						

Note, After Diviſion, if any thing remain it is of the ſame Name with your Diviſor, as in the firſt Example following, at 3 *qrs.* per Pound, we divide by 16, the Quotient is 53 Shillings, and there remains 6, that is 6*d.* 3 Farthings, or 4½*d.* the like is to be obſerved of any other.

$\frac{1}{16}$	854 at 3*q.* *per lib.*		
$\frac{1}{20}$	53 *s.*		
	l.	*s*	*d.*
Anſwer	2	13	4½

$\frac{1}{4}$	90½ at 5¼*d.* *per oz.*			
$\frac{1}{6}$	1	2	7	2
$\frac{1}{48}$		15	1	0
		1	10	2½
Anſwer	1	19	7	0½

---

			d:
$\frac{1}{4}$ *and* $\frac{1}{24}$	750 at 3½ *per C.*		
$\frac{1}{14}$	187		6
$\frac{1}{24}$	31		3
$\frac{1}{20}$	218		9
	l.	*s.*	*d.*
Anſwer	10	18	9

$\frac{1}{4}$	414½ at 9½*d.* *per yd.*		
$\frac{1}{2}$	5	3	7½
$\frac{1}{24}$	10	7	3
		17	3¼
Anſwer	16	8	1¾

When the given Price is 2 *s.* double the Figure in the Unites Place for Shillings, and the reſt are Pounds.

Queſt. 5. 680 at 11¼*d.* *per* Pound ?
Anſwer 31*l.* 17*s.* 6*d.*
Queſt. 6. 576½ at 13¼*d.* *per* Yard ?
Anſwer 31*l.* 16*s.* 6*d.* 2½*q.*
Queſt. 7. 207¼ at 19*d.* *per* Day ?
Anſwer 16*l.* 8*s.* 1¾*d.*
Queſt. 8. 863 at 22½*d.* *per* Gallon ?
Anſwer 80*l.* 18*s.* 1½*d.*
Queſt. 9. 845½ at 2*s.* 3¼*d.* *per* Ell ?
Anſwer 95*l.* 19*s.* 11*d.* 3½*q.*
Queſt. 10. 760¾ at 2*s.* 5½*d.* *per* Buſhel ?
Anſwer 93*l.* 10*s.* 2*d.* 0½*q.*
Queſt. 11. 639 at 2*s.* 10¾*d.* *per* Quart ?
Anſwer 92*l.* 10*s.* 5½*d.*
Queſt. 12. 707½ at 3*s.* 2¼*d.* *per* Foot ?
Anſwer 112*l.* 15*s.* 1*d.* 3½*q.*

How

How to caſt up Expences, the Price of Goods, &c. by divers eaſy Rules.

1. By knowing your weekly Expence, to know what it comes to in the Year.

RULE.

The Farthings in each Week doth make appear,
The Shillings and the Pence ſpent in the Year.

Queſt. 1. If you pay 12*qrs.* or 3*d.* to the Poor, *per* Week, what doth that come to in the Year ?

Anſwer 12*s.* and 12*d.* = to 13*s.*

By knowing your daily Expence, to know what it comes to in a Year.

RULE.

Number the Pennies of each Day's Expence,
So many Angels, Groats and Pence
You ſpend within the Year's Circumference.

Queſt. 2. If a Labouring Man Earn 9*d.* *per* Day, what is that *per Annum ?*

Solution.

			l.	*s.*	*d.*
9	Pounds	is	9	0	0
	Angels		4	10	0
	Groats		0	3	0
	Pence		0	0	9
	Anſwer		13	13	9

Noie, If the Daily Expence be Shillings and Pence, reduce all into Pence, then Work as above.

3. By knowing your yearly Expence, to know wh it coſts per Day.

RULE:

The third Part of the Pounds you yearly have,
Yields Two Pence every Day to ſpend or ſave,

Note, For every one that remains add Three Farthings but this Rule will give a little too much in the Anſwer, however it may be of Service to ſome who do not underſtand Figures.

EXAMPLE.

If the yearly Rent of a Houſe be 6*l.* what is that pe Day?

Anſwer 4*d.* by the Rule, but in Truth 'tis only 3*d.* $3\frac{285}{365}$.

4. By knowing the Price of a Pound Weight of any Comodity in Farthings, to know the Price of the grea Hundred, or 112*lib.*

RULE.

For every Farthing that one Pound doth coſt,
Reckon Two Shillings and a Groat.

EXAMPLE.

At 7*qrs.* per Pound, what 112 *lib.*

Anſwer 16*s.* 4*d.*

5. Another Rule by Half Pence, to know what 112 Pounds comes to.

RULE.

As many Half Pence that each Pound doth coſt,
So many Crowns the Hundred, bating ſo many Groats.

EXAMPLE.

EXAMPLE.

At $3\frac{1}{2}d.$ per Pound, what coſt 112 *lib.*

Solution.

$3\frac{1}{2}$ = 7 Half-pence.
5
35 *s.* = 1*l.* 15*s.* the Anſwer.

6. Another Rule by Half-pence.

Each Half-penny a Pound that's ſold or bought,
Yields four times Shillings, twice as many Groats.

EXAMPLE.

At $8\frac{1}{2}d.$ per Pound, what coſt 112 *lib.*

Solution.

$8\frac{1}{2}d.$ = 17 Half-pence   17
4   2
68 Shillings   34 Groats = 11*s.* 4*d.*
Add 11 : 4*d.*
20) 79 (3*l.* 19*s.* 4*d.* the Anſwer.

7. By knowing the Price of 112 Pounds, in Pounds and Shillings to know what one Pound coſt.

RULE.

Take half the Number of Shillings that 112 *lib.* coſt, and as often as 7 is contained in that half, take ſo many Farthings from that half Sum, and what is left are Farthings, the Price of one Pound.

EXAMPLE.

EXAMPLE.

If 112*lib*. coſt 6*l*. 1*s*. what coſt 1 *lib*.

Solution.

6*l*. 1*s*.
20

121
7) 60 (8
Quot. 8
Rem. 52 Farthings = 13*d*.
This generally gives too much.

XII. *Of Barter.*

DEFINITION.

BARTER among Merchants, is the Exchanging Wares for Wares, or one Comodity for another, and informs them ſo to proportion their Goods that neither may ſuſtain Loſs.

Queſt. 1. Two Merchants, A and B Barter, A hath Nutmegs at 4*s*. 6*d*. per Pound, and B hath 246 Pounds of Pepper, at 2*s*. 4*d*. per Pound, I demand how much Nutmegs A muſt give B for his Pepper?

Firſt find what the Pepper comes to thus,

lib. *d*. *lib*. *d*.
If 1 : 28 : : 246 : 6888

d. *lib*. *d*. *lib*.
2dly, If 54 : 1 : : 6888 : $127\frac{30}{54} = \frac{5}{9}$
Anſwer $127\frac{5}{9}$*lib*. of Nutmegs.

Queſt. 2. A hath $3\frac{1}{2}$ *C.* of Pepper, at $13\frac{1}{2}$*d.* per Pound, B hath Ginger, at $15\frac{1}{4}$*d.* per Pound, I demand how much Ginger muſt be delivered for the Pepper?

	lib.		*d.*		*C.*		*l.*	*s.*
Say, If	1	:	$13\frac{1}{2}$	: :	$3\frac{1}{2}$	:	22	1

	d.		*lib.*		*l.*	*s.*		*lib.*
2dly,	$15\frac{1}{4}$	:	1	: :	22	1	:	$347\frac{1}{61}$

Anſwer $347\frac{1}{61}$*lib.*

Queſt. 3. A hath 14*C.* of Sugar at 6*d.* per Pound, for which B gives him 1*C.* 3*q.* of Cinnamon, I demand how B rated the Cinnamon per Pound?

	C.	*q.*		*d.*		*C.*		*s.*
If	1	3	:	6	: :	14	:	4

Anſwer 4 Shillings.

Queſt. 4. A hath 4 Tuns of Brandy, worth 37*l.* 16*s.* per Tun, ready Money, but in Barter he hath 50*l.* 8*s.* per Tun, and B gives A 21*C.* 2*q.* $11\frac{1}{3}$*lib.* of Ginger for his Brandy, I deſire to know how B ſold his Ginger in Barter, per Hundred, and how much it is worth in ready Money?

	T.		*l.*	*s.*		*T.*		*l.*	*s.*
If	1	:	37	16	: :	4	:	151	4
If	1	:	50	8	: :	4	:	201	12

	C.	*q.*	*lib.*		*l.*	*s.*		*C.*		*l.*	*s.*	*d.*	*q.*
If	21	2	$11\frac{1}{3}$	:	201	12	: :	1	:	9	6	7	$3\frac{3674}{7258}$

Laſtly, To know how much it was worth in ready Money.

	C.	*q.*	*lib.*		*l.*	*s.*		*C.*		*l.*	*s.*	*d.*	*q.*
If	21	2	$11\frac{1}{3}$	:	151	4	::	1	:	6	19	11	$3\frac{4570}{725}$

		l.	*s.*	*d.*	*q.*
Anſwer	It was worth in Barter.	9	6	7	$3\frac{3674}{7258}$
	Worth ready Money.	6	19	11	$3\frac{4570}{7258}$

Queſt. 5. A and B Barter, A hath 320 Dozen of Candles, at 4*s.* 6*d.* per Dozen. for which B gives him 30*l.* in Money, and the reſt in Cotton, at 8*d.* per Pound, I demand how much Cotton he muſt give him more than the 30*l.*

	D.		*d.*		*D*		*l.*		*l.*		*l.*
If	1	:	54	::	320	:	72	—	30	=	42

Now ſay,

	d.		*lib.*		*l.*		*C.*	*q.*	
If	8	:	1	::	42	:	11	1	the Anſwer.

Queſt. 6. A and B Barter, A hath 608 Yards of Broad Cloth, worth 14*s.* per Yard, for which B gives him 125*l.* 12*s.* ready Money, and 85*C.* 2*q.* 24*lib.* of Bees Wax, I do demand how he reckon'd his Wax per Hundred ?

	y.		*s.*		*y.*		*l.*	*s.*		*l.*	*s.*		*l.*
If	1	:	14	::	608	:	425	12	—	125	12	=	300

	C.	*q.*	*lib.*		*l.*		*C.*		*l.*	*s.*
Then if	85	2	24	:	300	::	1	:	3	10

Queſt. 7. A hath Sugar at 8*d.* per Pound, ready Money, but in Barter he requires 10*d.* per Pound, and B hath Thread at 2*s.* 3*d.* per Pound ready Money, at what Price ought B to deliver his Thread at per Pound, to be equal in Barter with A ?

	d.		*d.*		*d.*		*s.*	*d.*	*q.*	
If	8	:	10	: :	27	:	2	9	3	the Anſwer.

Queſt. 8. A hath Cheeſe at 25*s.* per *C.* ready Money, but in Barter demands 32*s.* B delivered in Barter Linnen Cloth, at 18*d.* per Yard, what did the Cloth coſt in ready Money?

	s.		*s.*		*d.*		*s.*	*d.*	*q.*
If	32	:	25	: :	18	:	1	2	$0\frac{96}{384}$

Queſt. 9. A hath 6 Tuns of Wine, at 40*l.* per Tun, ready Money, and in Barter requires 44*l.* per Tun, beſides he requires $\frac{1}{3}$ Part of his over Price in ready Money, B hath 50*C.* of Tobacco, at 3*l.* 1*s.* per*C.* ready Money, how ought he to ſell it in Barter?

See the Work.

	l.		*l.*	*s.*	*d.*		*l.*	*s.*	*d.*	*l.*	*s.*	*d.*
$\frac{1}{3}$ of	44	=	14	13	4	From	40	0	0	44	0	0
						Sub.	14	13	4	14	13	4
						Rem.	25	6	8	29	6	8

	l.	*s.*	*d*		*l.*	*s.*	*d.*		*l.*	*s.*		*l.*	*s.*	*d.*	*q.*
If	25	6	8	:	29	6	8	: :	3	10	:	4	1	0	$3\frac{1363}{6072}$

Anſwer 4 1 0 $3\frac{1368}{6072}$ per Hundred.

XIII. *Of Rebate, or Diſcount.*

DEFINITION.

REBATE, is the Difference betwen a certain Quantity of Money, due at a certain Day, and the preſent Value or Worth of it; or it is when one Man gives another Man Credit for the Goods he ſells, but the Debtor is willing to pay ready Money, with this Condition, that he may have lawful Intereſt for the Money he pays, and for ſo long Time as he pay it before it becomes due, and this amongſt Tradeſmen is called Rebate, or Diſcount.

How to State the Queſtion.

Firſt, See what the Intereſt of 100*l.* comes to for the Time demanded.

Secondly, Add that Intereſt to 100*l.* which muſt always be the Firſt Term in the Rule of Three, and 100*l.* the Second, and the Sum to be rebated the Third.

Queſt. 1. A ſells to B a Watch for 20*l.* to be paid in 6 Months, but B is willing upon an after-agreement to pay preſent Money, upon Rebate, after 5*l.* per *Cent. per Annum*, Simple Intereſt, I demand the Sum paid and rebated ?

	m.		*l.*		*m.*		*l.*	*s.*
1ſt, If	12	:	5	::	6	:	2	10

	l.	*s.*		*l.*		*l.*		*l.*	*s.*	*d.*	*q.*
2dly, If	102	10	:	100	::	20	:	19	10	2	$3\frac{1450}{2050}$

	l.	*s.*	*d.*	*q.*
Sum agreed for	20	0	0	0
The preſent Money	19	10	2	$3\frac{1450}{2050}$
Sum rebated	0	9	9	$0\frac{600}{2050}$

Note, But if you would find the rebated Sum, ſay, As the Hundred Pounds with it's Intereſt is to the Intereſt of 100 for the given Time, ſo is the preſent Sum to the Rebate.

See the Work of the above Queſtion

	l.	*s.*		*l.*	*s.*		*l.*		*s.*	*d.*	*q.*
If	102	10	:	2	10	::	20	:	9	9	$0\frac{600}{2050}$

Anſwer 9*s.* 9*d.* $0\frac{600}{2050}$*q.* as above.

Queſt. 2. Delivered a Bond to a Creditor of 264*l.* 13*s.* 4*d.* which will be due 8 Months hence, but upon Rebate at 5*l. per Cent. per Annum*, I am willing to make preſent Payment, how much is the preſent Payment, and alſo the Diſcount.

	m.		*l.*		*m.*		*l.*	*s.*	*d.*
1ſt, If	12	:	5	: :	8	:	3	6	8

		l.	*s.*	*d.*		*l.*		*l.*	*s.*	*d.*	*q.*
2dly, If		103	6	8	:	100	: :	264	13	4	0
Anſwer	Preſent Money							256	2	6	$3\frac{216}{248}$
	Rebate							8	10	9	$0\frac{32}{248}$

Queſt. 3. What's the Diſcount of one Pound for one Year, at 5*l. per Cent. per Annum?*

	l.	*s.*	*d.*	*q.*
Preſent Money	0	19	0	$2\frac{30}{105}$
From	1	0	0	0
Rebate - -	0	0	11	$1\frac{75}{105}$

By this it appears that one Pound in a Year is decreaſed to 19*s.* 0*d.* $2\frac{30}{150}$*q.* at 5*l. per Cent.*

Queſt. 4. What's the preſent Money, and Diſcount of 58*l.* 16*s.* 3*d.* for 172 Days, at 5*l. per Cent. per Annum*, Simple Intereſt?

		l.	*s.*	*d.*	*q.*
Anſwer	Preſ. Sum	57	9	2	$0\frac{11904000}{35865600}$
	Diſcount	1	7	0	$3\frac{23961600}{35865600}$

Queſt. 5. What's the preſent Money, and Diſcount of 76*l.* 8*s.* 4*d.* for 239 Days, at 5*l. per Cent. per Annum* Simple Intereſt?

		l.	*s.*	*d.*	*q.*
Anſwer	Preſ. Money	73	19	10	$2\frac{128352}{361872}$
	Rebate	2	8	5	$1\frac{233420}{361872}$

XIV. *Of Profit and Loſs.*

DEFINITION.

THE Title itſelf is a ſufficient Explanation, for in Trade it is only to know what you gain or loſe, by Buying and Selling any Comodity. In which are four Varieties.

1. To know what is gain'd or loſt *per Cent.*

2. To know what it ſhall be ſold for, to gain or loſe ſo much *per Cent.*

3. Having gain'd or loſt ſo much *per Cent.* to know what it coſt.

4. *Laſtly,* There being ſo much gain'd *per Cent.* when ſold at ſuch a Price, to know what is gain'd *per Cent* when ſold for more, or what is loſt *per Cent.* when ſold for leſs.

Queſt. 1. If 1*lib.* of Tobacco coſt 18*d.* and is ſold for 21*d.* I demand what is gain'd *per Cent.*

	d.		*d.*		*l.*		*l.*	*s.*	*d.*
If	18	:	3	: :	100	:	16	13	4

That is; by laying out of 100*l.* you will gain that Sum, *viz.* 16*l.* 13*s.* 4*d.*

Queſt. 2. If 90 *Engliſh* Ells of Cambrick coſt 60*l.* for how much muſt one Yard be ſold to gain 18*l. per Cent.*

Firſt find what one Yard will come to at the given Price.

If 90 *E.* : 60*l.* : : 1*Y.*
5 4
450 : 60 : : 4 : 10*s.* 8*d.*

Then to - - 100*l.*
Add - - 18

If 100*l.* : 10*s.* 8*d.* :: 118 : 12*s.* 7*d.*

Queſt. 3. If 276 Fother of Leads (each $19\frac{1}{2}$) be ſold for 256*l.* at 5 Months Credit, and I gain 11*l. per Cent. per Annum.* The Queſtion is, how much the Whole coſt ready Money?

See the Work.

To - - - - 100*l.*
Add - - - - 11

111

l. *l.* *l.* *l.* *s.* *d.* *q.*
If 111 : 256 :: 100 : 230 12 7 $1\frac{45}{111}$

Queſt. 4. If Cloth ſold at 12*s.* per Yard, be 10*l. per Cent.* Profit, what Gain or Loſs *per Cent.* ſhould I have had I ſold the ſame for 7*s.* per Yard?

Operation.

s. *l.* *s.* *l.* *s*
If 18 : 110 : 7 : 96 5 Sub.
100 0 from

Anſwer loſt - - 3 15 *per Cent.*

A General Rule.

Where there is Gain *per Cent.* add the Gain *per Cent.* to 100*l.* but where there is Loſs *per Cent.* ſubtract as much as you loſe *per Cent.* from 100*l.* the Sum or Difference is the third Number in the Rule of Three.

XV.

XV. *Of Exchange.*

DEFINITION.

EXCHANGE is to pay Money in one Country, and receive the like Sum or Value in another, with Conſideration of either Loſs or Gain.

Note, The Par of Exchange is the fix'd and Standard Value of Foreign Coins, *&c.* expreſſed in Sterling Money of our own; 'tis ſo called, becauſe in Exchange one equal Value for another is given.

The Courſe of Exchange is the Current Price of Exchange, always unſettled, being ſometimes above, and ſometimes below the Par, according to the various Circumſtance and Accidents of Trade and Nations

Before I give any Examples it will be proper to give the Value of Gold and Silver Coin, both of our own and other Nations: And firſt of our own as weigh'd in Air and Water. *Troy Weight.*

	Weigh'd in Air.		Water.	
	pw.	*gr.*	*pw.*	*gr.*
A Guinea	5	9	5	1$\frac{3}{4}$
A Crown	19	8$\frac{1}{2}$	17	11$\frac{1}{5}$
Half a Crown	9	16$\frac{1}{4}$	8	17$\frac{3}{5}$
A Shilling	3	20$\frac{9}{10}$	3	11$\frac{4}{5}$
Six Pence	1	22$\frac{1}{2}$	1	17$\frac{9}{10}$

	l.	*s.*	*d.*
One Pound of Gold is worth	48	0	0
One Ounce is worth	4	0	0
A Penny-weight is worth	0	4	0

One

	l.	*s.*	*d.*
One Grain is worth	0	0	2
One Pound-weight of Silver	3	2	0
One Ounce is worth	0	5	2
One Penny-weight is worth	0	0	$3:0\frac{2}{5}$
One Grain is worth	0	0	$0:0\frac{1}{2}$

Scotch Money.

	l.	*s.*	*d.*
A Pound	0	1	8
A Mark	0	1	$1\frac{1}{2}$
A Noble	0	0	$6\frac{3}{4}$

Iriſh Money.

A Pound	0	15	0
A Harper	0	0	9
An Obb, or Cob	0	0	$4\frac{1}{2}$

Note, 100*l.* in Gold, weighs 2 *lib.* 10 *oz. Troy*.

And 100*l.* of Silver weighs 26 *lib.* 4 *oz.* And 22*d.* in Copper weighs one Pound *Averdupoiſe*.

Dutch Money.

A Dutch Stiver	0	0	$1\frac{1}{5}$
6 Stivers, or 1 Skilling *Flemiſh*	0	0	$7\frac{1}{5}$
20 Stivers, 1 Gilder	0	2	0
A *Flemiſh* Pound, —— 33*s.* 4*d.* =	1	0	0
A Zealand, or common Dollar	0	3	0
A Duccatoon,	0	6	$3\frac{3}{5}$
A Specie Dollar	0	5	0
A Croſs Dollar	0	4	2
Old *Philip*'s Dollar	0	5	0
Ferdinando's Dollar	0	4	3
Prince of *Orange*'s Dollar	0	4	4
Leopald's Dollar	0	4	3
Rudolphus's Dollar	0	4	4
Maxamilian's Dollar	0	4	5
Holland Rider	1	5	9

German Money.

	l.	s.	d.
A Rix Dollar of the Empire	0	4	$3\frac{1}{4}$
A Gilder of *Noremberg*	0	7	1
A Mark Lubs	0	16	0

French Money.

A Denier	0	0	$0\frac{1}{40}$
A Soulz = 12 Deniers	0	0	$0\frac{3}{10}$
A Liver = 20 Soulz	0	1	6
A Crown = 3 Livers	0	4	6

Spaniſh Coins.

13 Malvadies =	0	0	$0\frac{1}{4}$
A Rial =	0	0	$6\frac{3}{4}$
8 Rials, or 1 Piece of Eight	0	4	$6\frac{1}{4}$
A Piece of Eight Mexico	0	4	6
A Piece of Eight *Peru*	0	4	5
A Piece of Eight *Seville*	0	4	6
A Doubloon of *Caſtile*	0	5	6
A *Velentia* Ducat	0	5	3
Saragoza Ducat	0	5	6
Barcelona Ducat	0	6	0
Barcelona Crown	1	2	0
Pattavoon	0	4	8

Portugal Coin.

42 Rees	0	0	1
A Vintine,	0	0	3
A Rial	0	0	6
A Teſtoon	0	1	3
An old Cruſado	0	5	0
A Mill Ree = 1000 Rees	0	6	8

Italian Coin.

A Liver at *Leghorn*	0	0	9
A Current *Florence* Crown	0	5	3
A Ducat *de Banco Venice*	0	4	4
A St *Mark*	0	2	10

	l.	s.	d.
A Julio, or Julier	0	0	6
A Dollar	0	4	6
7 Livers = to 1 Ducat	0	5	9
The Ducat of *Rome*	0	7	6
The Crown at *Rome*	0	7	6
The Crown of *Placentia*	0	7	6
The *Naples* Ducat	0	5	0
The Crown Current	0	5	6
The Torri	0	1	0
A Bajocke	0	0	0½

Poland and *Ruſſian* Coin.

At *Dantzick*.

90 Groſz = 1 Rix Dollar	0	4	6
A Gilder	0	1	6
The Stiver or Dollar	0	2	3
Silver Mark or Gilder	0	1	6

Sweden Coin.

Silver Dollar	0	2	3
Silver Mark, or Gilder	0	1	6

Arabian and *Perſian* Coin.

The Abaſſis	0	1	4
A Mamada	0	1	0

At *Bantam*.

1000 Patees of Copper	0	5	0

At *Aleppo* and *Scanderoon*.

16 Shekees = 1 Piece of Eight	0	4	6
14 Shekees = 1 Lyon Dollar	0	4	0
A Dina	1	10	0
A Soraphat Goa,	0	4	6

At *Siam*.

A Gold Coin worth	0	10	7
Silver Coin	0	2	5

At *Surret.*

	l.	s.	d.
The Aſper	0	0	$1\frac{1}{4}$
The Larin	0	4	0
The Saraphim	0	4	6
The Rupie	0	2	0
The Fenon, or Fanam	0	0	3
A Pagod	0	9	0
A Saltanin	0	8	0

Queſt. 1. A Merchant at *Leghorn* delivered 500 Livres to receive at *London* in Sterling Money, how much muſt he receive?

	Liv.		d.		Liv.		l.	s.
If	1	:	9	: :	500	:	18	15

The Courſe of Exchange is publiſhed Weekly, which compared with the Par, will ſhew whether it is above or below at any time. In *December* 1735, the Courſe of Exchange ſtood thus.

	s.	d.
Amſterdam	35	10
Ditto at Sight	35	6
Rotterdam $2\frac{1}{2}$ Uſance	36	
Antwerp	36	6
Hambourgh $2\frac{1}{2}$ Uſance	36	
Paris	$31\frac{7}{8}$	
Bourdeaux $\frac{1}{2}$ Uſance	$30\frac{5}{8}$	
Cadiz	$40\frac{1}{2}$	
Madrid	$40\frac{1}{4}$	
Bilboa	$40\frac{1}{4}$	
Leghorn	$50\frac{3}{8}$	
Genoa	$52\frac{1}{2}$	
Venice	$50\frac{3}{8}$	
Lisbon	5	$6\frac{7}{8}$
Porto	5	$3\frac{3}{8}$
Dublin		$12\frac{7}{8}$

The Reader muſt here take Notice, that the Courſe of Exchange is in a conſtant Flux, they continue not at a ſett Standard as our Sterling Money doth, but are ſometimes raiſed, and ſometimes depreſſed.

Queſt. 2. A Merchant in *London* delivers 481 *l.* Sterling, to receive the ſame at *Paris* in *French* Crowns, at 4 *s.* 6 *d.* I demand how many *French* Crowns he ought to receive ?

	s.	*d.*		*C.*		*L.*		*Crowns.*
If	4	6	:	1	:	481	::	$2137\frac{42}{54}$

Queſt. 3. If 180 *d.* at *London,* be equal to 2 Ducats at *Rome,* and 5 Ducats at *Rome,* equal to $16\frac{18}{27}$ Dollars at *Danzick,* how many Pence at *London* are equal to 10 Dollars at *Danzick ?*

State it as has been taught in the Rule of Three, p. 36.

Pence at *London* 180 = 2 Ducats at *Rome.*
Ducats at *Rome* 5 = $16\frac{18}{27}$ Dollars at *Danzick.*
London 10 = *d.* Dollars at *Danzick.*

900)243000(270

Anſwer 270 *d.*

XVI. *Of Single Poſition.*

DEFINITION.

THIS Rule of Falſe Poſition takes it's Name from working by any falſe or feign'd Number we find out a true Anſwer. And this is the Rule.

As the Parts of the Poſition
Is to the Poſition itſelf,
So is the given Number by gueſs,
To the true Number ſought.

EXAMPLE.

Queſt. 1. A certain Perſon being demanded how old he was, to avoid a direct Anſwer, ſaid, If the Half, the Third, the Fourth, and the Sixth Parts of his Age, were added together they would make juſt $52\frac{1}{2}$, now how old was he?

Take any Poſition at Pleaſure, that will be divided into the Parts above-mentioned, as ſuppoſe 18.

$$\text{Then } \left\{\begin{array}{c}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\\ \frac{1}{6}\end{array}\right\} \text{of } 18 \text{ is} = \left\{\begin{array}{l}9\\ 6\\ 4\frac{1}{2}\\ 3\end{array}\right.$$

$$22\tfrac{1}{2} \text{ the Sum.}$$

Now ſay,

As 22.5 : 18 : : 52.5 : 42.

PROOF.

$$\left.\begin{array}{c}\frac{1}{2}\\ \frac{1}{3}\\ \frac{1}{4}\\ \frac{1}{6}\end{array}\right\} \text{of } 42 \text{ is} = \begin{array}{r}21\\ 14\\ 10.5\\ 7.\end{array}$$

Sum 52.5

Anſwer 42.

Queſt. 2. Three Men, A, B, and C, make a Purchaſe of 3000*l*. B is to ſhare double with A, and C to ſhare triple with B, now what are their reſpective Shares?

$$\text{Anſwer} \left\{\begin{array}{lrrr}A & 333 & 6 & 8\\ B & 666 & 13 & 4\\ C & 2000 & 00 & 0\end{array}\right.$$

Proof 3000 00 0

Queſt. 3. One who had waſted $\frac{2}{3}$ and $\frac{1}{5}$ of his perſonal Eſtate had 36 Pounds remaining, now I would know what was his Eſtate?

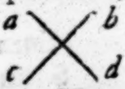
Anſwer 270*l*.

XVII. *Of Double Poſition.*

DEFINITION.

THIS is when we have two falſe Poſitions given at Pleaſure, to find a true Anſwer to a Queſtion propounded.

When a Queſtion is propounded, make ſuch a Croſs as this,



and make choice of any Number, which call your firſt Poſition, and place it at the Top of the Croſs, where *a* ſtands: then work according to the Nature of the Queſtion, and if the true Number come out, 'tis done, otherways what it differs call the Firſt Error, which place at *c*. Make choice of another Poſition, which place with *b*, work with this as if it were the true Number, and if it differ from the Truth, ſet the Difference at *d*, and call it the Second Error, which Errors mark with *Pluſs* +, and *Minus* —, and thus the Poſitions will ſtand at the Top of the Croſs, and the Errors at the Bottom.

Then multiply the Firſt Poſition by the Second Error, (that is, Croſs-wiſe) and the Second Poſition by the Firſt Error, and reſerve the Products.

Then, if the Errors be both too much, or both too little, the Difference of the Products is the Dividend, and the Difference of their Errors is a Diviſor, the Quotient ariſing from that Diviſion is the Anſwer ſought.

But if the Errors be one too much and the other too little, the Sum of the Products, is a Dividend, and the Sum of the Errors is a Diviſor, the Quotient of this Diviſion is the Anſwer.

EXAMPLE.

Queſt. 1. Three Perſons, A, B, and C, thus diſcourſed together concerning their Age, quoth A I am 18 Years of Age, quoth B I am as old as A and half as old as C, and quoth C, I am as old as you both if your Years were added together, now I deſire to know the Age of each Perſon?

Solution.

Suppoſe the Age of C to be 40
Then B muſt be - - 20
A's Age add to B - , - 18

For the Age of B - 38
Add - - - 18

Sum - - - 56
Subtract - - 40

Error + - 16

2dly, Suppoſe C 60
The Half - 30
Add - - 18

B - Sum 48
18

66
60

Error + 6

40 60
+ +
16 6
60 40

960 240
240

10)720(72

Anſwer 72 Years, the Age of C.
36 is half of 72
Add 18 A.

Sum 54 B.

Queſt. 2. Two Men, *viz.* A and B diſcourſed thus together, A ſaid to B, I think you have this Year 20 Geeſe, no ſaid B to him again, that I have not, but if I had as many as I have, and half ſo many, and two Geeſe and a half, then I ſhould have 20, now I demand how many Geeſe he had?

Anſwer 7.

Queſt. 3. A Man going to an Alehouſe, ſaid to the Hoſt, if you'll give me as much Money as I have in my Purſe I will ſpend my Sixpence, the Hoſt did ſo, and he ſpent his Sixpence; he went to a Second Alehouſe, and ſaid, if you'll give me as much Money as I have in my Purſe, I will ſpend my Sixpence, the Hoſt did ſo, and he ſpent his Sixpence; he went to a Third Alehouſe and did likewiſe, and when he had ſpent his Sixpence he had nothing left, now the Queſtion is, what the Tipler had in his Purſe at firſt?

Anſwer 5¼*d.*

Queſt. 4. There is a Ciſtern with Four Cocks, which holds 8 Barrels of Water, the Firſt Cock will run it all out in 6 Hours, the Second in 4 Hours, the Third in 3 Hours, and the Fourth in 2 Hours, now I demand in what Time they will all run it out, if they were all ſet a running at once?

Anſwer in 48 Minutes.

Queſt. 5. A Gentleman hired a Workman for 20 Days, at 18*d.* per Day for every Day he worked; but for every Day he played he was to bate 12*d.* At the End of 20 Days he received 8 Shillings, which was his full Due. Now I demand how many Days he wrought, and how many Days he played?

Anſwer 11*d.* 2*h.* 48′
And play'd 8 19 12′

Queſt. 6. A and B receive 100*l.* for the Ranſom of a Priſoner, fall at Variance in parting the Money, and after each had ſnatch'd what he could (upon Agreement) B gave A ⅕ of what he had ſnatch'd from him, and A gave B ⅓ of what he had ſnatch'd from him, which done, each of them had 50*l.* now how much Money did each Man ſnatch up?

		l.	*s.*
Anſwer	A	37	10
	B	62	10

Sum 100 Proof.

Queſt. 7. A Thief breaking into an Orchard, Stole from thence a certain Number of Pears, and at his coming

ing forth he met with Three Men, one after another, who threatned to accuſe him of Theft; and for to appeaſe them, he gave unto the Firſt Man, half the Pears that he had Stole, who return'd him back 12 of them; then he gave unto the Second half of them he had remaining, who return'd him back 7 of them; and unto the Third Perſon he gave half the Reſidue, who return'd him back 4. and in the End he had ſtill remaining 20 Pears, now the Queſtion is, how many Pears he had ſtole in all?

Anſwer 76.

Queſt. 8. A Gentleman hired a Servant for a Year, at 6*l.* Sterling and a Livery Cloak, valued at a certain Rate, but when $\frac{7}{12}$ of the Year was expired, they fell at variance, and the Gentleman put away his Servant, gave him the Cloak together with the 50*s.* of the Money agreed for, which was the Servant's full Due for the Time of his Service, the Queſtion is, to find what the Cloak was valued at?

Anſwer 2*l.* 8*s.*

Queſt. 9. *If to my Age you add,*
The Four Fifths of Three Fourths, and Two Fifths more,
The Number Sixty Eight will then be had,
What was my Age in Years above a Score?

Anſwer 34 = 14 above 20.

XVIII. *Of Compound Intereſt.*

DEFINITION.

As for Simple Intereſt, it is ſufficiently explained in the Golden Rule of Five Numbers. Compound Intereſt, is when a Sum of Money is put out to Intereſt, and the Intereſt thereof becoming due, is ſtill continued in the Hand of the Debtor, ſo as to become Part of the Principal, Intereſt being reckoned for it, from the Time it became due, for which Reaſon it is called Intereſt upon Intereſt. And as Simple Intereſt increaſeth by a Series of Arithmetical Proportionals continued, ſo doth Compound

Intereſt increaſe by a Rank or Series of Geometrical Proportionals.

And *Note*, That the Simple Intereſt of One Hundred Pounds being known, the Compound Intereſt of any other Sum for any Number of Years may be likewiſe found out by ſo many ſingle Rules of Three, as there are Years. For as 100*l.* is to it's Intereſt for one Year, ſo is any other Sum, to it's Intereſt for any other Time.

And ſo is the Firſt Year's Intereſt to the Second, and the Second Year's Increaſe to the Third, *&c.*

EXAMPLE.

What doth 10*l.* amount unto at the End of 7 Year's Compound Intereſt at 5*l.* *per Cent.* Say,

l. *l.* *l.* *l.*
As 100 : 105 : : 10 : 10.5

Here I have given the Fourth Term in a Decimal mix'd Number, which is 10 *l.* 10 *s.* which is the beſt Method of anſwering Queſtions in Compound Intereſt; for by what follows you may eaſily conceive, that it is only the Principal Sum, multiply'd into it's Increaſe for one Year, ſo often as the Time requireth. As in the Example 105, being what 100*l.* is increaſed to for one Year, is multiplied 7 Times into the Principal Sum, 10*l.* *&c.* and the laſt Product is the Anſwer to the Queſtion.

100*l.* is increaſed in one Year to - -	105
It's Principal Sum - - - - -	10
Firſt Year's Increaſe - - -	10.5
	105
Second Year's Increaſe - - -	11.025
	105
	55125
	11025
Third Year's Increaſe - -	11.57625
	105
	5788125

	5788125
	1157625
Fourth Year's Increaſe - -	12.1550625
	105
	607753125
	121550625
Fifth Year's Increaſe - -	12.762815625
	105
	63814078125
	12762815625
Sixth Year's Increaſe -	13.40095640625
	105
	6700478203125
	1340095640625
Seventh Year's Increaſe -	14 0710042265625

The Value of this Decimal of a Pound Sterling, is 1*s.* 5*d.* 0.1640575*q.* ſo that by letting out 10*l* for 7 Year, without taking up any Intereſt, it will grow in that Time to 14*l.* 1*s.* 5*d.* 0.1640575 Parts of a Farthing Intereſt.

But how expeditiouſly this may be done by Logarithms, will beſt appear by the following Work. For to the Logarithm of the Increaſe of one Hundred Pounds for one Year, *viz.* 105, you add the Logarithm of the Principal Sum 10*l.* and from the Sum of thoſe two Logarithms Subtract the Logarithm of 100*l.* the Remainder is the Logarithm of the Firſt Year's Increaſe 10*l.* 10*s.* which Logarithm of one Year's Increaſe multiply by 7, the Number of Years, omitting the Index, gives the Logarithm of the Increaſe of 10*l.* at the End of 7 Years, Compound Intereſt.

See the Work.

100*l.* in one Year is increaſed to	105*l.*	2.0211893
Principal Sum - -	10.	1.0000000
Sum of the Logarithms - -		3.0211893
100*l.* - - - - -		2.0000000
Firſt Year's increaſe - -	10.5.	1.0211893
Number of Years - - - - -		7
Anſwer - -	14.071..	1.1483251

You ſee we have got the three firſt Decimal Places of a Pound, which is near enough the Truth.

EXAMPLE II.

What will 4 *l.* amount unto in 10 Years, at 5 *l.* *per Cent.*

Operation by the Logarithms.

Logarithm of { 105 = 2.0211893
{ 100 = 2.0000000
Remains Logarithm 0.0211893
Years multiply - - - 10
0.2118930
Principal Sum 4*l.*=Logar. .6020600
Anſwer 6.5155785 Logar. .8139530

The Value of the Fraction is 10*s.* 3*d.* 2.95536*q.* So that we ſee 4*l.* being put out 10 Years at Compound Intereſt, it will be increaſed to 6*l.* 10*s.* 3*d.* 2.95536*q.*

XIX. *Of the Square Root.*

DEFINITION.

THE Square of any Number is, when it is multiplied into itſelf, as the Square of 4 is 16, the Square of 5 = 25, *&c.* therefore having any Square Number given to find it's Root, do thus, *viz.* begin at the Unites Place, and there make a Dot, and ſo over every other Figure; now as many Dots as there are, ſo many Places or Figures you will have in the Root. By pointing of it thus, the given Square is divided into Periods, then ſee what is the neareſt Leſs Square of the firſt Period to the Left Hand, and place it's Root in the Quotient, which Square of that Root place under the firſt Period, and ſubtract, ſet the Remainder under the Line, and thereto bring down the next Period, then double the Root, and place it on the Left Hand a crooked Line for a Diviſor. See how often that Diviſor is contained in the Dividend laſt brought down and placed under the Line, always making allowance of one Place in the Dividend, becauſe the ſame Figure you place in the Root, the ſame muſt be placed in the Unites Place in the Diviſor. Repeat this Work as often as there are Periods or Pairs of Figures to bring down: And at laſt, if any Thing remain, add two Cyphers, and for every two ſo added, you will have one Decimal place in the Root.

The following Examples will make it plain to the diligent Reader.

What's the Square Root of 2401 ?

2401 (49 Root.
16
89) 801
801
Rem. 0

PROOF.

To prove the Work, multiply the Root in itſelf, and to the Product add the Remainder, if any, this laſt Sum will be equal to the given Square.

More Examples for Practice.

459684 (678 Root.
36
127) 996
889
1348) 10784
10784
Rem. 0

814602573 (28541.24 Root. A mixt Number.
4
48)414
384
565) 3060
2825
5704)23525
22816
57081) 70973
57081
570822) 1389200 Rem.
1141644
5708244) 24755600
22836976
Rem. 1918624

Note, The Square Root is of excellent Uſe in ſolving Arithmetical and Geometrical Queſtions, which I ſhall exhibit as follows.

1. To find a mean Proportional between any two Numbers.

RULE.

Multiply the two given Numbers together, and extract the Square Root of the Product, gives the Anſwer.

EXAMPLE.

Queſt. 1. What's the Geometrical Mean, between 4 and 6?
Anſwer 4.89.
Queſt. 2. What's the Geometrical Mean between 20 and 30?
Anſwer 24.49.

2. To find the Diameter of a Circle that ſhall be equal in Area to an Ellipſis whoſe Tranſverſe and Conjugate Diameter are given.

RULE.

RULE.

Multiply the Two Diameters of the Ellipſis together, and extract the Square Root of the Product will give the Diameter of a Circle equal.

EXAMPLE.

Queſt. 3. Let the Tranſverſe Diameter be 36, and the Conjugate 23.5 Inches, what is the Diameter of a Circle equal?

Anſwer 29.08 Inches.

3. Any two Sides of a right lin'd plain Triangle being known, the Third is eaſily ſound.

For in the 47th Propoſition of the Firſt Book of *Euclid*'s *Elements* it is Demonſtrated, that the Square of the Hypothenuſe, is equal to the Square of the other Two Sides.

EXAMPLE.

Admit three Towns, as *London*, *Landaff*, and *Boſton*; *Boſton* lieth from *London* directly North 89 Miles, and *Landaff* lieth directly Weſt from *London* 131 Miles, now I would know how far *Landaff* is from *Boſton*?

Anſwer 158.37 Miles.

4. Admit two Ships ſet Sail from one Port, the one ſails directly North 89 Miles, and the other directly Weſt, and at the End of their ſailing they were diſtant 158.37 Miles, I demand how far the Second Ship ſailed?

Anſwer 131 Miles.

5. Two Men Travel from the ſame Place, A Travels Weſt 131 Miles, B goes directly North 'till he be diſtant from A 158.37 Miles, I demand how far B travell'd?

Anſwer 89 Miles.

6. The Area of a plain Superficies given, to find the Side of the Square that ſhall be equal thereto?

RULE.

The Square Root of the Area is the Anſwer.

EXAMPLE.

EXAMPLE.

Let the Area of a plain Superficies be 576 Inches, what's the Side of the Square that bounds it?

Anſwer 24 Inches.

In like Manner, if you extract the Square Root of the Content or Area of a Circle, *Pentagon*, *Hexagon*, *&c.* whether the Figure be regular or irregular, it will give the Side of a Square equal to that Superficies.

7. Given, the Side of a Square, to make other Squares, Two, Three, Four Times, more or leſs, greater or leſſer, than that Square propoſed.

RULE.

Square the Side of the given Square, and it gives the Area thereof, which multiply by 2, 3, 4, *&c.* the Square of the Product will give the Side of whoſe Area ſhall be 2, 3, 4, *&c.* times greater than the given Square.

EXAMPLE.

Let the Side of a Square be 12 Inches, and let it be required to make three other Squares, to contain Double, Triple, and Quadruple the given Square?

Given Square's Area is	144		144		144
Multiply by - -	2	and	3	and	4
	288(16.97		432(20.78		576(24

$$\text{Anſwer Side Square}\left\{\begin{array}{l}16.97\ \text{Double.}\\ 20.78\ \text{Triple.}\\ 24.\ \text{Quadruple.}\end{array}\right\}\text{in Area.}$$

The like is to be obſerved of Circles.

8. In a plain right lined Oblique Angled Triangle having the Sides given ſeverally, to find where the Perpendicular, let fall from the obtuſe Angle, will fall upon the Baſe.

9. The Height of a Mountain being known, to find how far it ſhall be ſeen at Sea, or on plain Ground.

10. Given the Semidiameter of the Earth 3984.58 Miles, with the Diſtance of a Mountain ſeen at Sea, to find the Height of that Mountain, this is the Converſe of the 9th.

RULE.

Multiply the Two Diameters of the Ellipſis together, and extract the Square Root of the Product will give the Diameter of a Circle equal.

EXAMPLE.

Queſt. 3. Let the Tranſverſe Diameter be 36, and the Conjugate 23.5 Inches, what is the Diameter of a Circle equal?

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Multiply by - -	2	and	3	and	4
	288(16.97		432(20.78		576(24

$$\text{Anſwer Side Square}\left\{\begin{array}{l}16.97\ \text{Double.}\\ 20.78\ \text{Triple.}\\ 24.\ \text{Quadruple.}\end{array}\right\}\text{in Area.}$$

The like is to be obſerved of Circles.

8. In a plain right lined Oblique Angled Triangle having the Sides given ſeverally, to find where the Perpendicular, let fall from the obtuſe Angle, will fall upon the Baſe.

9. The Height of a Mountain being known, to find how far it ſhall be ſeen at Sea, or on plain Ground.

10. Given the Semidiameter of the Earth 3984.58 Miles, with the Diſtance of a Mountain ſeen at Sea, to find the Height of that Mountain, this is the Converſe of the 9th.

11. The Diameter of a Pipe of a Cock given, to find the Diameter of another Pipe that ſhall fill the ſame in half the Time, more or leſs.

EXAMPLE.

There is a Pipe whoſe Diameter is $1\frac{1}{2}$ Inch, will fill a Ciſtern in 2 Hours, I demand the Diameter of another Pipe that will fill it in 1 Hour?

Anſwer 2.12 Inches.

12. The Circumference of a Cable being $7\frac{1}{4}$ Inches, and that one Fathom thereof doth weigh $16\frac{1}{4}$ Pounds, I demand how many Pound that Rope doth weigh whoſe Circumference is $11\frac{3}{4}$ Inches?

Operation.

ſquare *pounds* *ſquare* *pounds.*
As 52.5625 : 16.25 : : 138.0625 : 42.6828

Anſwer 42 *lib.* 10 *oz.* 14.7968. *dr.*

13. If the Length of a Caſk be 40 Inches, Bung 28, and Head 24, what's the Diagonal Line?

Anſwer 32.8 Inches.

14. In a right lin'd Triangle, let one Side be 40 Inches, the next 26, and the leaſt 22, what's the Area?

Anſwer 264 Inches.

15. A Company of Men drinking 'till the Reckoning came to 6*s.* $0\frac{1}{4}$*d.* I demand how many Perſons there were in Company, and what they paid a piece?

Anſwer 17 Perſons paid $4\frac{1}{4}$*d.* a piece.

16. A Company of Men Drinking, 'till the Reckoning came to 30*s.* 1*d.* I demand how many there were in Company, and what they paid a piece?

Anſwer 19 Men, paid 19*d.* a piece.

17. A Company of Men drinking 'till the Reckoning came to 17*s.* $6\frac{1}{4}$*d.* I demand how many there were in Company, and what they paid a piece?

Anſwer 29 Men paid $7\frac{1}{4}$*d.* a piece.

18. A Company of Men Drinking, 'till the Reckoning came to 6*l.* 16*s.* $8\frac{1}{4}$*d.* I demand how many there were in Company, and what they paid a piece?

Anſwer 81 Men, paid 1*s.* $8\frac{1}{4}$*d.* a piece.

19. In an Army of 75625 Soldiers, how to place them in ſquare Battalia?

75625

75625 (275 Men in Rank and File.
4
47) 356
329
545) 2725
2725
Rem. 0

20. To place any Number of Men, ſo that the Number of Men in Rank may be double to thoſe in File.

Suppoſe 35912 Men
Half 17956 (134 File
2
268 Rank.
1
23) 79
69
264) 1056
1056
0

For Proof, 268 × 134 = 35912.

21. Suppoſe 25000 Soldiers were to be martialed in a ſquare Battalia of Ground, in ſuch Sort that their diſtance in File ſhould be 8 Feet, and the Rank 3 Feet, how many Men in Rank, and how many in File ?

As 8 : 3 :: 25000 : 9375 (96 File) 25000 (260

Anſwer { 96 File / 260 Rank } beſides 40 Men over and above.

22. If 47748 be the Number of Men to be martial'd in Battel Array in ſuch Sort the Number of Men in File to thoſe in Rank, ſhould be as 8 to 20 ?

Anſwer 345 Rank, 135 File, beſides 138 over.

XX. *Of the Cube Root.*

DEFINITION.

A Cube is a Geometrical Figure; and as a Square has only Length and Breath, ſo a Cube has Length, Breadth and Depth: And in order to a right Underſtanding of this Work, it will be neceſſary for the Learner to get by Heart this Table of the Squares and Cubes of the Nine Digits.

Cubes.	1	8	27	64	125	216	346	512	729
Squares.	1	4	9	16	25	36	49	64	81
Roots.	1	2	3	4	5	6	7	8	9

1. Point out the given Cube Number, beginning with the Unites Place, and ſo over every third Figure towards the Left Hand; and as many Points or Periods as there are, ſo many Figures there will be in the Root.

2. Find the neareſt leſſer Cube in the firſt Period towards the Left Hand, which found, place in the Root on the Right Hand the crooked Line, cube this Figure (placed in the Root) and ſet it under the firſt Period, draw a Line and ſubtract, place the Remainder under the Line, to which bring down the next Period, and call this the Reſolvend.

3. To find a Diviſor, Triple the Root, and ſet it under the Line drawn under the Reſolvend; then ſquare the Root, and triple the Square, ſet this Product down as in common Multiplication, *viz.* Unites under the Place of Tens, draw a Line and add theſe two Numbers together for the Diviſor.

4. To find how often the Diviſor is contained in the Reſolvend, you muſt ſet aſide the Figure in the Unites Place of the Reſolvend, that as often as you can have the

Diviſor in that Part of the Reſolvend, ſet it in the Root, and obſerve theſe Steps.

1. Cube the Figure placed laſt in the Root.

2. Square the ſame, and multiply that Square by the Triple Root, (which Triple Root was found when you found the Diviſor) placing Unites under Tens as in common Multiplication.

3. *Laſtly*, Multiply the Figure laſt placed in the Root, by the Triple Square of the laſt Root, placing Unites under Tens as before; draw a Line under them and add theſe three laſt Lines of Figure into one Sum, and ſubtract it from the Reſolvend; to the Remainder bring down the next Period of three Figures, or Cube, to which find a Diviſor, and proceed in all the Steps as above directed, until all the Cubes are brought down and wrought, if any thing remain at laſt, add three Cyphers, and you will have a Decimal in the Root. The following Examples (being well minded) will make all plain.

EXAMPLE I.

Let the Cube Root of 1728 be required?

$\dot{1}72\dot{8}$ (12 Root.

1 Cube of 1

728 Reſolvend.

3 Triple Root 1.

3 Triple □ of Root 1.

33 Diviſor.

8 Cube of Root 2:

12 Square of 2 by Triple 1.

6 Triple □ of Root 1.

Sum 728 Subtract from a Reſolvend.

Rem. 0

EXAMPLE II.

⁝ · ·
45499293 (357
27 Cube of 3.
1849 Reſolvend.
9 Triple of the Root 3.
27 Triple □ Root 3.
279 Diviſor.
125 Cube of the Root 5.
225 □ of the Root 5, by Triple Root 3.
135 Root 5, by Triple □ Root 3.
Sum 15875 Sub.
2624293 Reſolvend.
105 Triple Root 35.
3675 Triple □ Root 35.
36855 Diviſor.
343 Cube of 7.
5145 □ Root 7, by Triple Root 35.
25725 Triple □ Root 35, by Root 7, *viz.* 3675
2624297 Subt. from Reſolvend.
Rem. 0

In the like Manner the Cube Root of 9876543210 will be found to be 2145. and of 1603393328631365.740 to be 117043.1 *ferè*.

The Uſe of Cube Root.

Note, All Solids are Proportional, one to another, as the Cube of their Diameters.

Given, the Side of a Cube, to make another Cube, 2, 3 or 4 times greater or leſſer.

RULE.

1. To make it greater than the given Cube, multiply the Cube of the Side by the Quantity you intend it to be, and the Cube Root of the Product is the Side of the Cube required.

2. To

…an the given Cube, divide the Cube of the Side by any Quantity, and the Cube Root of the Quotient is the Side of the Cube ſought.

EXAMPLE.

Admit the Side of the Cube be 12 Inches, the Solidity of which is 1728, I would have three other Cubes that ſhall contain 2, 3, and 6 times greater and leſſer than the given Cube?

Operation.

Side 12 Inches.	1728	1728
12	3	6
144	5184(17.3	10368(21.8
12		
1728		
2		
3456(15.1 Cube Root.		

Anſwer the Sides of the greater Cubes are $\left\{\begin{array}{l}15.1\\17.3\\21.8\end{array}\right\}$ Inches.

3. Alſo the Sides of a Cube leſſer by 2, 3, and 6 times than the given one.

The Sides will be $\left\{\begin{array}{l}9.5\\8.3\\6.6\end{array}\right\}$ Inches.

4. To three Numbers given, to find a Fourth in a Duplicate Proportion.

DEFINITION.

The Nature of this Propoſition, is to diſcover the Proportion of Lines to Superficies, and Superficies to Lines, for like Planes are in a duplicate Ratio; that is as the Quadrat of their Homologal Sides.

RULE.

RULE.

Multiply the Square of the Third Term by the Second Term, and divide the Product by the Square of the Firſt Term, the Quotient is the Anſwer.

EXAMPLE.

Suppoſe 3, 4, and 5 be given, to find a Fourth in a Duplicate Ratio?

$$\begin{array}{ccccc} 3 & : & 4 & : & 5 \\ 3 & & & & 5 \\ \hline 9 & & & & 25 \\ & & & & 4 \\ \hline & & & 9) & 100(11\frac{1}{9} \end{array}$$

5. Three Numbers given, to find a Fourth in a Triplicate Proportion.

DEFINITION.

The Nature of this Propoſition is to diſcover the Proportion of Lines to Solids, and Solids to Lines: For like Solids are in a Triplicate Ratio, that is, to the Cubes of their Homologal Sides.

RULE.

Multiply the Cube of the Third Term by the Second Term, divide the Product by the Cube of the Firſt Term, the Quotient is the Anſwer; or which is all one, As the Cube of the Diameter of the given Bullet or Sphere, is to it's Weight or Solidity, ſo is the Cube of the Diameter of another Bullet, *&c.* to the Weight or Solidity thereof.

EXAMPLE.

Admit an Iron Bullet, whoſe Diameter is 6¼ Inches, do weigh 9¾ Pounds, what ſhall an Iron Bullet weigh, whoſe Diameter is 15½ Inches?

See the Work Decimally performed.

6.25	15.5
6.25	15.5
3125	775
1250	775
3750	155
39.0625	240.25
6.25	15.5
1953125	120125
781250	120125
7343750	24025

As 244.140625 : 9.75 *lib.* : : 3723.875

9.75

1861937 5

2606712 5

33514875

244.140625) 36307.78125 (148.71

Anſwer 148 *lib* : 11 *oz.* : : 5.76 *dr.*

6 Admit the ſolid Content of a Globe be 428307596 Inches, what is the Side of a Cube which ſhall be equal in Content?

Anſwer the Cube Root of the Globe 973.7 Inches is = to the Content.

7 Admit one of the Sides of a Cube, or the Diameter of a Globe to be 8$\frac{3}{4}$ Inches, and the Side of another Cube, or Diameter of another Globe be 11$\frac{1}{4}$ Inches, I demand the Side of another Cube, *&c.* that ſhall be equal in Content to them both?

RULE.

The Cube Root of the Sum of the Two Cubes is the Side of another Cube equal to them both, which in this Example will be found to be 12.79 Inches.

8. Given the Diameter of the Concaves of two Guns, and the Quantity of Powder that will Charge one of them

to find the Quantity of Powder that will ſuffice to Charge the other.

RULE.

As the Cube of the Diameter of the Gun given, is to the Quantity of Powder that will Load the ſame, ſo is the Cube of any other Gun's Diameter, to the Quantity of Powder that will Load the ſame.

EXAMPLE.

Admit a Cannon Royal whoſe Diameter at the Bore is 8 Inches, require 26 Pound of Powder to Charge it, what Quantity of Powder will charge a Piece whoſe Diameter is $1\frac{1}{2}$ Inch ?

Cube of 8 = 512, and Cube 1.5 = 3.375

Cube. *lib.* *Cube.* *lib.* *oz.* *dr.*
As 512 : 26 : : 3.375 : .171 = 2 11.776

9. Given, the Diameter of a Granado Shell, to find the Weight of the Metal, and the Content of the Concavity of the Shell in Cubic Inches.

RULE.

Firſt, Find the whole ſolid Content of the Shell, as if it was really ſo, which is done by multiplying the Cube of the Shell's Diameter by 11, and dividing the Product by 21, the Quotient is the ſolid Content in Cubic Inches of the Whole Shell; and alſo find the ſolid Content of the Concavity, which ſubtracted from the whole Content, gives the Content of the Metal, which divide by 4.422979 the Weight of a Cubic Inch of Common Iron *Averdupoiſe*, the Quotient are the Ounces, which divide by 16 gives Pounds.

EXAMPLE.

Admit a Granado Shell, whoſe Diameter from outſide to outſide is 12 Inches, and the Diameter within $7\frac{1}{2}$ Inches, ſo the thickneſs of the Metal is 2.25 Inches, I demand the Content of the Metal in ſolid Inches, and alſo it's Weight in Pound *Averdupoiſe*.

Operation.

Cube of 12 = 1728. Cube of 7.5 = 421.875

11 905.143 11

21) 19008 (220.982 Subt. 21)4640.625

4.422979)684.161000000(154.683

16) 154.683 (9.667

Anſwer 684.161 Solid Inches.
154.683 Ounces.
9.667 Pounds Weight.

10. **Given the Mould and Burthen of one Ship, to build another of the ſame Mould of any aſſign'd Burthen greater or leſſer.**

R U L E.

Firſt, The Dimenſions taken in a Ship are, the Length of the Keel, the Breadth on the Midſhip Beam, and the Depth in the Hold: Thereſore if the Cubes of theſe ſeveral Dimenſions be multiply'd by 2, 3, 4, *&c.* the Cube Roots of theſe ſeveral Products will be the Dimenſions of another Ship, whoſe Burthen will be 2, 3, 4, *&c.* times greater than the Ship given.

E X A M P L E.

Admit a Merchant's Ship, whoſe Burthen is 72 Tuns, the Length of the Keel be 45 Feet, the Breadth on the Midſhip Beam 17 Feet, and the Depth in the Hold 9 Feet, and it is required to build another Ship whoſe Burthen ſhall be five Times greater than the other Ship, I demand what muſt be the Dimenſions of the Ship required?

Anſwer Keel 76.9 Feet.
Breadth 29.0
Depth 15.3

XXI.

XXI. To Extract the Square Root of a Fraction, either Vulgar or Decimal.

1. FIRST, If the Fraction propounded be not in it's leaſt Terms, reduce it into it's leaſt, and then by the Rules foregoing, Extract the Square Root of the Numerator, and alſo of the Denominator, ſo ſhall this new Fraction be the Square Root of the Fraction propounded, ſo the Square Root of $\frac{49}{64}$ is $= \frac{7}{8}$.

Note, But many times the Numerator and Denominator of a Vulgar Fraction hath not a perfect Square Root, to find which infinitely near, reduce it to a Decimal, by adding Cyphers to the Numerator, and dividing by the Denominator, and always be ſure to let the Decimal have an even Number of Places, as 2, 4, 6, *&c.* and then extract it's Square Root as if it were a whole Number.

EXAMPLE.

Let $\frac{14}{25}$ be given, it is reduced to this Decimal .56 and by adding five Pair of Cyphers to it, I find it's Square to be .748331.

2. If your Vulgar Fraction be a mixt Number, reduce it into an improper Fraction, then work as before. But if the Fractional Part be not an exact Square Number, it is beſt to reduce it into a Decimal mixt Number.

As for Example.

If the Square Root of $6\frac{3}{4}$ be required, it will be $\frac{21}{4}$ and it's Root $\frac{5}{2} = 2\frac{1}{2}$, which is not exact; therefore it will be this Decimal, 6.75, and it's Root 2.598, nearer the Truth.

XXII.

XXII. *The Extraction of the Cube Root of a Vulgar Fraction.*

EXTRACT the Cube Root of the Numerator for a new Numerator, and the Cube Root of the Denominator for a new Denominator, and this new Fraction ſhall be the Cube Root of the given Fraction. So $\frac{27}{343}$ being given, it's Root will be found $\frac{3}{7}$.

If you want the Cube Root of a mixt Number, it is beſt to reduce it to a Decimal. As ſuppoſe $8\frac{4}{5}$ = Decimal 8.8 it's Cube Root is 2.08.

XXIII. *The Extraction of Roots by Logarithms.*

1. TO Extract the Square Root by Logarithms, is no more than to take the Logarithm of the given Number, and the Half thereof ſhall be the Logarithm of the Square Root ſought.

EXAMPLE.

Suppoſe 576 were a Square to be extracted, it's Logarithm is 2.7604225, the Half of which is 1.3802112 the Logarithm of 24, the Square Root ſought.

The Extraction of the Cube Root, by the Logarithms.

RULE.

Seek the Logarithm of the given Cube, and divide it by 3, and the Quotient is the Logarithm of the Cube Root ſought.

EXAMPLE

Admit it were required to extract the Cube Root of 421875, it's Logarithm is 5.6251839, one third of which is 1.8750613 the Logarithm of 75, the Cube Root ſought.

And after this Manner it is eaſy to extract the Biquadrate Root, the Surſolid, the Squared Cube, the Seventh, Eighth, or any other Power by the Logarithms, as I will ſhew by Example by and by.

That the Extracting all Sorts of Roots is ſometimes neceſſary, and that no Method is ſo eaſy as that by Logarithms, are Truths beyond all poſſibility of denial.

The only Thing that allays their Excellency, is that the largeſt Canons (as thoſe of *Sherwin*'s) are ſo ſcanty, that without ſome other farther Invention, it will not furniſh us with the Logarithm of the Cube of 343 = 40353607, but by the following Method the Root of any Power may be extracted infinitely near.

RULE.

Seek the Logarithm of 7 or 8 Places to the Left Hand of the Number given, and prefix to that Logarithm it's proper Index or Characteriſtic, anſwerable to the Number of Places in the whole given Number, that is, leſs by one, than the Number of Places of the abſolute Number, and divide it by the Index or Exponent of the Power whoſe Root you ſeek, the Quotient is the Logarithm of the Root ſought. That is, if you ſeek the Square Root, divide by 2; if the Cube Root, by 3; if the Biquadrate Root, divide by 4, *&c.* I ſhall here begin with the Root 2, and proceed to the Ninth Power, prefixing their Logarithms, by which the Learner may ſee by underſtanding this eaſy Proceſs, how to extract a greater Root, with as much facility as he can wiſh.

Given the Root	2	Logar.	0.3010300
Multiply by	2		
2d Square	4	-	0.6020600
	2		
3. Cube -	8	- -	0 9030900
	2		
4. Biquadrate	16	- -	1.2041200
	2		
5. Surſolid	32	- -	1.5051500
	2		
6. Square cubed	64		1.8061800
	2		
7th Power	128	-	2.1072100
	2		
8th Power	256	.	2.4082400
	2		
9th Power	512	-	2.7092700

And ſo on *ad infinitum.*

Here you ſee if the Logarithm of 2 = 0.3010300 be multiply'd by the 9th Power, it will produce the Logarithm of 512 = 2.7092700, conſequently if we divide the Logarithm of 512 by it's Exponent or 9th Power, we ſhall gain the Logarithm of it's Root 2.

EXAMPLE I.

Mr *Ward* has extracted the Biquadrate Root of 4857532416, and finds it to be 264, let us ſee how this agrees with our Method?

Hence, becauſe the abſolute Number has ten Places, the Characteriſtic of it's Logarithm muſt be 9. Then in *Sherwin*'s Logarithm, I find the Logarithm of the five leading Figures next to the Left Hand, *viz.* 48575 to be 4.6864128 with the Difference 89.4. which multiplied by the remaining Figures of the abſolute Number, *viz.* 32416, the Product 28.979404, then, becauſe of the great Fraction I call 28, 29, and add it to the Logarithm of 48575 = 4.6864128 makes .6844157 to which prefix it's proper Characteriſtic 9; I find the Logarithm of 4857532416 to be 9 6864157 which divided by it's Exponent, or Power 4, gives 2.4216039 the Logarithm of 264, the Root ſought.

After this Manner let us extract the Root of the 9th Power of this Number 4722366482869645213696? Here being twenty two Places, the Characteriſtic muſt be 21, and the Logarithm of it is 21.6741597, which divided by the Power, or Exponent 9, gives the Logarithm 2.4082399, whoſe abſolute Root is 256, the Root ſought.

But if you would have the Square Root of this vaſt Number of 22 Places, you cannot go to it immediately, for the Logarithm 21.6741597 being halfed, gives 10.83707985 for the Logarithm of the Square Root: And this having for it's Index 10, ſhews that the Number to which it belongs muſt needs conſiſt of 11 Places, which no Canon of Logarithms can come near.

But this ſeeming Difficulty is ſoon removed: For if you ſeek the Root of any higher Power, compound of ſuch a Power as in this Caſe is the Square ſquared, ſquared Cube, or ſquaredly ſquared Square (*See Geometrical Pro-*

greſſion, page 33) the Root primarly intended, is by Multiplication ſoon gained.

As for Example.

If you divide the Logarithm 21.6741597 by 4, the Exponent of the ſquared Square, or 4th Power, it gives 5.4185399 the Logarithm of 262144, and that being the ſquared Square Root, or the Square Root of the Square Root. Therefore ſquare this Root 262144 (that is multiply it in itſelf) it will produce 68719476736 the Square Root of 4722366482869645213696. Or if the ſaid Logarithm 21.674159 had been divided by 6, the Index or Exponent, the Quotient would have been 3.61235995, the Logarithm of 4096, which being cubed, will be 68719476736, the Square Root, becauſe the ſquared Cube Root, is but the Cubic Root of the Square Root.

XXIV.

I Shall here add three uſeful Problems concerning Intereſt and Annuities in the Logarithmetical Way.

PROBLEM I.

To find the Time wherein a given Sum of Money will be increaſed to any Sum required at any Rate of Compound Intereſt.

The RULE.

Subtract the Logarithm of the leſs Sum, out of the Logarithm of the greateſt, and from this Difference ſubtract the Logarithm of one Pound and it's Intereſt for a Year, if any thing now remain add Cyphers, and you will have the Quotient in Years, and Decimal Parts of a Year.

EXAMPLE.

EXAMPLE.

In what Time will 4*l.* be increaſed to 6.5155785*l.* at 5*l.* *per Cent. per Annum* ?

Operation.

Logarithm of $\begin{cases} 6.5155785 & \text{is } 0.8139530 \\ 4. & \text{is } 0.6020600 \end{cases}$

Logar. of 1.05 = .0211893)0.2118230(10
.211893.

Rem. 0

Anſwer 10 Years.

PROBLEM II.

To find the Time when a yearly Payment given will raiſe a required Stock at any Rate of Compound Intereſt.

The RULE.

Suppoſe the Yearly Payment to the Intereſt of a Sum of Money, and find by the Rule of Three a Correſpondent Principal; which being done, add that Principal to the Stock, and from the Logarithm of their Sum, ſubtract the Logarithm of the ſaid Principal; that difference divided by the Logarithm of the Rate, ſhew the Time.

EXAMPLE.

In what Time will 30*l.* *per Annum*, raiſe a Stock of 400*l.* at 5*l.* *per Cent.* Compound Intereſt ?

Operation.

If 5*l.* : 100*l.* : : 30*l.* : 600*l.*
Add 400

Sum 1000

Logarithm 1000 - 3.0000000
Logarithm 600 - 2.7781513
Logarithm 1.05 =.0211893)0.2218487(104.7
•• 12

Month 5.64
30

Days 19.20

Anſwer 10 *y.* : 5 *m.* : 19.2 *d.*

PROBLEM III.

To find the Time wherein a yearly Payment will pay off a Debt, at any Rate of Compound Intereſt.

The RULE.

As in the former Problem, find the Principal, anſwering the yearly Rent at the Intereſt propoſed, and ſubtract the Debt out of the Correſpondent Principal, and the Logarithm of the Difference, from the Logarithm of the Correſpondent Principal, this laſt difference, divide by the Logarithm of the Rate, ſhews the Time.

EXAMPLE.

In what Time will a yearly Rent of 58*l.* pay off a Debt of 740*l.* at 5*l. per Cent.* Compound Intereſt?

Operation.

If 5*l.* : 100*l.* : : 58*l.* : 1160*l.*
Debt Subtract - 740

420

Logar. of 1160 = 3.0644580
Logar. of 420 = 2.6232493
Logar. of 1.05 =.0211893)0.4412087(20.82
•• 12

9.84
30

25.20

Anſwer 20*y.* : 9 *m.* : 25.2 *d.*

XXV.

XXV. *Ceres and Virginum:*

I Shall conclude this Chapter with the Rule called *Ceres* and *Virginum*, which teaches only how to reſolve Merry or Sporting Queſtions, to exerciſe young Practitioners in Numbers.

Queſt. 1. A Lady's Caterer bought 10 Birds of two Sorts, *viz.* Turkeys and Geeſe, for 24*s*. the Turkeys coſt 4*s*. and the Geeſe 2*s*. a Piece, how many did he buy of each ?

RULE.

Multiply the whole Price by the leaſt Price, ſubtract the Product from the whole Price, the Remainder divide by the Difference between the two Prices, the Quotient is the Number of Turkeys, *viz.* $2 \times 4 = 8$*s*. and 8 Geeſe $\times$ 2*s*. $= 16 + 8 = 24$*s*.

Queſt. 2. Suppoſe 21 Perſons in Company, Men, Women, and Children, and amongſt them they ſpent 26*s*. and ſo that every Man ſpent 2*s*. every Woman 1*s*. and every Child 6*d*. how many were there of each ?

Anſwer	8 Men,	at 2*s*. each	16*s*.
	7 Women	1 -	7
	6 Children	6*d*.	3
	21		26

Or they may be

9 Men,	at 2*s*. each	18*s*.
4 Women	1	4
8 Children	6*d*.	4
21		26

Or they may be

7	Men, at 2*s*. each	14*s*.	
10	Women 1	10	
4	Children 6*d*.	2	
21		26	

Laſtly,

7¾	Men, at 2*s*. each	15*s*.	6*d*.
7¾	Women 1	7	9
5½	Children 6*d*.	2	9
21		26	0

Queſt. 3. Admit 900*l*. to be diſtributed according to the Will of a deceaſed Friend to thirty ſeveral Men, A, B, and C, ſo that each A may have 60*l*. each B 40*l*. and each C 20*l*. how many Men muſt there be of each Sort ?

Anſwer						
A	may be either	7,	6,	5,	4,	3.
B		1,	3,	5,	7,	9.
C		22,	21,	20,	19,	18.

30

Or			
2	A at 60*l*.	each	120
11	B at 40		440
17	C at 20		340
		Sum	900

Queſt. 4. Suppoſe 10 Perſons of ſeveral Countries, *Engliſh*, *Dutch*, *French*, and *Spaniards*, to pay a Debt of 1000*l*. ſo that every *Engliſhman* pays 50*l*. every *Dutchman* 130*l*. every *Frenchman* 70*l*. and every *Spaniard* 150*l*. how many are there of each Country ?

Anſwer				
4	*Spaniſh*,	at 150*l*. each		600*l*.
1	*Dutch*,	at 130	-	130
1	*French*,	at 70	-	70
4	*Engliſh*,	at 50	-	200
10				1000

Or thus,

2	*Spaniſh*, at	150*l*. each	300*l*.
3	*Dutch*, at	130	390
3	*French*, at	70	210
2	*Engliſh*, at	50	100
10			1000

Queſt. 5. Suppoſe you buy 12 Loaves of Bread for 12*d*. ſo that ſome might be Two Penny, ſome Penny, ſome Half Penny, and ſome Farthing Loaves, I demand how many there muſt be of each Sort?

Now, becauſe of 12 Loaves for 12*d*. the Mean Price is one, but one of the Particulars being alſo one, there ſhould be no Penny Loaves, becauſe there is no difference betwixt the Mean Price and one Penny. But it may be thus, *viz*. either,

4	Two Penny Loaves	=	8	Pence.
2	Penny Loaves	=	2	
2	Half-penny Loaves	=	1	
4	Farthing Loaves	=	1	
12			12	

Or elſe it may be,

3	Two Penny Loaves	=	6*d*.	0*q*.
4	Penny Loaves	=	4	0
3	Half-penny Loaves	=	1	2
2	Farthing Loaves	=	0	2
12			12	0

XXVI. *Of Decimal Arithmetic.*

DEFINITION.

THE Word *Decimal* is derived from *Decem*, Ten; this kind of Numbering ſuppoſeth the Integer to be divided into Ten equal Parts: As ſuppoſe a Pound, a Mile, a Year, or any other thing whatſoever; and as whole Numbers increaſe from the Right Hand towards the Left, in a Decuple Proportion from Unity, ſo Decimals decreaſe from Unity in the ſame Proportion from the Left Hands towards the Right, as is more evident from this Table.

Unites	1.	Unites, one Integer.
Primes	.1	One tenth Part.
Seconds	.01	One Hundred Part.
Thirds	.001	One Thouſand Part.
Fourths	.0001	One ten Thouſand Part.
Fifths	.00001	One hundred Thouſand Part.
Sixths	.000001	One Million Part.
Sevenths	.0000001	One ten Million Part.
Eighths	.00000001	One hundred Million Part.
Ninths	.000000001	One Thouſand Million Part.

Here you ſee Decimal Fractions are of ſeveral Denominations, as Primes, Seconds, Thirds, &c. and the Numerator is always wrote alone, and the Denominator of every Decimal is ſo many Cyphers with Unites annexed, as there are Decimal Places, thus, $\frac{25}{100}$, or $\frac{123}{1000}$, &c. and is produced by parting of an Unite into ten equal Parts, as this Table ſhews.

One	Unites	makes 10	Primes.
	Primes		Seconds.
	Seconds		Thirds.
	Thirds		Fourths.
	Fourths		Fifths.
	Fifths		Sixths.
	Sixths		Sevenths.
	Sevenths		Eighths.
	Eighth		Ninths.
	Ninth		Tenths.

I. *Of Reduction of Decimal Fractions.*

To reduce a Vulgar Fraction to a Decimal.

RULE.

Add to the Numerator of the Vulgar Fraction a competent Number of Cyphers, and divide that Sum by the

Denominator of the ſame Vulgar Fraction, the Quotient is the Decimal equivalent to the given Vulgar Fraction.

For as the Denominator of the given Vulgar Fraction is to it's Numerator, ſo is an Unite with ſo many Cyphers annexed as you intend your Decimal ſhall have Places to the Decimal required.

But here Note, That in every Quotient that ariſes in finding a Decimal Fraction, there muſt be as many decimal Places as you added Cyphers to the Numerator of the given Vulgar Fraction; for what the Work falls ſhort of that, muſt be ſupplied by placing Cyphers to the left Hand of the Decimal, as you will the better perceive by the following Examples.

Reduce $\frac{1}{4}$ to a Decimal?

4)100(.25

Reduce $\frac{1}{2}$ of any thing to a Decimal?

2)10(.5

Reduce $\frac{3}{4}$ to a Decimal?

4)300(.75

Reduce $\frac{2}{3}$ of $\frac{3}{5}$ of $\frac{5}{8}$ to a Decimal?
Firſt it is reduced to this ſingle Fraction $\frac{1}{4}$. *See page* 48.
Anſwer .25
Reduce $\frac{1}{4}$ of $\frac{1}{2}$ of $\frac{3}{4}$ to a Decimal?
It is this ſingle Fraction $\frac{3}{32}$

32)300000(.09375
. . . .

In finding the Decimal of Money, Weight, Meaſure, *&c.* you muſt reduce what you are ſeeking the Decimal of into the loweſt Name mentioned, and divide by what you deſign for the Integer reduced into the ſame Name, the Quotient thence ariſing is the Decimal ſought.

What's the Decimal of 1*s.* one Pound the Integer?

20)100(.05

What's the Decimal of 6*d.* one Pound the Integer?
It is this Vulgar Fraction $\frac{6}{12}=\frac{1}{2}$ of $\frac{1}{20}=\frac{1}{40}$

40)1000(.025

What's the Decimal of 11 *d.* one Pound the Integer?
It is this Vulgar Fraction $\frac{11}{12}$ of $\frac{1}{20}=\frac{11}{240}$

240)11.000000000(.045833333
• • • • • • • • •

What's the Decimal of 3*qrs.* one Pound the Integer?
It is this Compound Vulgar Fraction $\frac{3}{4}$ of $\frac{1}{12}$ of $\frac{1}{20}$.

960)3.000000(.003125
• • • •

After this Manner are the Decimal Tables calculated in Books treating of Decimals.

To find the Value of a Decimal Fraction in the known Parts of Money, Weight, Meaſure, Time, &c.

RULE.

Multiply the given Decimal by the Denominitive Parts of it, and cut off to the Right Hand of the Product, ſo many Places as the given Decimal has places, and thoſe on the Left Hand are the Value of the given Decimal.

Examples following.

What's the Value of this Decimal .748 of a Pound Sterling?

```
             .748
               20
            ------
Shillings  14.960
               12
            ------
Pence       11.52
                4
            ------
Farthings    2.08
```

Note, Cyphers on the Right Hand of any Decimal are of no Uſe.

What's the Value of .74415513 of a Year
Anſwer 271*d*, : 19*h*. : 15′ : 50″

A TABLE

A TABLE *of the Decimal Parts of a Foot.*

Incr.	Decimal.	Incr.	Decimal.	Incr.	Decimal.
¼	.0208333	¼	·3541666	¼	.6875
½	.0418666	⅓	·375	½	·708333
¾	.0625	¾	·3958333	¾	·7291666
1	.083333	5	·4166666	9	75
¼	.104166	¼	·4375	¼	·7508333
½	.125	½	·458333	½	·7916666
¾	.145833	¾	·479166	¾	.8125
2	.166666	6	·5	10	.833333
¼	·1875	¼	·520833	¼	.8541666
½	.208333	½	·541666	½	.875
¾	.229166	¾	·5625	¾	.895933
3	.25	7	·583333	11	.916666
¼	.270833	¼	.6041666	¼	·9375
½	.291666	½	.625	½	·958333
¾	·3125	¾	.6158333	¾	·979166
4	·333333	8	.6666666	12	. ——

ADDITION.

This is the very ſame with Addition of whole Numbers, in which you muſt obſerve to place Primes under Primes, Seconds under Seconds, Thirds under Thirds, *&c.* and caſt the Sum up all one as if it were Whole. And when you have done, ſeperate from the Total ſo many Decimals to the Right Hand, as the greateſt Decimal hath Places. As theſe Examples will better inform you.

Fractions to be added.

	.473	.870461
	.3506	.725
	.30751	.604318
	.29	.5
	.1806	.327
Sum	1.60171	3.026779

Mixt Numbers to be added.

	l.	*l.*	*s.*	*d.*	*q.*
	7.8104	7	16	2	1.984
	6 104	6	2	0	3.84
	5.0081	5	0	1	3.776
	4.9102	4	18	2	1.792
	3.20759	3	4	1	3.2864
	2.071	2	1	5	0.16
	1.8	1	16	0	0.
Sum	30.91129	30	18	2	2.8384

20

Shill. 18.22580

12

Pence 2.7096

4

Farthings 2.8384 The ſame as on the Right Hand.

Of SUBTRACTION.

This is the very ſame with that of whole Numbers, only remember to ſeperate to the Right Hand of the Remainder, ſo many Figures for Fractions, as there are Decimal Places in the greateſt Number, and to place Primes under Primes, *&c.* as in theſe Examples.

Fractions.

Fractions.	Mixt Numbers.
l.	Ells.
·57	641.
·379	·3765
.191	640.6235

In ſubtracting Integers and Decimals, obſerve the following Order.

	l.		*l.*	*s.*	*d.*	*q.*
Lent	1681.176	-	1681 :	3 :	6 :	0.96
Paid	946.754	-	946 :	15 :	0 :	3.84
Pounds	734.422		734 :	8 :	5 :	112
	20					
Shillings	8.44					
	12					
Pence	5.28					
	4					
Farthings	1.12					

Multiplication of Decimals.

This in all Reſpects is the ſame with whole Numbers, only when the Work is done, obſerve to cut off to the Right Hand as many Decimal Places as are in both the Factors, that is in the Multiplicand and Multiplier, and if the Product doth not produce ſo many Places, then you muſt ſupply that defect by placing Cyphers to the Left Hand of the Product. Here are three Caſes, which mark well.

Caſe 1. A Decimal Fraction multiply'd by a Decimal Fraction.

Multiplicand	·75	- - -	·685
Multiplier	·25		.125
	375		3425
	150		1370
Product	.1875		685
			·085625

Caſe 2. A Decimal Fraction multiply'd by a mix'd Number.

Multiply me	.943	and - -	.6104
By	7.35	by - -	29.18
	4715		48832
	2829		6104
	6601		54936
			12208
Product	6.93105		17.811472

Caſe 3. A decimal Fraction multiply'd by a whole Number.

Multiply me	.7306
By	5062.
	14612
	43836
	36530
Product	3698.2972

Contractions.

Let it be required to multiply 7.03215 by 4.06032, and to have only four decimal Places in the Product in the Firſt, three in the Second, two in the Third, and one in the Fourth.

Note, The Multiplier muſt be tranſpoſed in the Way of Working, and the Unit's Place of the Multiplier is ſet under that Place of the Multiplicand, which you deſign to retain in the Product.

See the Operations.

I.

7.03215
230604

281286
4219
21

28.5526

II.

7.03215
230604

28128
421

28.549

III.

7.03215
230604

2813
42

28.55

IV.

7.03215
230604

28.1

Note, You muſt always have Regard to the Increaſe of the two next Figures on the right Hand.

Queſtions in Multiplication of Decimals.

What will 3 *s.* 11 *d.* produce being ſquared, a Pound the Integer?

Anſwer 9 *d.* 0.81654 *q.*

Queſt. 2. What's the Square of 3 *s.* 11 *d.* a Shilling the Integer?

Anſwer 15 *s.* 4 *d.* 0.33 *q.*

Queſt. 3. What's the Square of 2 *l.* 11 *s.* 7 *d.* ¾ *q*, a Pound the Integer?

Anſwer 6 *l.* 13 *s.* 4 *d.* 1 *q.* 497.

Queſt 4. What will 1 *s.* 6. *d.* produce being ſquared, Pound the Integer?

Anſwer 1 *d.* 1 *q.* 4.

But if a Shilling be made the Integer, the Anſwer will be 2 *s.* 3 *d.*

Note, Queſtions of this Nature admit of infinite Anſwers according to what you make the Integer.

Diviſion of Decimals.

You are to proceed in this in every Reſpect as if they were whole Numbers, only when the Work is done you are to find out the Value of the Quotient, by ſubſtracting the Number of decimal Places in the Diviſor from the Number of Places in the Dividend, and the Remainder are the Number of Decimal Places to be prick'd off in the Quotient. Here are Nine Caſes, which take in the following Order.

Caſe 1. A whole Number divided by a whole Number.

283)154605.(546.307
• • •

Note, If any Thing remain, add a Cypher, and ſo on, for every Cypher you add, there will be a Decimal in the Quotient. Here are three Cyphers added.

Caſe 2. A whole Number divided by a mixt Number.

28.3) 154605.000(5463.07
• • •

Caſe 3. A whole Number divided by a decimal Fraction.

.283) 154605.000(5463.07.
• • •

Caſe 4. A mixt Number divided by a whole Nnmber,

283.)1546.05(5.46
• • •

Caſe 5. A mixt Number divided by a mixt Number.

28,3)1546.05(54.6
• • •

Caſe 6. A mixt Number divided by a decimal Fraction.

.283)1546.050(5463.
: · : :

Caſe 7. A decimal Fraction divided by a whole Numb.

283.).154605(.000546
· · ·

Caſe 8. A decimal Fraction divided by a mixt Number.

28.3).154605(.00546
· · ·

Caſe 9. A decimal Fraction divided by a decimal Fract.

.283).154605(.546
· · ·

Contractions.

Becauſe the Way of dividing Decimals often proves tedious, when it is required to continue the Diviſion till the Value of the Remainder be ſmall; for this Reaſon I ſay, the following Method was invented.

As many Decimals as you intend your Quotient ſhall contain, retain ſo many in your Dividend, and then begin to divide with the firſt Figure as in the common Way, and prick off one Place in the Diviſor to the right Hand, and begin with the next Figure, in the Quotient to multiply it by the Figure in the Diviſor which ſtand over the Dot, but do not ſet it down, but carry it's Increaſe to the next Figure, and ſet it down for the firſt Place of Work under the Dividend, proceed thus, by dotting under the Diviſor, by which Means you drop a Place every time, which will much ſhorten your Work.

EXAMPLE.

Let it be required to divide 31.41592 by 579.268 and to retain five decimal Places in the Dividend.

```
579.268)31.41592(.0542338
 · · · · ·  2899340
            -------
             245252
             231707
             ------
              13545
              11585
              -----
               1960
               1738
               ----
                222
                173
                ---
                 49
                 45
                 --
                  4
```

Example

EXAMPLE II.

Let it be required to divide 406.371082 by 617.26 and to retain three decimal Places in the Dividend ?

```
617.26)406 371|082(.65835
 . . ..  370356|
         ------|
          36015|
          30863|
          -----|
           5152|
           4937|
           ----|
            215|
            185|
            ---|
             30|
             30|
             --|
              0|
```

When you cannot have the Diviſor in the Dividend, drop a Figure in the Diviſor by dotting under it, and carry it's Increaſe to the next Product of the Diviſor, as for Example, Divide 21.83 by 9.706 here two Places of Decimals are retained.

```
9.706)21.83(.225
 . . .  1941|
        ----|
         242|
         194|
         ---|
          48|
          48|
          --|
           0|
```

Again,

```
97.06)218.3(2.225
          .

        1941|
         242|
         194|
         ---|
          48|
          48|
          --|
           0|
```

The

The Single Rule of Three Direct in Decimals.

OF Ordering the Numbers and ſtating the Queſtion is in all Reſpects as in whole Numbers, which we have taught in Chap. IV. therefore we ſhall proceed to give ſome Examples.

Queſt. 1. If 20 $\frac{1}{2}$ Yards, of Cloth coſt 36 *l.* 15 *s.* what will 64 $\frac{1}{4}$ Yards coſt.

In Decimals it will ſtand thus, a Pound the Integer.

```
      y.       l.        y.
If   20.5  :  36.75  : : 64.25
                         36.75
                        ------
                         32125
                        44975
                       38550
                       19275
                  ------------
          20.5)   2361.1875(115.179
                  . . . . . .
                  205
                  ---
                   311
                   205
                   ----
                    1061
                    1025
                    ----
                      368
                      205
                      ----
                      1637
                      1435
                      ----
                       2025
                       1845
                       ----
                        180
```

Remainder 180, to which add 3 Cyphers, and then there will remain 10. and the Quotient 115.179878 *l.* =115 *l.* 3 *s.* 7 *d.* 0.68288 *q.*

Queſt. 2. If 64.25 Yards of Cloth coſt 115 *l.* 3 *s.* 7 *d.* 0.68288 *q.* what will 20.5 Yards coſt.

Anſwer 36 *l.* 15 *s.*

Queſt. 3. If for 36 *l.* 15 *s.* I buy 20.5 Yards of Cloth, how many Yards may I buy for 115 *l.* 3 *s.* 7 *d.* 0.68288 *q.*

Anſwer 64.25 Yards.

Queſt. 4. If 115 *l.* 3 *s.* 7 *d.* 0.68288 *q.* will buy 64.25 Yards of Cloth, how many Yards may I buy for 36.75 *l.*

Anſwer 20.5 Yards.

And thus you ſee how every Queſtion may be varied four ſeveral Ways, and one is a ſure Proof of another.

Queſt. 5. If a Labourer earn 2 *s.* $1\frac{1}{2}$ *d. per* Day, what doth that come to in the Year? a Pound the Integer.

State it thus.

	D.		*L.*		*D.*		*l.*	*s.*	*d.*
If	1.	:	.1062499	::	365	:	38	15	$7\frac{1}{2}$

But make a Shilling the Integer it will ſtand thus.

If 1 *d.* : 2.125 *s.* :: 365 *d.*

Of Duodecimal Arithmetic.

THIS is what is called Croſs Multiplication amongſt Workmen and Artificers; they generally caſt up all their meaſured Work by this Way, and therefore it claims a Place in this Treatiſe.

Firſt then, Feet multiply'd by Feet produce Feet.
2. Feet by Inches, and divided by 12 are Feet.
3. Feet by Parts, and divided by 12 are Inches.
4. Inches by Feet, and divided by 12 are Feet.
5. Inches by Inches, and divided by 12 are Inches.
6. Inches by Parts, and divided by 12 are Parts.
7. Parts by Feet, and divided by 12 are Inches.
8. Parts by Inches, and divided by 12 are Parts.

9. Parts by Parts, and divided by 12 are Seconds. That is a Name next leſs than Parts: As in this Table.

Factors.	Feet.	Inches.	Parts.
Feet.	Feet	are Inches ÷ by 12 are Feet.	are Parts ÷ by 12 are Inches.
Inches.	are Inches ÷ by 12 are Feet.	are Parts ÷ by 12 are Inches.	are Secon. ÷ by 12 are Parts.
Parts.	are Parts ÷ by 12 are Inches.	are Secon. ÷ by 12 are Parts.	are Thirds ÷ by 12 are Secon.

EXAMPLE,

Let 5 Feet, 3 Inches and 6 Parts, be multiplied by 2 Feet, 4 Inches and 6 Parts, what will be produced?

See the Work.

	F.	I.	P.	
Multiply	5	3	6	
By	2	4	6	
	10	6	0	
	0	1	0	
	1	8	0	
	0	1	2	
	0	2	6	
			1	6
				3
Product	12	6	9	9

In adding the ſeveral Products together, you muſt ſet down all above 12, and carry the Twelves to the next Denomination.

More Examples.

	F.	I.	P.
Multiply	7	8	0
By	0	9	3
	5	3	0
		6	0
		1	9
			2
Product	5	10	11

	F.	I.	P.	
	0	11	9	
	0	5	0	
	0	4	7	
	0	0	3	9
Prod.	0	4	10	9

	F.	I.	P.		
Multiply me	49	11	9		
By	28	5	3		
	1372	0	0		
	25	8	0		
	1	9	0		
	20	5	0		
		4	7		
			3	9	
			3	0	
	1	0	3	0	
			2	9	
				2	3
Product	1421	3	4	8	3

The ſame performed decimally from the Table, Page 109.

49.97916
28.4375
24989580
34985412
14993748
19991664
39983328
9995832

Feet 1421.282362500

Feet

Feet	1421.282362500
	12
Inches	3.3883500
	12
Parts	4.66020
	12
Seconds	7.9224
	12
Thirds	11.0688

This Kind of Multiplication may be rationally performed by beginning to multiply by the leaſt Denomination towards the right Hand, and ſetting down all above 12, carry the Twelves to the next Figure, and add them all up by 12 except the Feet, which muſt be done by Tens, as in this Example.

					F.	I.	P.	S.
Multiply me					9	10	3	9
By					8	11	9	6
				4	11	1	10	6
			7	4	8	9	9	
		9	0	5	5	3		
	7	8	10	6	0			

Anſwer 88 : 6 : 9 : 1 : 2 : 7 : 6

That is 88 Feet, 6 Inches and 9 Parts, *&c.* of an Inch, the other Figures being of ſo ſmall a Value that they are not worth mentioning.

CHAP. II.

Of GEOMETRY.

DEFINITION.

GEOMETRY Originally ſignifies the Art of meaſuring the Earth, or any Diſtances or Dimenſions on or within it. But 'tis now uſed for the Science of Quantity, or Extenſion, Magnitude, abſtractly conſidered, without any regard to Matter.

Geometry very probably had it's firſt riſe in *Egypt*, where the *Nile* annually overflowing the Country, and covering it with Mud, obliged Men to diſtinguiſh their Lands one from another, by the Conſideration of their Figure; and to be able alſo to meaſure the Quantity of it, and to know how to Plot it, and to lay it out again in it's juſt Dimenſions, Figure and Proportion: After which 'tis likely a farther Contemplation of thoſe Draughts and Figures. help'd them to diſcover many excellent and wonderful Properties belonging to them, which Speculation continually was improving, and is at this very Day. Before I proceed, I ſhall firſt explain ſome Terms.

1. *Axiom*, is a Principal in any Art, ſo evident, that it needs nothing but the Light of Reaſon to demonſtrate it.

2. *Conſtruction*, isthe drawing of Lines and framing of Figures, on preparing the Propoſition for a Demonſtration.

3. *Corollary*, is a conſequent Truth gained from preceding Demonſtration.

4. *Definition*, is the unfolding, or explicating of the Nature and Affection of a Thing in a few Words.

5. *Demonſtration*, is the proving of a Thing by Definitions and Axioms, and ſo from ſeveral Arguments drawing a Concluſion, that it has that Affection the Propoſition did aſſert.

6. *Hypotheſis*, is when a Thing is ſuppoſed, or given to be ſo.

7. *Lemma*, is the Demonſtration of ſome Premiſe, in order to ſhorten a following Demonſtration.

8. *Problem*, is when ſomething is propoſed to be done.

9. *Propoſition*, is uſed promiſcuouſly, either for a Problem or Theorem.

10. *Poſtulata*, is a grantable Requeſt, or ſuch a Demand as reaſonably cannot be denied.

11. *Scholium*, is a ſhort critical Expoſition, gained from a former Demonſtration, or a Corollary, wanting an Explanation.

12. *Theorem*, is when ſomething is propoſed to be demonſtrated.

II. *Of Geometrical Definitions.*

1. A Point is that which hath no Parts, and is the very Beginning of Magnitude.

2. A Line is generated by the Motion of a Point, and is ſuppoſed to have Length without Breadth or Thickneſs.

3. A plain Superficies is generated by the Motion of a right Line, and has Length and Breadth only.

4. *Convex Superficies*, is generated by the Motion of a Curve, or crooked Line, and is well repreſented by the outſide of a Bowl, or Baſon.

5. A *Concave Superficies*, is alſo generated by the Motion of a Curve, or crooked Line, and truly repreſented by the Concavity of the Heavens, or by the inſide of a Bowl.

6. A *Wavey Superficies*, may be repreſented by the Surface of the Sea in Rough Weather.

7. An Arch, or part of a Circle, flows from a Point deſcribed on ſome particular Centre, and is uſed in meaſuring Angles.

8. A plain or right lin'd Angle, is made by the meeting of two right Lines in a Point; and if one of the Lines be continued beyond the Point where the other meets it, there will then be two Angles formed: But if both the

Lines are continued beyond the Point, where they firſt touched they will then form four Angles.

9. Angles may alſo be either Acute Right, or Obtuſe; that is in Quantity leſs than 90, juſt 90, or more than 90 Degrees.

10. A Triangle is a Figure made by three Lines, and if right Lines, 'tis called a right lin'd Triangle: And in reſpect of the Angles, may be either Acute, Right, or Obtuſe.

11. Every plain Triangle may be conſidered, with relation either to it's Sides, or to it's Angles; as to it's Sides, it may be either Equilateral, Iſoſceles, or Scalenous.

12. An Epuilateral Triangle, is that which has all it's Sides equal to one another, every Angle of which is 60 Degrees.

13. Iſoſceles, Equicural, or equal leg'd Triangle, is that which hath only two Sides equal: The Angle at the Baſe are equal.

14. A Scalenous Triangle is that which has no two Sides equal.

15. A Spheric Angle is made by the Interſection of two Curve Lines, or by the meeting of two Arches of two great Circles of the Sphere.

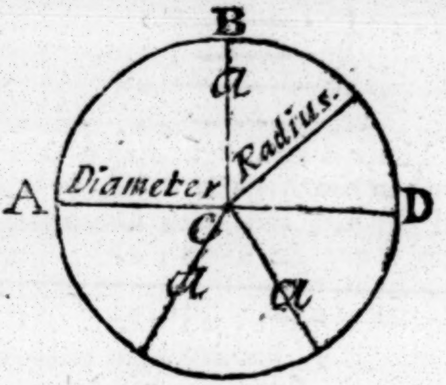
16. An Oblique Spheric Angle is made by the meeting of two Arches of two great Circles of the Sphere, and contains more than 90°, or a Quadrant.

17. A mix'd Angle is when one Leg is ſtreight and the other curved, or crooked, and this you often meet with in the Stereographic Projection of the Sphere, that is, when the right Line cuts the Concave Side of the Arch. But if the Angle be made on the Convex Side of the Arch, being a Tangent to that Arch, then it may be called the Angle of Contact, which is leſs equal and greater than the Truth, as *Euclid* demonſtrates in the 16th Propoſition of his third Book; and conſequently the Angle of Contact is not any known Quantity.

18. A Spheric Triangle is made by the Interſection of three great Circles of the Sphere, and the Sum of the three Angles of a Spheric Triangle is greater than two right Angles.

19. A Circle hath one Line which encloſes it's Space, which is made by the Motion of a Point round ſome other

Point, at ſome certain diſtance, and this curved Line when moved round is called the Circumference, or Periphery, and the Point exactly in the middle on which it was deſcribed is called the Centre, from whence all Lines drawn to the Circumference are equal, and are called Radius, whole Sine or Semidiameter, whoſe double is the Diameter or Chord of 180°. Conſequently all Circles have in them 360 Degrees.



20. A Semicircle, is half the Whole Circle, being contained betwixt the Diameter, and that Part of the Circumference cut off by it. From which an Inſtrument in Surveying takes it's Name. It has in it 180 Degrees, as A B D.

21. A Quadrant is exact one Quarter of a Circle, or 90 Degrees, being bounded by a fourth Part of the Circumference, and two Semidiameters cutting each other at a right Angle in the Center as A C B.

22. A Segment of a Circle is a Part cut off by a right Line, (leſs than the Diameter) drawn within the Circle, repreſented by the Chord or right Line A *d* in the following Figure, and this Segment may be greater than a Semicircle, as A I *d*, or leſſer, as A *b c d*.

23. A Sector is formed by Part of a Circle, and two Semidiameters drawn from the Center to the Circumference, it's Area is equal to the Superficial Content of a right Cone.

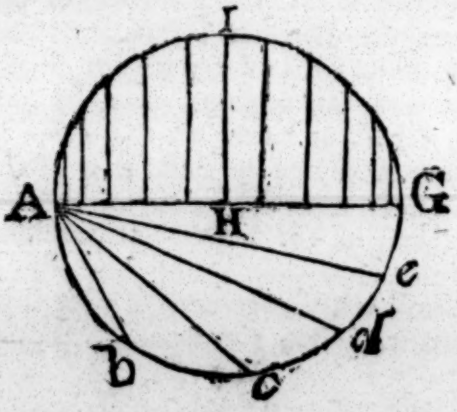
24. Every Circle, whether great or ſmall is ſuppoſed to be divided Mathematically into 360 equal Parts called Degrees.

and every Degree into 60 equal Parts called Minutes, every Minute into 60 other equal Parts, called Seconds, *&c.* the Reaſon of dividing the Circle into that Number is, becauſe it will admit of more equal Parts than any other Number, as by the Work is plain.

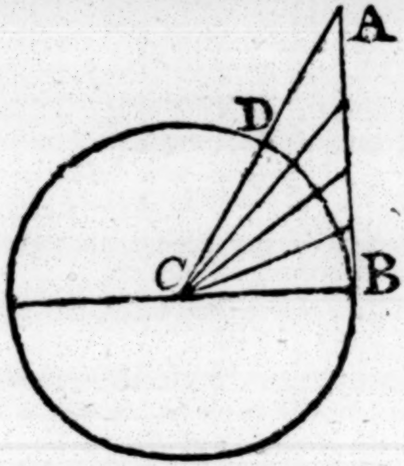
2) 360 (180 Degrees.
3)———(120
4)———(90
5)———(72
6)———(60
8)———(45
9)———(40
10)———(36

25. The Meaſure of a right Angle is an Arch of a Circle deſcribed on the Angular Point, by the Sweep or Chord of 60 Degrees, and lieth intercepted betwixt the two Sides that forms the Angle ; and as many Degrees as there are in the Arch, ſo much is the Meaſure of the Angle, when that opening is applied to the Line of Chords, which is the moſt expeditious, but any right lin'd Angle may be meaſured by it's Sine, Tangent, or Secant.

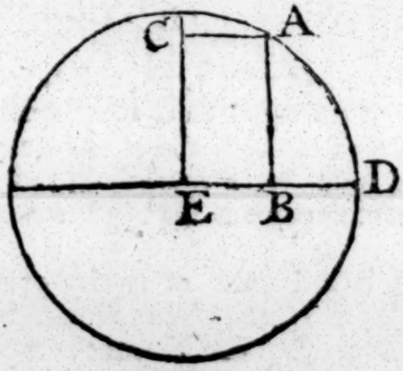
26. The right, or natural Sine of an Arch or Angle, is a Line perpendicular to the Diameter and continued till it touch the Periphery of the Circle ; as H I is the Sine of the Quadrant, A I = G I, and likewiſe the other Perpendiculars are Sines of their reſpective Arches, *&c.* The Chord of an Arch is like the String of a Bow, and leſs than the Diameter, (for the Diameter is the Chord of 180°) being drawn to touch the Circle in two Points any where, as A *b*, A *c*, A *d*, A *e*, are all Chords of their reſpective Arches.



27. The Tangent of an Arch, is a Line perpendicular to the Diameter, and touching the Circumference of the Circle, meeting with a Line iſſuing from the Center, called the Secant: So that A B is the Tangent, and A C is the Secant of the Arch D B, and ſo of the others in this Figure, where the Secant cuts the Circle and meets with the Tangent, is determined the Tangent and Secant of that Arch.



28. The Complement of an Arch or Angle is what the Angle wants of a Quadrant, or of a Semicircle; as ſuppoſe you have the Sine of 60=to A B, then it's Complement to a Quadrant is A C, and B D is the verſed Sine: Now 'tis plain from the Figure that the verſed Sine and Coſine, or Sine Complement, are always equal to Radius, for C A = E B, and E B + B D = E D the Radius.



N. B. The Chord, Sine, Tangent and Secant of every Arch leſs than 90, is alſo the Chord, Sine, Tangent and Secant of ſo much more than 90 as it's Complement is ſhort of 90 Degrees.

29. A Geometric Square, has four equal Sides, and the Angles all right, or 90 Degrees.

30. A Parallelogram, oblong or long Square, has it's oppoſite Sides parallel and equal and the Angles right.

31. A Rhombus has all it's Sides equal, and the oppoſite Angles equal, but none of them right, two being acute and two obtuſe.

32. A Rhomboides, hath it's oppoſite Sides and oppoſite Angles equal, but not right angled.

33. All four-ſided Figures whoſe Sides are not equal, are called Trapeziums, and it's Angles are alſo unequal.

34. The Diagonal of any Figure is a right Line drawn from the oppoſite Angles.

35. If Figures have above four Sides, and thoſe unequal, they are called irregular Polygons.

36. Regular Polygons are thoſe that have more than four Sides, in which the Sides and Angles are equal; ſo that if it has five equal Sides and Angles it is called a Pentagon, every Angle of the Circumference is 108 Degrees, and at the Center 72°.

A Hexagon has ſix equal Sides and Angles, every Angle at the Circumference is 120 Degrees, and at the Center 60 Degrees.

A Heptagon has ſeven equal Sides and Angles, the Angle at the Circumference is 128.5714° and at the Center 51.42857°.

An Octagon hath eight equal Sides and Angles, every Angle at the Circumference is 135°, and at the Center 45°.

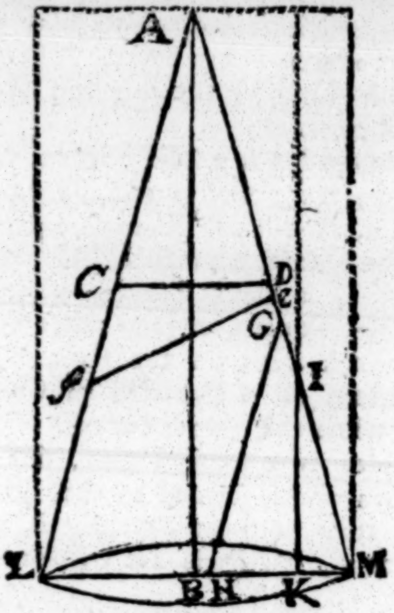
A Nonagon hath nine equal Sides and Angles, every Angle at the Circumference is 140°, and at the Center 40°.

A Decagon hath ten equal Sides and Angles, every Angle at the Circumference is 144°, and at the Center 36°.

An Undecagon hath eleven equal Sides and Angles, every Angle at the Circumference is 147.27° and at the Center 32.72 Degrees.

A Dodecagon is a Figure of twelve equal Sides and Angles; every Angle at the Circumference is 150, and at the Center 30 Degrees. From hence it is plain, that all regular Polygons derive their Name from their Number of Sides, be they ever ſo many, ſo that 'tis needleſs to mention any more of them.

37. A Cone is a ſolid Figure, riſing from a Circle by the Rotation of a Triangle round one of it's Sides and terminates in a Point at the Top, called it's Vertex; a Cone is equal to one third of it's circumſcribing Cylinder, as is plain from the Figure.



Every Cone hath five Sections. (1.) If the Cone be cut by a Plain thro' it's Axis A B, that Section will be a Triangle. (2.) If the Cone be cut any where by a Line parallel to the Baſe, as C D, this Section will be a Circle. (3.) If the Cone be cut in any oblique Poſition, as f *e*, this Section will be an Ellipſis. (4.) If the Cone be cut by a Line as G H, parallel to one of the Sides, this Section will be a Parabola. (5.) If the Cone be cut by a Plain as I K, parallel to the Axis A B, this Section will be an Hyperbola.

Note, That every Triangle is half it's circumſcribing Parallelogram, the Diameter of every Circle, is the Side of that Square that circumſcribes it, which is double to that Square inſcribed in the ſame Circle.

The Ellipſis is a Geometrical Mean between two Circles deſcribed on the Tranſverſe and Conjugate Diameters. The Parabola is $\frac{2}{3}$ of it's circumſcribing Parallelogram. The Hyperbola, is $\frac{5}{12}$ of it's circumſcribing Parallelogram.

38. There are three Kinds of Magnitudes, *viz.* Length, Breadth, and Thickneſs ; that is to ſay, a Line, a Superficies, and a Solid.

A Line being generated by the Motion of a Point, a Superficies by the Motion of a Line, and a Solid by the Motion of a Superficies ; and which Way ſoever a ſolid is moved, it ſtill produces but a Solid. Therefore a Solid is that which hath Length, Breadth and Depth, and it's Bounderies are Superficies.

39. A Parallepipedon is a ſolid Figure, contained under ſix Parallelograms, the oppoſite which are parallel.

40. A Priſm, is a ſolid Figure, contained under ſeveral Plains, whoſe Baſes are regular Polygons.

41. A Pyramid is a ſolid Figure, whoſe Baſe may be either a Triangle, Square, or any other Polygon, whoſe ſeveral Planes meet in a Point at the Top.

42. The Fruſtum of a Pyramid or Cone, is only the remaining Part when the Top is cut off.

43. A Cylinder is a ſolid Figure, like the Rowling Stone of a Garden, and is formed by the right Line moving round parallel to itſelf, and conſequently is in all Places of the ſame Diameter, it's Ends are Circles and parallel to each other.

44. A Spheroid is a ſolid Figure, made by the Motion of a Semi-Ellipſis, the tranſverſe Diameter remaining fix'd.

45. A Sphere or Globe, is a Solid, perfectly round every where, produced by the Rotation of a Semi-circle, the Diameter remaining fix'd.

46. The Segment of a Sphere, or Globe, is a Piece cut off leſs than half the Globe ; and the greater Part of it is called a Fruſtum

47. A Parabolic Spindle, is a Solid, formed by the Rotation of an acute Parabola, about it's greateſt Ordinate, remaining fixed.

48. A Parabolic Conoid, is made by the Rotation of a Semi-parabola, the Abſciſſa remaining fixed.

49. An Hyperbolic Conoid, is generated by the Motion of a Semi-Hyperbola, the tranſverſe Diameter remaining fix'd.

50. The Tetraedron, is one of the Pythagorian, or Platonic Bodies, and is a Solid Figure, contained under four equal and equilateral Triangles, which may be deem'd a little Pyramid.

51. The Hexahedron, or Cube, is another of the Platanick Bodies, and is a ſolid Figure, contained under ſix equal Sides, or Geometric Squares.

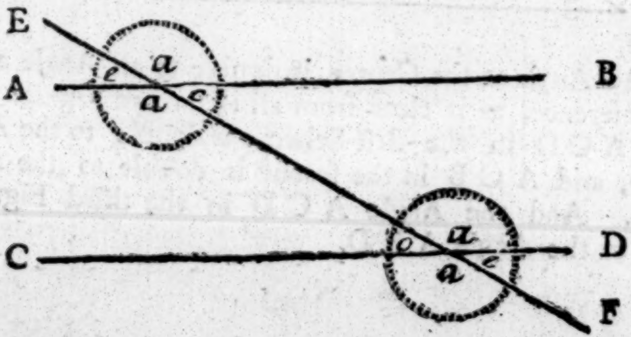
52. The Octahedron is another of the Platonic Bodies, and is a Solid, contained under eight equal and equilateral Triangles.

53 The Dodecahedron is another of the Platonic Bodies, and is a Solid, contained under twelve equal equilateral, and equiangular Pentagons.

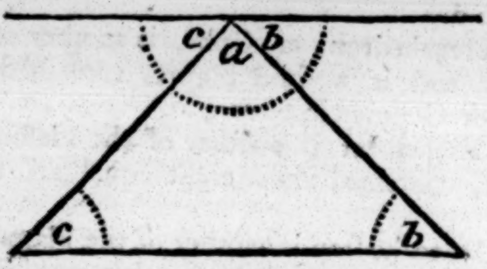
54. An Icoſahedron, is alſo one of the Pythagorian Bodies, and is a ſolid Figure, contained under twenty equal equilateral Triangles.

III. *Of Geometrical Theorems.*

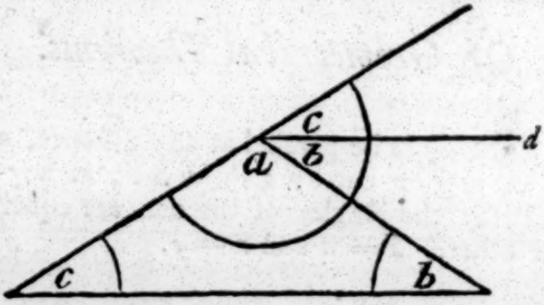
1. IF two right Lines be parallel at any diſtance, as A B and C D, and be cut by a third E F, at any Angle whatſoever, I ſay the Angles of the one are equal to the Angles of the other. $a = a$ and $e = e$.



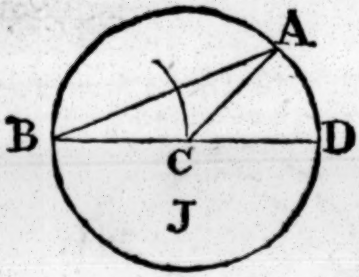
2. A right lin'd Triangle, made between two parallel Lines, the internal Angle is equal to the external; and that all the three Angles taken together are equal to a Semicircle. *b* = *b*, and *c* = *c*, and that *a* is = to Complement *b* and *c*.

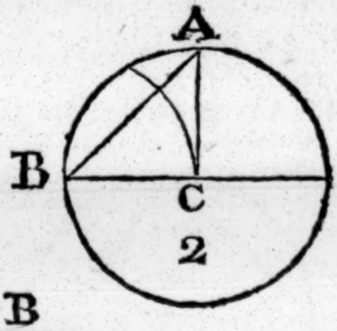


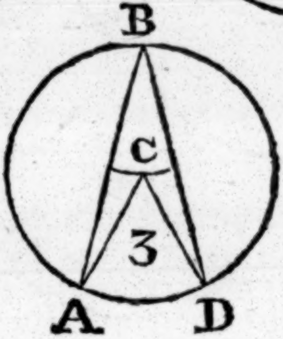
3. Any ſide of a right lin'd Triangle, being produced, the external Angle is equal to the Sum of the two acute internal Angles. And further, if we draw a Line parallel to the Baſe as *a d*, it will divide the external Angle into two other Angles, which will be in Quantity equal to the two internal Angles ſeperately.



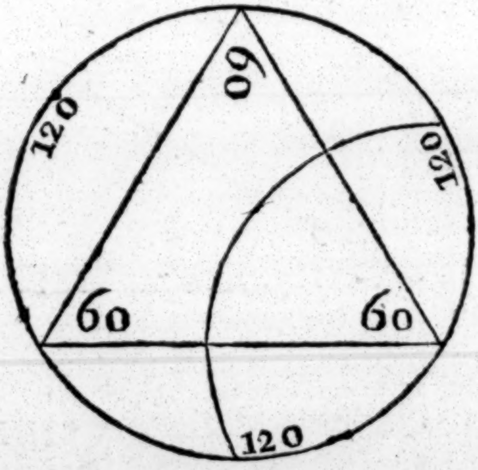
4: An Angle at the Centre, is double to an Angle at the Circumference, as is clear from all the three Figures; the Angle A C D in the firſt Scheme is double to the Angle A B D, and A C B in the ſecond is double to the Angle A B C. And the Angle A C D in the third Figure is double to the Angle A B D.







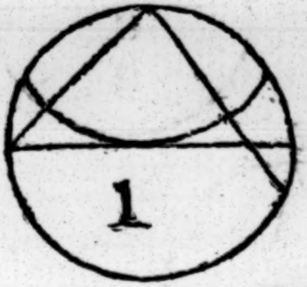
5. Every Equilateral Triangle inſcribed in a Circle, the internal Angles in the Triangle are equal to a Semicircle, and the external to the whole Circle, or 360 Degrees, as per Figure. For 60 × 3 = 180. And 120 × 3 = 360.

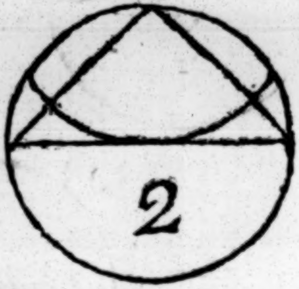


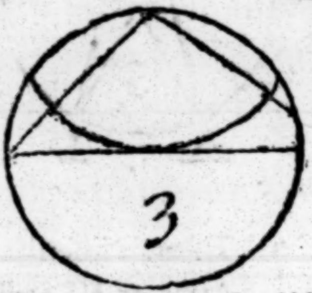
6. An Angle in an Arch greater than a Semicircle, is leſſer than a Quadrant, or 90°. *Euclid* 31, 3. As in Fig. 1.

7. An Angle in a Semicircle is equal to a Quadrant, or 90, as in Fig. 2.

8. An Angle in an Arch leſſer than a Semicircle is greater than a Quadrant or 90 Degrees, as in Fig. 3.

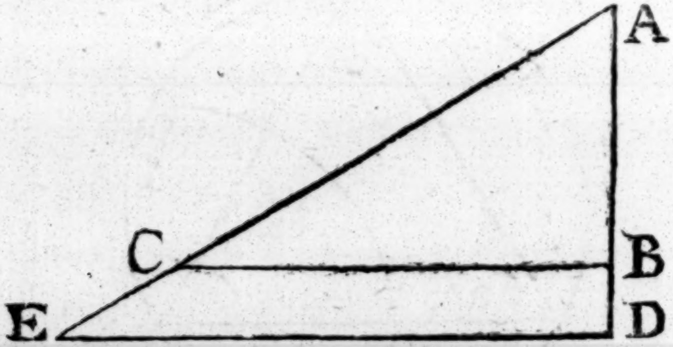






9. Equiangled Triangles have the Sides about the equal Angles proportional. For as,

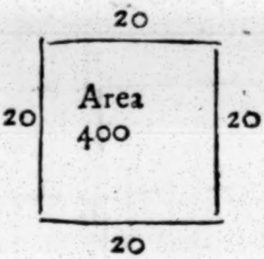
A B : A D :: B C : D E, or as A B : B C :: A D : D E.

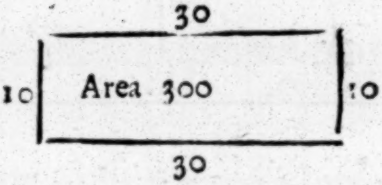


This is uſeful in Mercators Sailing.

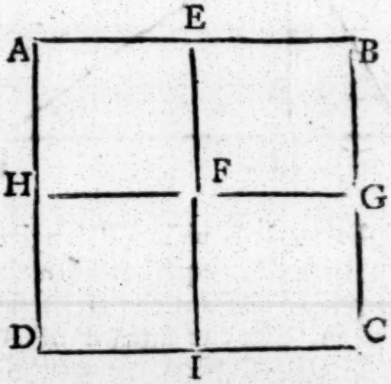
10. The Area of ſuperficial Figures are proportional at the Square of their Sides that bound them.

For in theſe two Figures the Area of the Square, exceeds that of the Parallelogram, proportional to the Square of their Sides; although they be 80 each all round, yet the Area of the Square exceeds the Area of the Oblong by 100.

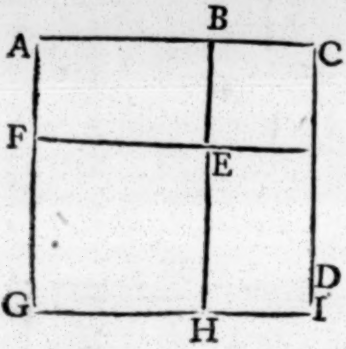




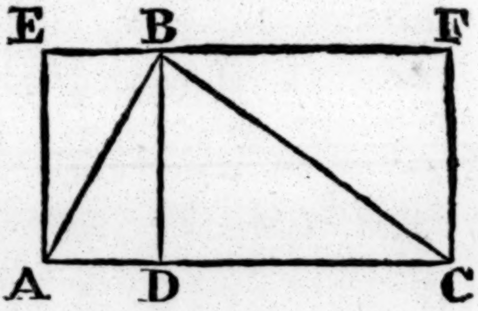
11. When a right Line is divided into two equal Parts, the Square made of thoſe Parts are equal but to $\frac{1}{4}$ of the Square made of the whole Line A B, whoſe Square is = A B C D. And the Square of A E, is = A E F H, one fourth Part of the great Square. But the Squares made of A E and E B, are equal to half the great Square.



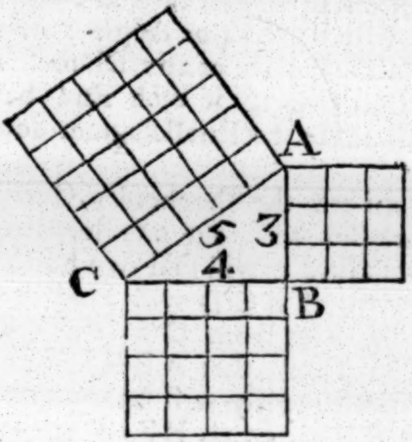
12. When a Line is divided by chance into two unequal Parts, the Square of the whole Line is equal to both the Squares made of the Parts, and to two Parallelograms comprehended under the ſame Parts alſo: For the Square B C D E, and the Square F E H G, with the Parallelogram A B E F, and E D I H are equal to the Square A C I G.



13. Every Triangle is half the Square or Oblong that circumſcribes it. In the Triangle A B C, let fall the Perpendicular B D. Then 'tis plain from the Figure that A B C is half of A E F C.

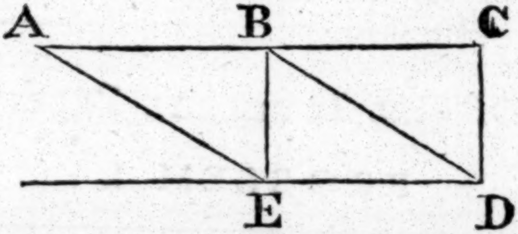


14. In all right angled plain Triangles, the Square made of the Hypothenuſe, is equal to both the Squares made of the two Legs which contain the right Angle. For by the Figure it is plain, that the Square of A B $3=9$, and the Square of B C $4=16+9=25=$ the Square of A C.



15. The diagonal Line of Every geometric Square, is double in Power to the Side of the ſame Square.

16. Parallelograms, which ſtand on the ſame Baſe, or on equal Baſes, and in the ſame Parallels, are equal one to the other. For B C D E = A B D E.



17. Every Polygon, is equal to a Parallelogram, whoſe Length is equal to half the Perimeter or Circumference thereof, and Breadth to a Perpendicular drawn from the Centre to the middle of any Side of the ſame. *Euclid* 41, 1.

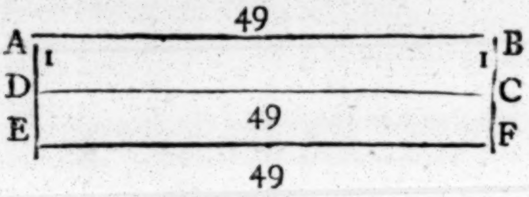
18. Every Circle is nearly equal to a Parallelogram, whoſe Length is equal to half the Circumference, and Breadth to the Semidiameter.

19. Every Circle's Sector is nearly equal to a Parallelogram, whoſe Length is equal to the Semidiameter of the

Circle, and Breadth to half the Curve or Arch-line thereof. Or whoſe Length is equal to half the Semi-diameter, and Breadth of the whole Curve or Angle-Line thereof.

20. If any Parallelogram of any Dimenſions whatſoever have for it's End Unity, if each End be increaſed by Unity, that will make the Parallelogram double to what it was before.

From hence ariſeth the Shepherd's Queſtion : *viz.* If an hundred Hurdles will fold one hundred Sheep, one hundred and two Hurdles will fold two hundred Sheep. For if the Parallelogram A B C D will hold 100 Sheep, take another Hurdle and ſet from C to F, and another from D to E, 'tis then plain, that A B F E is twice as big as A B C D, and conſequently will hold double the Number of Sheep. For $AB = 49 + EF = 49 = 98. + BF\ 2, + AE\ 2 = 4 + 98 = 102$ Hurdles.



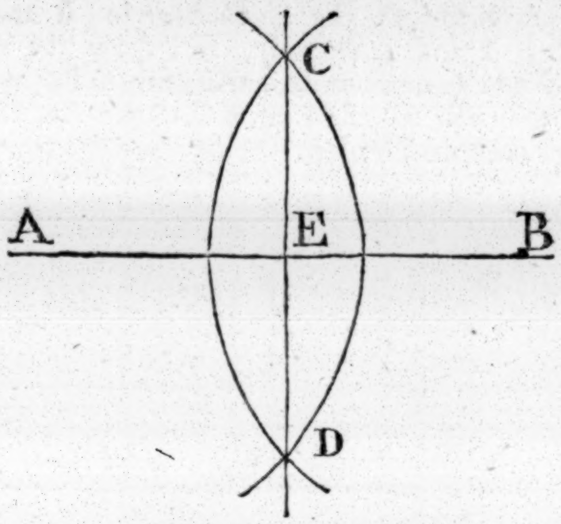
IV. *Of* GEOMETRICAL PROBLEMS.

PROBLEM I.

To divide a given Line into two equal Parts.

LET the Line given be A B, ſet one Foot of the Compaſſes in A, and open the other Foot to any Diſtance above half the Line, and ſweep the Arch C D, with that Extent or Opening of the Compaſſes, ſet one Foot at B and draw the Arch C D, lay a Ruler to the Interſection C D, and draw the right Line C D E and it will divide the given Line A B in E, into two equal Parts.

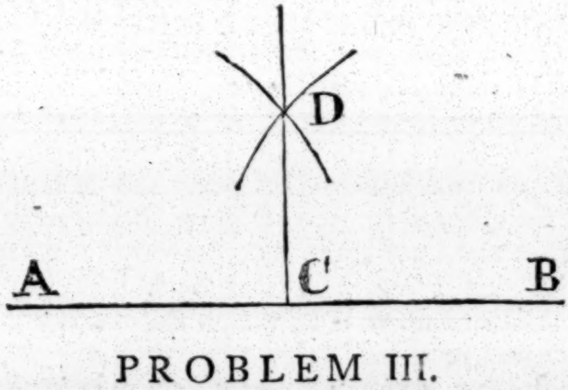
PROBLEM



PROBLEM II.

To erect a Perpendicular at the middle of a given right Line.

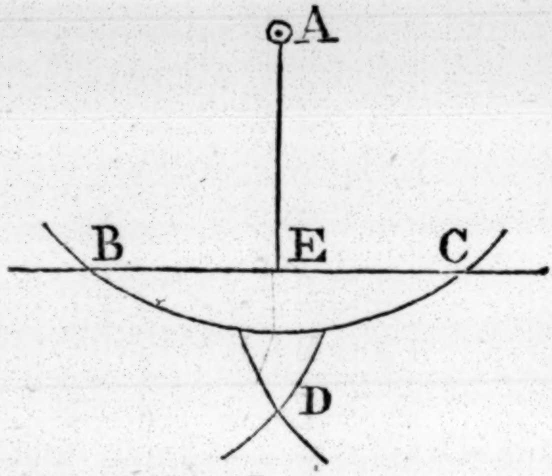
By the laſt Problem find the middle of the Line A B at C, ſet one Foot of the Compaſſes at A, and draw the Arch D, with the ſame Extent ſet one Foot in B, and draw another Arch at D, where theſe two Arches interſect at D lay a Ruler, and draw the Line D C, and 'tis done.



PROBLEM III.

To let fall a Perpendicular upon the Middle of a given Line.

Let the Point above be A, ſet one Foot of the Compaſſes in the given Point and open the other Foot and draw the Arch to cut the given Line in B and C, ſet one Foot in B and C ſeverally, and draw the Arches at D, lay a Ruler from A to D, and draw A E, which is a true Perpendicular to the Line B C; the Angles A E B, A E C are each 90 Degrees.



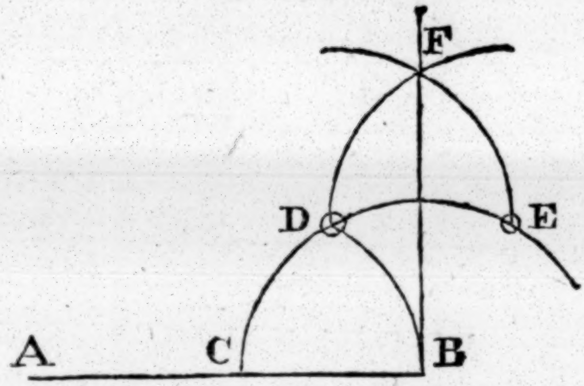
PROBLEM IV.

To erect a Perpendicular at the End of a given right Line.

There are ſeveral Ways to do this, but I ſhall content my ſelf with one, and leave the other Methods to be practiſed by the judicious Reader.

Let the given Line be A B, ſet one Foot of your Compaſſes at the End of the Line at B, and open the other Foot to any convenient Diſtance, and ſweep the Arch C D E, ſet one Foot in C and draw B D, carry that extent and ſet one Foot in D, and draw E F, with the ſame extent ſet one Foot in E and draw D F, lay a Ruler from F to B and draw F B, which ſhall be at right Angles to A B as was required.

See Theorem 2 of the 8. *page* 134. where this is performed according to *Euclid*.



PROBLEM V.

We have a Problem directing us how to draw Parallel Lines, but ſince the Invention of the Parallel Rules, I rather chuſe to adviſe the Young Practitioner to buy the Ruler, and make Uſe of that, before the Problem.

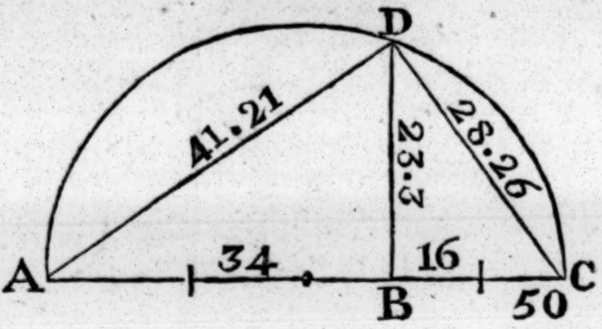
Alſo parallel or centric Circles are eaſily drawn without giving any Directions.

To find a Geometrical mean Proportion between any two given right Lines.

This we have ſhewn how to do in Numbers when we taught the Uſe of the Square Root; it now remains to ſhew the young Geometrician how to do it by Lines which will give him a bright Idea of many uſeful Things in Geometry.

Let the two Lines given be A B = 34, and B C = 16, draw an occult Line, and from a Scale of equal Parts take 34. and lay it from A to B, from the ſame Scale take 16 and ſet it from B to C, by which the firſt Problem biſect A C, and ſweep the Semi-circle A D C, from B erect the Perpendicular B D, which meaſured on the ſame Scale you will find to be 23.3, and that is a true geometrical mean Proportional between 16 and 34. Draw A D and C D, ſo ſhall the Angle A D C (by *Theorem* 8.) be a right Angle; and by *Theorem* 14 A D will be found 41.21, and C D 28.26. For as B C 16 : B D 23.3 : : B D = 23.3 : A B 34.

PROBLEM

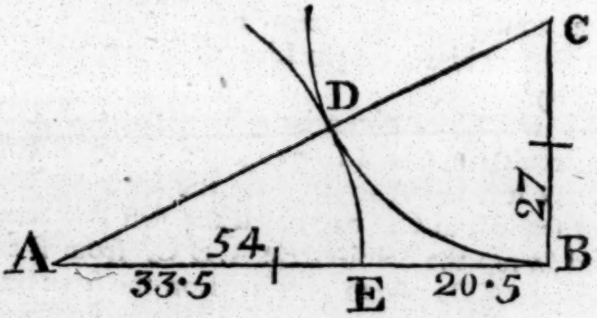


PROBLEM VI.

To divide a right Line in extream and mean Proportion.

Let the given right Line be A B = 54, from a Scale of equal Parts, or from the Line of Lines on the Sector, (which how to do, we ſhall ſhew when we come to treat of the Uſe of that Inſtrument) take half of A B = 27 and ſet it at right Angles from B to C, ſet one Foot of the Compaſſes in C, and draw the Arch B D, joyn A and C, then A D = A E meaſured on the ſame Scale will be found to be 33.5, then A B = 54 — A E 33.5 = EB 20.5

For as $\begin{cases} \text{E B.} & \text{A E.} & \text{A E.} & \text{A B.} \\ 20.5 : & 33.5 :: & 33.5 : & 54 \end{cases}$

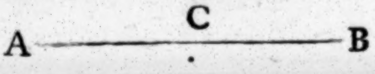


PROBLEM VII.

To divide a given right Line into any Proportion required.

Let the given Line be A B = 60, and I would have it divided in proportion as 40 to 50?

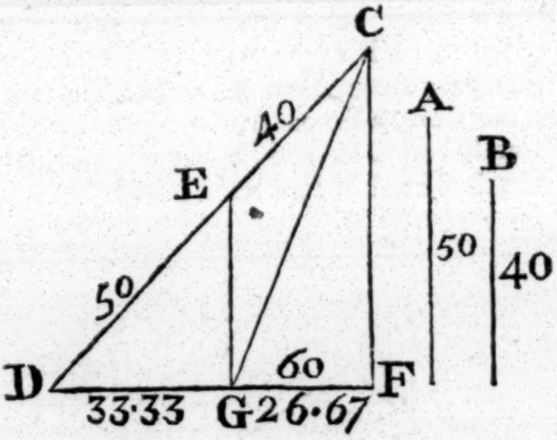
Add the two Numbers of your Proportion together, *viz.* 40 + 50 = 90, then ſay, as 90 : 60 : 50 : : 33.33 the greater Part of the Line. Then A B 60 — 33.33 = 26.67. the leſſer Part of the given Line.



Or for C B the leſſer Lines you may ſay

AC CB
As 50 : 40 : : 33·33 : 26.67.

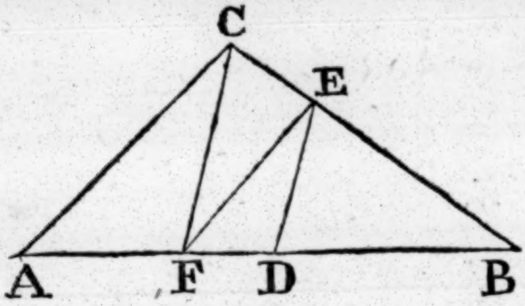
But this may be done Geometrically thus, Let D F be = 60 and be divided in the Proportion of A to B. Make the Angle C D F any Quantity at pleaſure, and draw D C, erect the Perpendicular F C, take the Line A and ſet from D to E, and B ſet from E to C, draw E G parallel to C F, ſo ſhall D G and G F meaſured ſeverally on the Line of equal Parts be 33.33 and 26.67, and in Proportion as 50 to 40; draw the Line G C, ſo ſhall the Triangle C D F, be to the Triangle C G F, as 50 to 40.



Note, From hence we learn to divide a Field into twſo equal Parts, ſo that each Perſon might have the Benefit o the Water (for his Cattle) which was placed on one Side thereof.

Let A C B be the Field, and a Baſon of Water at F be required to be ſo divided into two equal Parts that each might enjoy an equal Share of the ſaid Water.

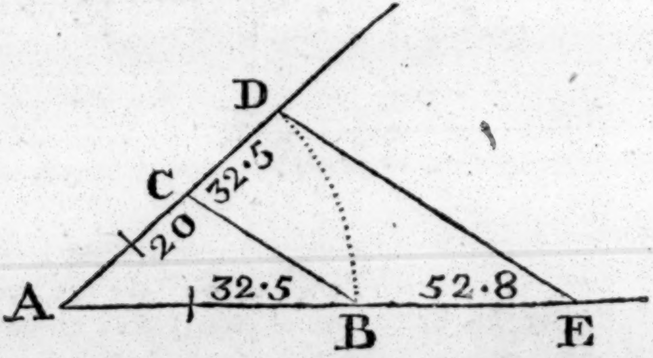
Firſt divide A B into two equal Parts at D, then draw F C, and D E parallel thereto, and laſtly draw F E, and 'tis done. Now the Triangles A C B and F E B are Similar, therefore A C E F = F E B.



PROBLEM VIII.

Two Lines given, to find a Third in Proportion.

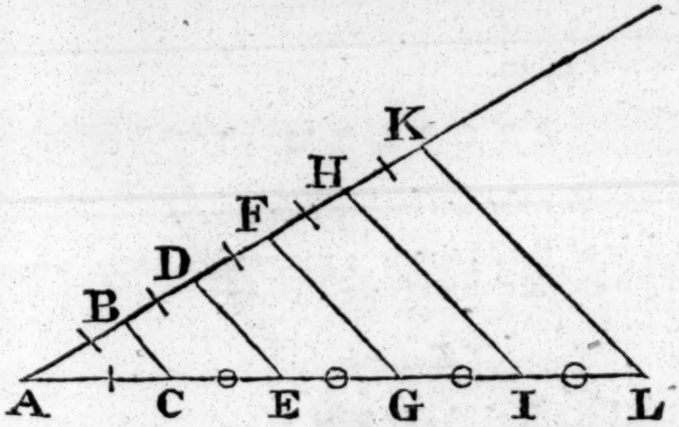
Let the two Lines be A B = 32.5 and A C = 20. Make the Angle D A B at pleaſure, and ſet off 20 from A to C, and 32.5 from A to B, draw C B, then with one Foot of the Compaſſes on A, deſcribe the Arch B D, draw D E parallel to C B, and 'tis done; for then it will be, As A C : A B : : A B : A E.



PROBLEM IX.

To Three given right Lines to find a Fourth, Fifth, Sixth, &c. *in Proportion.*

Make the Angle K A L any Quantity at pleaſure; let A B, A C, and A D be given to find others in Proportion; draw B C and D E parallel to B C. Then, as A B : A C : : A D : A E. Then from the ſame Scale of equal Parts that you laid off A B, ſet off A F, A H, and A K, and draw F G, H I, and K L parallel to B C, and you will have theſe Proportions, *viz.* As A B : A C : : A F : F G : and to H I and to K L, *&c.*



PROBLEM X.

To make a Square equal to a Triangle.

The Side of the Square is a geometrical Mean between the Baſe and half the Perpendicular.

Example.

Let the Baſe of a Triangle be 30, and the Perpendicular 40, what's the Side of a Square equal.

See the Work.

Baſe - - - 30
$\frac{1}{2}$ Perpendicular 20

600 (24.49

44) 200
176

484) 2400
1936

4889) 46400
44001

2399

Anſwer 24.49.

PROBLEM XI.

To reduce an Oblong to a Square.

The Side of the Square is a geometrical Mean between the Length and Breadth of the Oblong.

Example.

¶ Let the Length of the Oblong be 40, and Breadth 18, what's the Side of the Square equal?

Operation.

18
40

720 (26.8
. .

46) 320
276

528) 4400
4224

176

PROBLEM

PROBLEM XII.

To reduce a Rhombus to a Square.

The Side of a Square is a geometrical Mean between the Side of the Rhombus and it's Perpendicular.

Example.

Firſt, the Angles of the Rhombus are always the ſame, *viz.* 60, 60, and 120, 120, which all added together, make 360; and if we put 1 for the Side of the Rhombus, the Perpendicular will be found to be .866, and the Side of the geometrical Square equal is 93.

PROBLEM XIII.]

To reduce a Rhomboides to a Square.

The Side of the Square is a geometrical Mean between the longeſt Side and the Perpendicular.

Example.

Let the Side of the Rhomboides be 50, and the Perpendicular 22, What's the Side of the Square equal?

Operation.

```
       22
        50
      ----
      1100(33.16   Side Square equal
      9
      ---
   63)200
      189
  661)1100
       661
      ------
 6626) 43900
       39756
      ------
        4144
```

PROBLEM XIV.

To reduce a Trepezium to a Square.

Every Square's Side is a geometrical Mean between the Diagonal and half the Sum of the Perpendiculars.

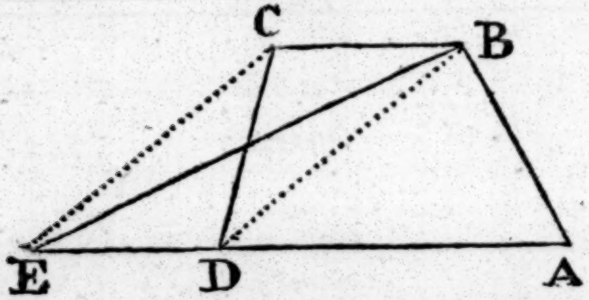
EXAMPLE.

Let the Diagonal be 60, and the half Sum of the two Perpendiculars 23, what's the Side of the Square equal? Anſwer 37.14.

PROBLEM XV.

To reduce a Trepezium to a Triangle.

Let the Trepezium be A B C D. Draw the Diagonal B D, and continue A D to E, draw C E parallel to B D, draw B E and 'tis done. So that the Triangle E B A = the Trepezium D C B A. See Theorem 16.

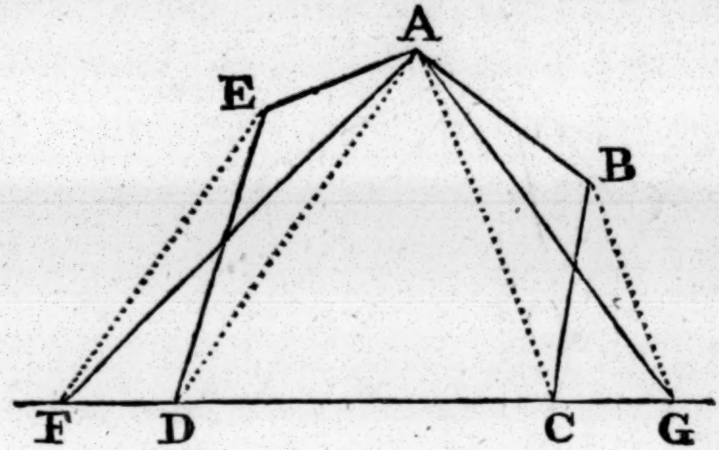


PROBLEM XVI.

To reduce an irregular Figure of Five Sides into a Triangle.

Let A B C D E be an irregular Figure. Firſt, continue the Side D C to F and G at pleaſure: Draw A C and A D, then draw E F and B G parallel to A D and A C. Laſtly, draw A F and A G, ſo ſhall the Triangle A F G be = to the irregular Figure A B C D E.

PROBLEM



PROBLEM XVII.

To reduce an irregular Figure of Six or Seven Sides into a Triangle.

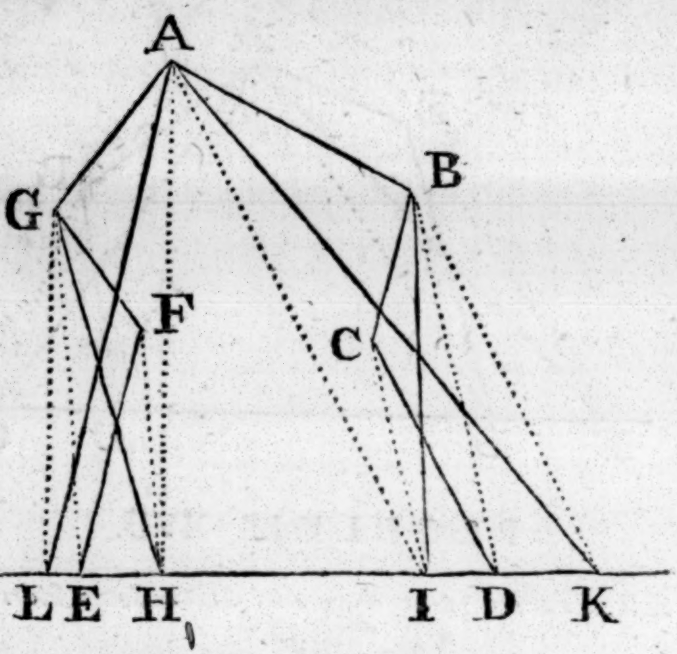
Let A B C D E F G be a Plot to be reduced to a Triangle.

Firſt, By the laſt Problem reduce the Seven Sides into Five, thus ;

Draw the Line G E, and parallel thereto F H, then draw the Line G H, whereunto the two Lines G F and F E are thereby reduced. Then draw the Line B D, and parallel thereunto C I, and B I, now are the two Lines B C and C D reduced to a ſtreight Line B I, and the whole Plot to a Figure of five Sides.

Secondly, To reduce this to a Triangle.

Produce the Side E D to K and L, and draw the Lines A I and A H, and parallel to them B K and G L, cutting the Line E D, (being extended) in K and L. Laſtly, Draw A K and A L, and 'tis done, for I ſay, the Triangle, A L K is = to the irregular Figure A B C D E F G.



PROBLEM XVIII.

To reduce a Pentagon to a Square.

The Side of every Square is a geometrical Mean between half the Perimeter and the Perpendicular. As ſuppoſe the Side of the Pentagon be 1, the Perpendicular is .6882, then what's the Side of the Square equal?

See the Work,

Perpendicular - - .6882
Half N° Sides - - 2.5

34410
13764

1.720500 (1.312 *fere.*

23) 72
69
261) 305
261
262) 4400

PROBLEM

PROBLEM XIX.

Given, a Square of any Dimenſion whatſoever, to make another bigger or leſſer, at pleaſure.

In Theorem 15, I have told you that the Diagonal of one Square is the Side of another Square double in Power to the ſaid given Square.

RULE.

Multiply the Square of the Side by 2, 3, 4, *&c.* the Square Root of the Product will give the Side of a Square whoſe Area ſhall be 2, 3, 4 times greater than the given Square, ſo that if the Side of one Square be 12, you will find the Side of another Square twice as big to be 16.97, and thrice as large to be 20.78, and four times as large, the Side will be 24.

If the Side of a Square be given to find the Tranſverſe and Conjugate Diameters of an Ellipſis, that ſhall be equal in Area to that of the Square.

Firſt, Find the Diameter of a Circle thus, *viz.* Multiply the Side of the Square by 1.128379, or divide it by .886227 the Product, or Quotient, is the Diameter of a Circle equal in Area: Then find the Circumference of it thus, Multiply the Diameter always by 3.14159 and the Product gives the Circumference, then half Circumference multiply'd by half Diameter is the Area or Superficial Content equal to that of the given Square: Then becauſe every Ellipſis is a geometrical Mean between two Circles, deſcribed on the two Diameters of the Ellipſis, thoſe Diameters may be taken at Pleaſure, and conſequently various Ellipſes may be found to anſwer in ſuperficial Content to that of the given Square: For the Ellipſis being a Figure that flows between the Circle and Parabola.

But here Note, That 1.128379 is the Diameter of a Circle, whoſe Area is equal to the Square, whoſe Side is Unity or 1. And .886227 is the Side of a Square equal when the Diameter of a Circle is one. And 3.141592 is the Circumference of a Circle, when the Diameter is one: Near the Truth.

PROBLEM XX.

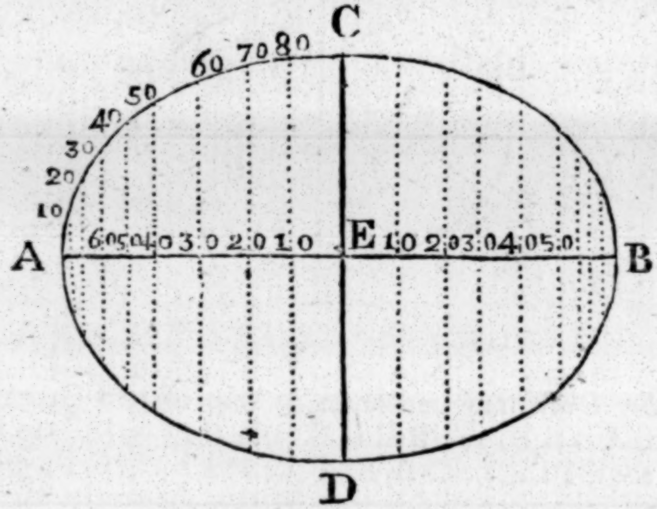
To lay down an Ellipſis by the Line of Sines on the Sector, having given the Tranſverſe and Conjugate Diameters.

Take A E = E B in your Compaſſes, and open the Sector at 90, 90 on the Line of Sines, and as the Sector now ſtands, take off the Sines 10, 20, 30, 40, 50, 60, 70, 80, and ſet them from E each Way towards A and B, draw occult Lines through thoſe Points in the tranſverſe Diameter parallel to the Conjugate, which done, ſet the Sector on the Sines 90, 90 to the Radius C E, and take in your Compaſs the Sine of 80, and ſet from 10 to 80, take the Sine 70, and ſet from 70 to 20 on each Side the Conjugate Diameter, the Sine 60, ſet from 30 to 60, the Sine 50 ſet from 40 to 50, the Sine 40 ſet from 50 to 40, the the Sine 30 ſet from 60 to 30, the Sine 20 ſet from 70 to 20, the Sine 10 ſet from 80 to 10, ſo will the Points 10, 20, 30, 40, 50, 60, 70, 80, C B D A be in the Ellipſis, which with an even Hand draw the Curve and 'tis done: Which is truly Mathematical, and has not any part of a Circle in it.

Note, The Ellipſis being delineated as above, upon Paſtboard and neatly cut out, you may then draw the Ellipſis by help of that Pattern Paſtboard, with Ink to a great Exactneſs, which is always my Method in drawing the Azimuths and Meridians in the Orthographick Projection of the Sphere.

Every Ellipſis has two Focus, or Navel Points, in the tranſverſe Diameter of the Ellipſis, through which Lines drawn at right Angles thereto are called the *Latus Rectum*, the Length of which may be found by this Proportion.

As the Tranſverſe Diameter is to the Conjugate, ſo is the Conjugate Diameter to the *Latus Rectum*.



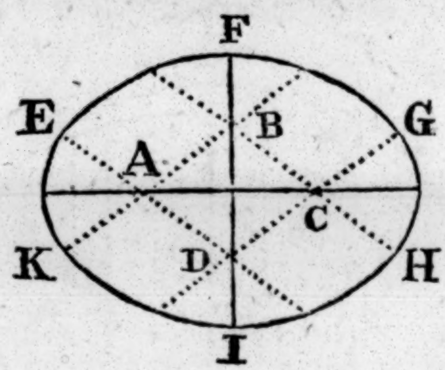
PROBLEM XXI.

Of Ovals:

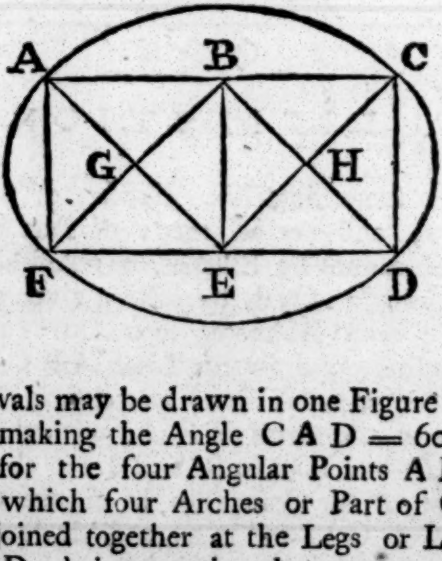
DIFINITION.

Ovals are ſomething like Ellipſes, of which there are divers Sorts, and are all made of Part of Circles, and conſequently cannot be Ellipſes, as noted above.

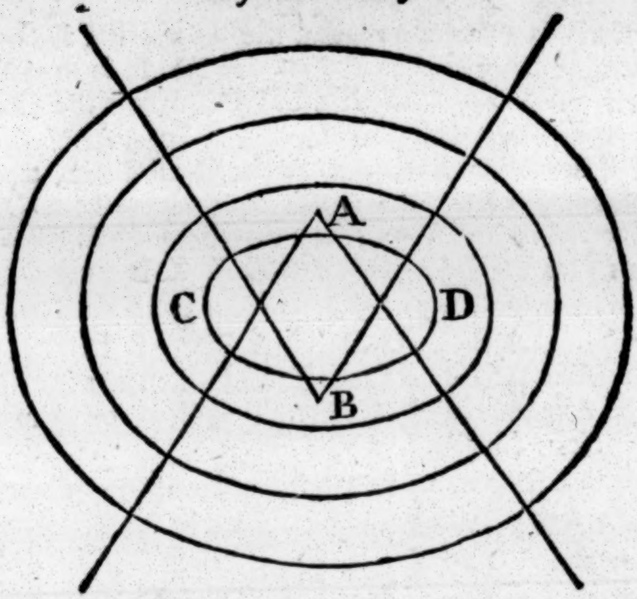
1. To draw an Oval, by having given the two Diameters, divide each Diameter into four equal Parts, and through thoſe Parts draw the Lines A B C D, then ſet one Foot of the Compaſſes in D and extend the other Foot to F, and draw the Arch E F G; with this Extent of the Compaſſes ſet one Foot in B, and draw the Arch H I K; ſet one Foot of the Compaſſes in A and C ſeverally, and draw the Arch G H and E K, and 'tis done.



2. An Oval may be drawn by help of two Geometric Squares A B E F, and B C D E, join the two Sides in B E, and draw the Diagonals A E, B F, and B D, C E, on the Centre E, with the Diſtance E A draw the Arch A C, on the Center B, with the Diſtance B F draw the Arch F D, and you will have a neat Oval.

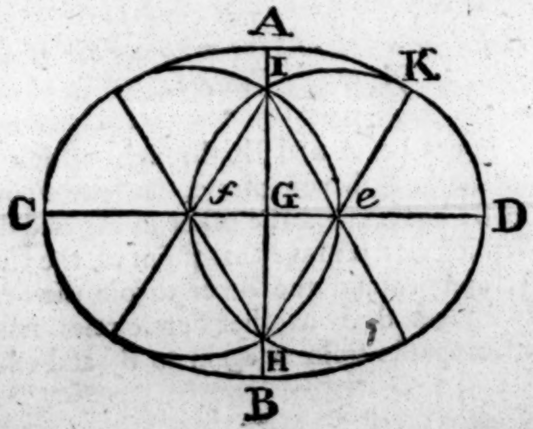


3. Several Ovals may be drawn in one Figure parallel to each other, by making the Angle C A D = 60 = to the Angle C B D, for the four Angular Points A B C D are the Center on which four Arches or Part of Circles are drawn, which joined together at the Legs or Lines A C, A D, B C, B D, being continued, you may draw as many Ovals as you pleaſe, as you may the better perceive by this Figure.



4. An Oval may be drawn (by having only one Diameter of the Oval given) on two Circles. If the Line C D be 1, then will A B be .756 thus proved. To avoid Fractions, ſuppoſe C D 12, then F I = 4 □ = 16. F G = 2 □ = 4. and 16 − 4 = 12 □ √ = 3.46 = G I. + G H 3.46 = 6.92 = I H. And H K = C E = F D = 8 − H I = 6.92 = 1.08 = A I + I H = 6.92 + H B 1.08 = A B 9.08. Then to reduce them to the Unity, ſay,

CD AB CD AB.
As 12 : 9.08 :: 1 : .756

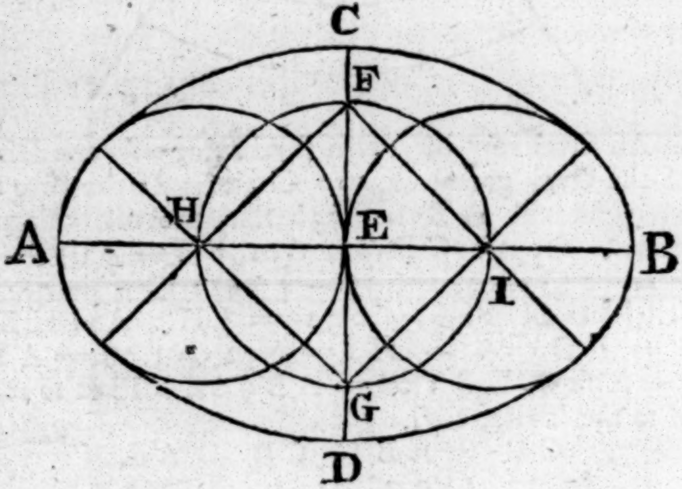


FIEH is a Rhombus, and the Angles F I E = F H E = 60, < H F I = H E I = 120°. H is the Center on which the Arch A K is drawn, and I is the Center on which the Arch B is drawn.

5. The longer Diameter of an Oval given, to draw it on three Circles. To avoid Fractions call the Diameter A B 40, then is E I = 10 □ = 100 + □ E F = 200 □ √ = 14.14 = F I = C E + E D = C D 28.28. To reduce which to Unity, ſay,

	AB		CD		AB		CD.
As	40	:	28.28	: :	1	:	.707

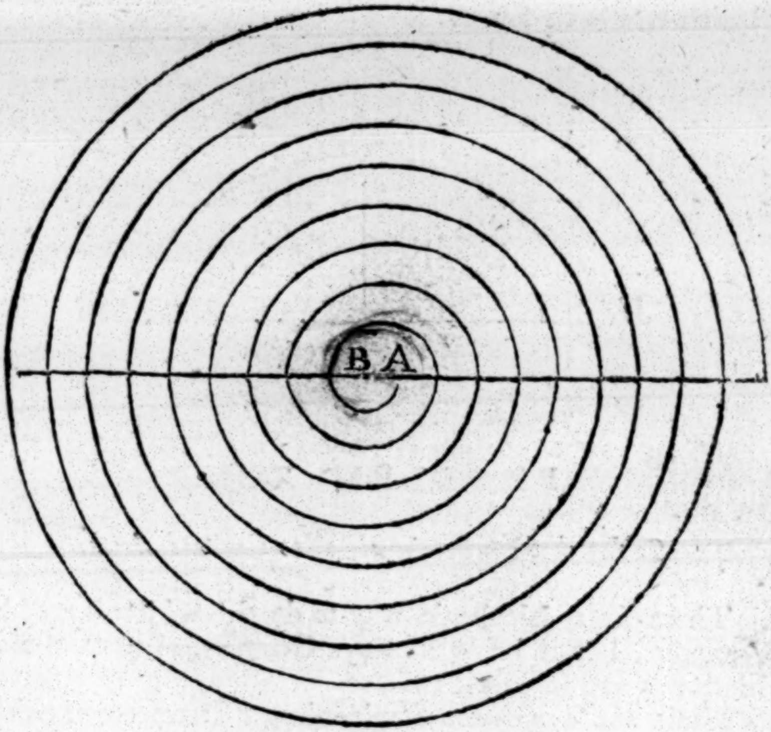
The Angles H, F, I, G, are all right, as being made in a Semicircle, *See Theorem* 7.



PROBLEM XXII.

To draw a Helix, or a Spiral Line with a Pair of Compaſſes from two Centers.

Let two Centers be A and B, through which draw a right Line, what Length you pleaſe, ſet one Foot of the Compaſſes in B, and extend the other to A, and draw the firſt little Semicircle; remove that Point of the Compaſſes from B to A, and extend the other to join the Semicircle uſt now drawn, and draw another Semicircle; remove the Point of the Compaſſes from A again to B, and extend the other Point to the laſt Semicircle, and there join it, and draw another Semicircle, do thus as long as you pleaſe, and you will have a delightful Spiral Line, rowling in ſeveral Circles, as per Fig.



PROBLEM XXII.

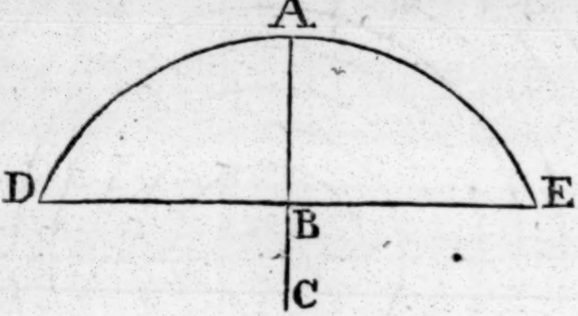
How to ſtrike an Arch to any Heighth or Breadth.

Let it be required to make an Arch four Foot high, = A B, and the width on the Baſe 12 Foot = D E ?

Now all the Matter lies to find the Center that will ſweep this Arch, for our Aſſiſtance herein, we muſt call in Prob. 5, by which we find that B E = BD is a geometrical Mean Proportional between A B and the reſt of the Diameter, therefore, divide the Square of B E = 36, by A B = 4, and the Quotient 9 is the other Part of the Diameter, then

AB 4 + 9 = 13, the whole Diameter of that Circle, whereof the Arch D A E is a Part of the Circumference. For,

AB DB DB.
As 4 : 6 :: 6 : 9 + 4 = 13 whoſe $\frac{1}{2} = 6\frac{1}{2}$ = the Semidiameter A C, ſo is C the Center of the Arch D A E which was to be found.



PROBLEM XXIII.

To lay down a Semioval, alias *Curteel Arch.*

There is a Mechanic way to do this which is uſed by Artificers, but it being no ways Geometrical, I ſhall only hint of it and ſo paſs it by.

Limit the Tranſverſe Semiconjugate Diameter of what you pleaſe.

The Baſe or Tranſverſe Diameter they divide into ſeven equal Parts, and the Semiconjugate into 15 (but not of the like Parts with the Tranſverſe) then they take 11 of thoſe 15 and ſet perpendicular to the Tranſverſe at the Diviſions 1 and 6, now have they three Points given to draw the Top of the Arch, to which three Points find a Center, and that Part is compleated, the Ends of the Arch are drawn at the Diſtance of two of the Diviſions of the Tranſverſe Diameter.

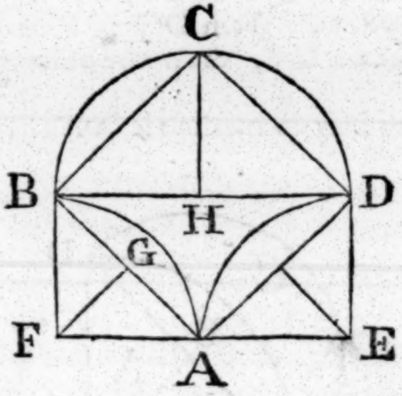
PROBLEM

PROBLEM XXV.

To draw a Pelicoides.

DEFINITION.

A Pelicoides is a Figure, made up, or compoſed of a Parallelogram, a Square, a Semi-circle, Quadrants and Triangles, which how to meaſure will be ſhewn in their proper Places. And the young Geometer cannot miſs laying of it down at the very Sight of the Figure.

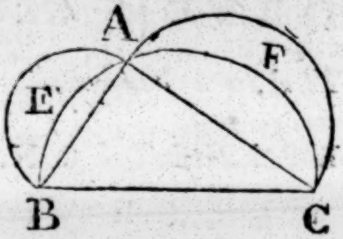


PROBLEM XXVI.

DEFINITION.

Lunes, or Lunulæ, are three Semi-circles deſcribed on the three Sides of a right lin'd right angled Triangle.

For the Triangle A B C is right angled for two Reaſons, firſt becauſe the Angles are made in a Semi-circle, (ſee Theorem 7.) and ſecondly, becauſe the Sides are in Proportion as 3, 4 and 5. How this Figure is meaſured, will be ſhewn by and by.

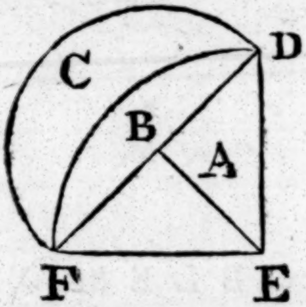


PROBLEM XXVII.

To deſcribe a Lune in a Quadrant.

Firſt draw the Triangle D E F, and on the Center E, deſcribe the Quadrantal Arch D F; upon the Middle of of the Hypothenuſe D F, draw the other Semi-circle and 'tis done.

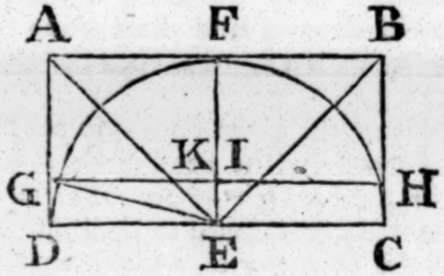
We ſhall ſhew how to meaſure it in it's proper Place.



PROBLEM XXVIII.

A Sphere is $\frac{2}{3}$ of a Cylinder circumſcribed.

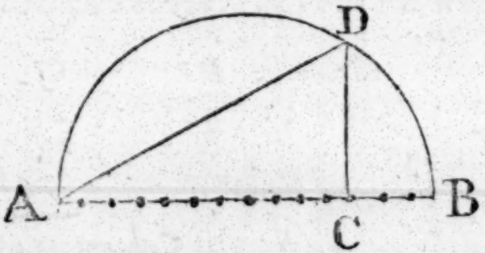
Let A B C D be a Cylinder, D F C a Hemiſphere, A E B an inverted Cone, to have the ſame Baſe and Altitude, and to be cut by infinite Planes all parallel to the Baſe, G H is one. I ſay the Square G I is every where equal to the Square F E (the Radius of the Sphere.) The $\square IE = \square IK$: and conſequently, ſince Circles are to one another as the Square of their Radius, all the Circles of the Hemiſphere will be equal to all thoſe of the Cylinder, leſs all thoſe of the Cone, *Q E D*.



PROBLEM XXIX.

To Reduce a Circle to a Square.

This Problem is grounded upon *Archimedes*'s Proportion of the Diameter of a Circle to the Circumference being as 7 to 22, this being not perfectly true, (ſee Page 151) but the neareſt in whole Numbers, and may ſerve well enough our preſent Purpoſe, therefore let the Diameter A B, be divided into 14 equal Parts, at 11 of thoſe Parts erect the Perpendicular C D, and draw A D, ſo is A D the ſide of the Square nearly equal in Content to the given Circle. And then if the Diameter be 14, the Side of the Square will be 12.41, as ſhall be ſhewn when we come to the Menſuration of Figures.

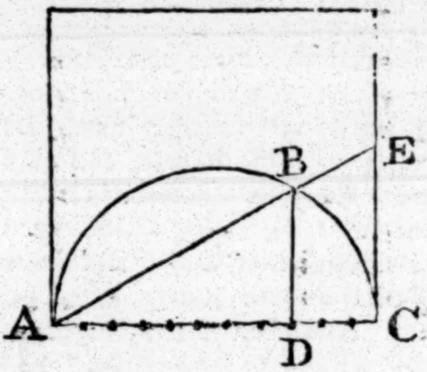


PROBLEM XXX.

To Reduce a Square to a Circle.

This is grounded upon the above Proportion for the Sake of whole Numbers. And here *Note*, That when the Side of the Square is 11, that then the Diameter of a Circle equal is 12.41.

Divide the Side of the given Square into 11 equal Parts, at 5.5 of thoſe Parts draw the Semi-circle A B C, and at 8 of the Parts on the Side of the Square erect the Perpendicular D B, draw A B continued to the Side of the Square at E, ſo is A E the Diameter of a Circle equal in Content to the given Square, and if the Side be 11, the Diameter as 12.41.



By theſe two laſt Problems, we have proved the Proportion that a Circle hath to a Square, whoſe Diameter and Sides are equal is as 11 to 14, in round Numbers.

	Di.		*Cir.*		*Di.*		*Cir.*	
As	7.	:	22	: :	14	:	44.	Side 14.
Squared = 196.							$22\frac{1}{2}$ *Cir.*	
							$7\frac{1}{2}$ *Diam.*	
							Content 154	

Now ſay, As 11 to 14, ſo is 154 the Content of the inſcribed Circle to 196, the Content of the circumſcribed Square: Becauſe you ſee the Diameter of the Circle inſcribed in the Square, is equal to the Side of the Square, *viz.* each 14 as above, both being equal to A C.

PROBLEM XXXI.

To Divide a Circle into any Number of equal Parts.

1. Draw a Circle of any Radius whatſoever, and draw the Diameter A B, this divides the Circle into two equal Parts.

2. Erect the Perpendicular F C, and that ſhall be the Side of an Hexagon, or the ſixth Part of the Circle = A D.

3. Set F C from A to D, and from D to E, draw A E, for the Side of an equilateral Triangle.

4. Draw A C for the Side of the Square inſcribed, or the fourth Part of the Circle.

5. Biſſext F B in G, and draw C G, make G H = G C, and draw C H for the Side of the Pentagon, or fifth Part of the Circle.

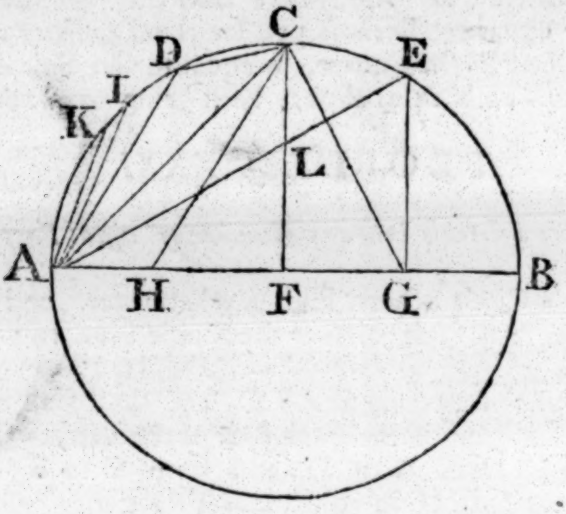
6. Joyn E G for the Side of a Heptagon, or one ſeventh Part of the Circle.

7. Biſſext the Arch A C in I, and draw A I for the Side of an Octagon, or an eighth Part of the Circle.

8. Divide the Arch A D E into three equal Parts in K, and draw A K for the ninth Side or Part of the Circle.

9. The Line H F is the Side of a Decagon, or a ten ſided Figure.

10. The Line F L is the Endecagon, or eleventh ſided Figure. And by doubling or tripling theſe Lines, the Circle may be geometrically divided into more = Parts at Pleaſure.



A Synopſis of the Scheme.

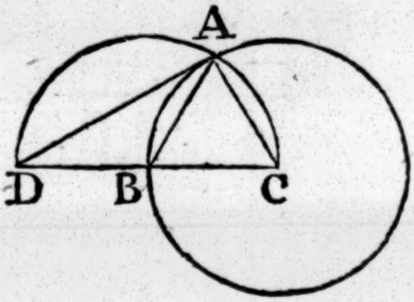
A B	is the	$\frac{1}{2}$	Part of the Circle.
A E		$\frac{1}{3}$	
A C		$\frac{1}{4}$	
C H		$\frac{1}{5}$	
A D		$\frac{1}{6}$	
E G		$\frac{1}{7}$	
A I		$\frac{1}{8}$	
A K		$\frac{1}{9}$	
H F		$\frac{1}{10}$	
F L		$\frac{1}{11}$	
D C		$\frac{1}{12}$	

PROBLEM XXXII.

To draw a Tangent to a given Circle.

Let the Circle be B A, and C the Center, and the Point of Contact A, take the Radius A C in your Compaſſes, and ſet one Foot in B, draw the Semi-circle D A C. Laſtly, draw D A, and it is a true Tangent to the Point of Contact, becauſe the Angle D A C as made in a Semi-circle, is a right Angle.

PROBLEM



PROBLEM XXXIII.

To deleniate a Parabola in Plano.

DEFINITION.

A Parabola is a conic Section, as mentioned in *page* 129 A B ſtanding at right Angles with the Baſe C D is called the Abſciſa, and *a b, c d, e f, g h, i k, n o,* are Ordinates.

To find the Latus Rectum L M,

It will always hold,

As { A B : C B : : C B : L M.
{ 70 37.5 37.5 20.089.

Note, The Focus through which the Latus Rectum muſt paſs, is diſtant from the Vertex A a quarter of the Latus Rectum, which in this Example is 5.022.

To find the Ordinates.

RULE.

The Rect Angle of the Latus Rectum, and it's intercepted Axe, is equal to the Square of the Ordinate.

In the Example above we have put 70 for the Abſciſa A B, and 75 for the Baſe C D, from which the Latus Rectum L M is found to be 20.089, one 4th of which is = 5 022 = A P, then 70 − 5.022 = 64 978 = P B, ÷ 7 = 9.282, the Diſtance of each Ordinate.

Now let us find the Ordinate N O, the Work ſtands thus.

$AP = 5.022 + Pq = Aq = 14.304$
$LM = 20.089$

128736
114432
28608

287.353056(16.951
1

26)187
156

329) 3135
2961

3385) 17430
16925

33901) 50556
33901

16655

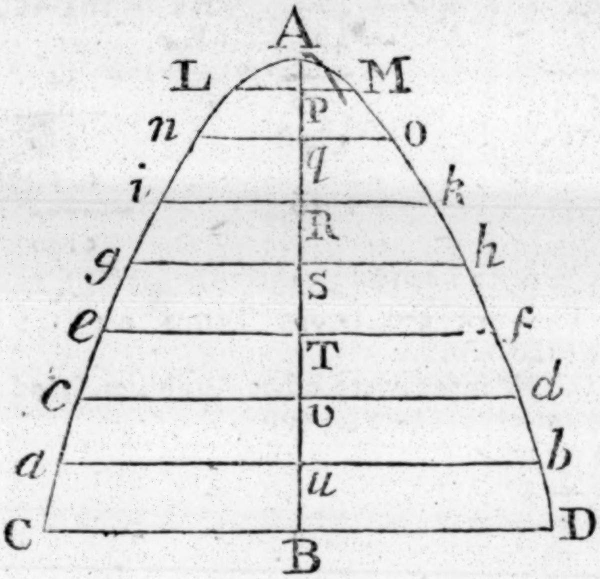
$Nq = 16.195$
$\times \quad 2$

$NO = 33.902$

And after this Manner of working the other Ordinates are found, as is here ſet down; which being laid off at right Angles to A B, will give the Points *a*, *c*, *e*, *g*, *i*, *n*, L, M, *o*, *k*, *h*, *f*, *d*, *b*, which Points lye in the Parabola.

A B = 70
L M = 20.089
N O = 33.902
i k = 43.532
g h = 51.39
e f = 58.196
c d = 64.286
a b = 69.842
C D = 75.

PROB-



PROBLEM XXXVI.

To delineate an Hyperbola in Plano.

DEFINITION.

An Hyperbola is the fifth Section of a Cone cut by a Plane parallel to the Axis, and continued 'till it meet the other ſide of the Cone beyond the Vertex, which is here repreſented by E.

Let A B be 92, E C 104, and D C 63, then if we take ſix Ordinates. *g a*, *h b*, *i c*, *k d*, *l e*, *m f*, every one will be 9, taking D C = 63. Then if A F be continu'd 'till it meet D E, D E will be 41, for E C 104 — D C 63 = D E 41. Then to find the Latus Rectum F D, it will hold,

As E F : *f* G : : E D : D F.

From which it is plain, that having the Latus Rectum and Latus Tranſverſum E D, the Ordinates in any Hyperbola may be drawn.

For as E C × D C : □ A C : : E *a* × D *a* : □ *a g*.

See the Work for the Ordinate $g\ a = a\ s$.

E C = 104	A C = 46	E C = 104 − C a 9 =
D C = 63	A C = 46	E a 95
		D a 54
312	276	380
624	184	475
6552	2116	5130

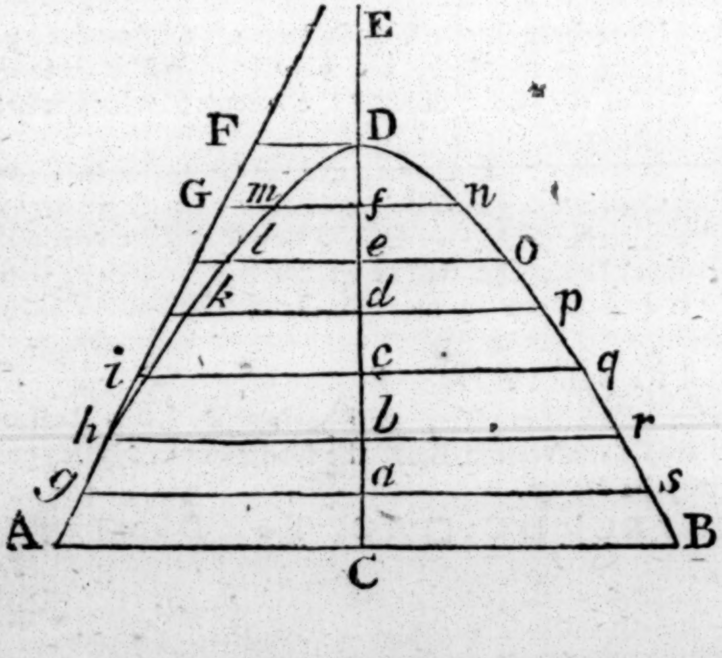
Now ſay, As 6552 : 2116 : : 5130 : 1656.76 □√= 40.7

Note, The firſt and ſecond Terms are the ſame through all the Work.

After this Manner were other Ordinates found, and their Quantities as is here ſet down.

$m\ n = 24.1$
$l\ o = 37.0$
$k\ p = 48.0$
$i\ q = 59.8$
$h\ r = 70.6$
$g\ s = 81.4$
$A\ B = 92.$

Now theſe Ordinates being drawn at right Angles to CD will give the Points *g, h, i, k, l, m, n, o, p, q, r, s,* which are in the Hyperbola.



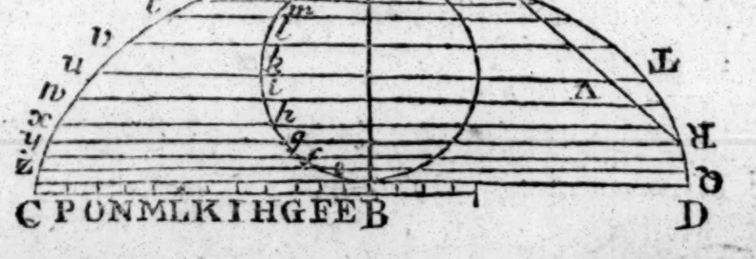
PROBLEM XXXVII.

To delineate a Cycloid, or Trochoid.

DEFINITION.

The Curve of this Figure is called by Mathematicians a Tranſcendant Curve, or a Line of an infinite Order, and may be beſt conceived to be generated by a Nail in a Coach Wheel; for in every Revolution of the Wheel, this Curve is deſcribed: And the whole Cycloidal Curve Line is equal to 4 Diameters of the generating Circle; and the Area of the Cycloidal Space is equal to three times that Circle.

Make the Diameter of the Circle A B of what Quantity you pleaſe, as ſuppoſe 60; then will the Circumference be 188.4954 (ſee *page* 151.) of the ſame Parts; and half the Circumference is 94.2477; therefore, ſet the Sector to 60 the Diameter of the Circle on the Line of Lines, (or on any Scale of equal Parts) and take off 94.2477, half the Circumference of the Circle, and ſet it from B to C and D; divide C B into any Number of equal Parts, as ſuppoſe 12, and alſo the Semi-circle A *k* B into the like Number of equal Parts, and thro' thoſe Parts in the Semi-circle draw Lines parallel to C D, take the Diſtance B E, and ſet from *p* to *q*, B F from *o* to *r*, B G from *n* to *s*, B H from *m* to *t*, B I from *l* to *v*, B K from *k* to *u*, B L from *i* to *w*, B M from *h* to *x*, B N from *g* to *y*, B O from *f* to *z*, *&c.* on each Side the Circle, ſo ſhall the Points *q*, *r*, *s*, *t*, *v*, *u*, *w*, *x*, *y*, *z*, be in the Curve of the Cycloid.



If the Cycloid be inverted, and a Body deſcend from Q R, or S, by it's own Gravity to A the loweſt Point o the Curve, the Times of theſe Deſcents ſhall be equal.

If a Body deſcend from R to S in the Curve R T S by it's own Gravity; the Times of it's Deſcent will b leſs than if it went in the ſtreight Line Q V S: or leſ than if it went in any other Line; therefore this is calle the Line of ſwifteſt Deſcent.

If the Curve C A D were cut into two equal Parts in the Line A B, and C B D were turn'd upwards, and a Pendulum hung at B, of a length of the Arch A C, counted from the Point of Suſpenſion to the Center of *Oſcillation*: I ſay that Center of *Oſcillation* will deſcribe the Cycloid DAC, and all the Vibrations, whether long or ſhort, ſhall be performed inthe ſame time; for which Reaſon it is called the *Iſochronal Curve*.

The firſt that applied Pendulums to a Movement, was the *Dutch* Man, Mr *Chriſtopher Hugens*, and ſince Pendulums are to each other as the Square of their Vibrations, and that a Pendulum vibrating Seconds, (or 60 times in a Minute) is, by Experience, found to be of the Length of 39.2 Inches; from hence it is eaſy, according to theſe Principals and Experiments on Pendulums, to eſtimate nearly the Depth of a deep Well, by the Fall of a Stone from the Mouth into the Water: Or the Diſtance that any Ship at Sea, or that any Fort is off, by the Times betwee ſeeing the Flaſh of the Powder, and hearing the Report o the Gun, or the Diſtance that any Thunder Cloud i off; For Sir *Iſaac Newton* found that a Sound moves 968 Feet in a Second of Time: So that counting the Second, (that is, the Swings of the Pendulum, whoſe Length is 39.2 Inches) between ſeeing the Flaſh of Lightning and hearing the Report, and thoſe multiply'd by 968 the Product are Feet, which divide by 5280, the Feet in an *Engliſh* Mile, will produce the Miles that the Cloud, Fort, *&c.* is diſtant from you.

Alſo, by the Vibrations of Pendulums, find the Length of any String, Chain, *&c.* that has a Weight hanging to it, without coming to meaſure it; or without making Uſe of any Quadrant, or ſuch like Inſtrument to take Heights. This may be put in Practice in taking the Heighth of Churches, Halls, *&c.* wherein hang Branches, or Braſs Candleſticks, being faſtned to the Roof of the Church, or Theatre, for if you hang up a String and Plummet of any known Length, (as ſuppoſe 3 Foot) and make the Candleſtick and Pendulum begin to ſwing both together, (which muſt be done by Help of a Correſpondent;) The Vibrations that the Candleſtic makes, while your Pendulum makes any competent Number, will eaſily help you to the Length of the String, Wire, or Chain that hold the Candleſtick, and conſequently the Heighth of the Roof of the Church will be known likewiſe.

EXAMPLE.

Suppoſe in the Quire, or Choir of St *Paul*'s Cathedral, *London*, one of the Candleſticks be made to ſwing 12 times while your Pendulum of a Yard long makes 96 Vibrations, and the Candleſtick hang 8 Foot from the Floor, how many Feet is it from the Floor to the Roof?

Operation.

Opera

```
  12              96
  12              96
 ----            ----
 144              576
                 864
               --------
          144) 9216 (64
                864
               ----
                576
                576
               ----
                  0
```

Anſwer 64 Yards = 192 Feet.

I ſhall conclude this Chapter with three Magick Squares.

$$\underbrace{\left.\begin{matrix}2&7&6\\9&5&1\\4&3&8\end{matrix}\right\}\qquad\qquad\left\{\begin{matrix}5&10&3\\4&6&8\\9&2&7\end{matrix}\right.}_{\begin{matrix}8&28&27&11\\23&15&16&20\\17&21&22&14\\26&10&9&29\end{matrix}}$$

The Firſt Square makes 15 every way, the Second makes 18 every way, and the third makes 74 every way. The Method of filling all Sorts of Magical Squares with the Magick Cubes, may be ſeen at large in the *Memoirs of the Royal Academy of Sciences*, for 1710, page 124, by Monſ. *Saurier*, in his *Conſtruction Generale, des Quarres Magiques.*

CHAP. III.

Containing the Menſuration of all Manner of Superficies.

And firſt of a Circle.

IN Page 151 we have ſhewn that if the Diameter of a Circle be Unity, or 1, that then the Circumference is 3.141592, ſo that if the Diameter of any Circle be multiply'd by 3.141592, the Product will be the Circumference: Or if the Circumference of any Circle be divided by 3.141592 the Quotient will be the Diameter. Now it is to be obſerved that the Dimenſions may be taken either in Inches, Feet, Yards, Poles, Chains, *&c.* and that there is often a neceſſity to reduce one Sort of Meaſure to another, as when the Dimenſions are taken in Inches, the Area in Inches divided by 144, the Square of 12, gives Feet, *&c.*

If the Diameter of a Circle be 1, the Area of that Circle will be .785398 which is called the Area of Unity: Then if Unity be divided by it's Area, the Quotient will be 1.273241, ſo that if the Square of the Diameter of any Circle be multiply'd by .785398, or divided by 1.273241 the Product or Quotient will be the Area of that Circle in Square Meaſure. *See p.* 22.

If 144 be multiply'd by 1.273241, the Product will be 183.3467, the Diviſor in Circular Meaſure: By which, if the Square of the Diameter of a Circle be divided by 183.3467, the Quotient is the Area in Feet.

The Square Root of 183.3467 is = 13.54, the Guage Point on the Sliding Rule, which is alſo the Diameter of a Circle, whoſe Content is 144 Inches, and the Circumference is = 42.5365. The Circle is the moſt Capacious Figure of all others, for if the Sides of a Square, Triangle, or any other Figure be equal to the Circumference of a Circle, it will not contain ſo much as the Circle doth. Suppoſe the Circumference of a Circle be 75.4 it's Area is 452.4, the Fourth Part of the Circumference is 18.85, which ſquared is only 355.3225, that being 97.0775 leſs than the Circle. And this is the Ground and Foundation of the Error of Meaſuring round Timber, by taking $\frac{1}{4}$ of the Girt for the Side of the Square equal; which falſe way I think ought no longer to have Place, but to be baniſhed from the Thoughts and Practice of all thinking Men, that have any thing to do with Menſurations. *See my* Royal Gauger, *p.* 61.

2. *To Meaſure a Square.*

Multiply the Side in itſelf, and the Product is the Superficial Content in the ſame Meaſure the Dimenſions were taken in.

3. *To Meaſure an Oblong.*

Multiply the Length by the Breadth, the Product is the Area, or Superficial Content.

4. *To Meaſure a Rhumbus.*

Multiply the Side by the Perpendicular Heighth, the Product is the Content.

5. *To Meaſure a Rhumboides.*

Multiply the Length by the Perpendicular Heighth or Breadth, and the Product is the Content.

6. *To Meaſure a plain Triangle.*

Multiply the Baſe, or longeſt Side, by half the Perpendicular, or the whole Perpendicular by half the Baſe, or the whole Baſe by the whole Perpendicular, and half this Product is the Content.

7. *To Meaſure a Trepezium.*

Multiply the Diagonal by half the Sum of the two Perpendiculars *& Contra*, and the Product is the Superficial Content.

8. *To Meaſure any irregular Figure.*

Reduce the irregular Figure into Trepezia and Triangles, and meaſure them as taught in the 6th and 7th hereof.

9. *To Meaſure any regular Polygon ; as a* Pentagon, Hexagon, Heptagon, Octagon, Nonagon, &c.

Multiply half the Sum of it's Sides into the Radius of the Circle inſcribed in the Figure, or half that Radius into the Sum of the Sides, the Product is the Area or Superficial Content.

Note, The Radius of a Circle inſcribed in any regular Polygon is equal to the Perpendicular of it, let fall from the Center to the Side of the Polygon.

I ſhall here ſubjoin a uſeful Table of regular Polygons ſhewing the Angles at the Center, Length of the Perpendicular, (or Radius of the Circle inſcribing it) and Area in Square Inches.

Names.

Names.	N° Sides.	Angle at Center. °	'	Per-pendi-cular.	Area in Square-Inches.
Trigon	3	120	00	.2872	.4308
Tetragon.	4	90	00	.566	1.0000
Pentagon	5	72	00	.688	1.72
Hexagon	6	60	00	.866	2.598
Heptagon.	7	51	26	1.038	3.633
Octagon.	8	45	00	1.207	4.828
Enneagon.	9	40	00	1.374	6.183
Decagon	10	36	00	1.539	7.695
Endecagon	11	32	44	1.702	9.361
Dodecagon	12	30	00	1.866	11.196

In framing of this Table, the Side of the Polygon is ſuppoſed to be 1. And as the Areas of like Figures are as the Squares of the Sides that bound them; therefore the Square of any Side multiply'd by the Area of the Polygon in the Table, will give the Area of the Polygon required, ſo that theſe Areas in the Table are common Multipliers.

10. *To Meaſure a Spheric Triangle.*

From the Sum of the three Angles ſubtract 180°, multi- the Superficies of the whole Globe by the Remainder, and divide by 720, the Quotient is the Superficial Content.

11. *To Meaſure a Semicircle.*

Multiply $\frac{1}{4}$ of the whole Circumference of the Circle by the Semidiameter, and you have the Area.

12. *To Meaſure a Quadrant.*

Multiply half the Arch of the Quadrant, by the Radius of the Circle, and it gives the Content.

To find the Length of an Arch of any Part of a Circle.

1. Find the Circumference of the whole Circle to the given Radius, as has been taught in p. 151, and alſo find the Angle at the Center made by the two Legs of the Sector. Then ſay,

If 360° give the whole Circumference in Inches, *&c.* what will the Angle at the Center give. Work by a direct Proportion and what comes out is the Length of the Arch Line in Inches, *&c.* or multiply the Chord of half the Arch by 8, and from the Product ſubtract the Chord of the whole Segment or Arch, divide the Remainder by 3, the Quotient is the whole Length of the Arch Line very near the Truth.

13. *To Meaſure the Sector of a Circle.*

Multiply half the Radius into the Arch, or half the Arch into the Radius, gives the Area.

14. *To Meaſure the Segment of a Circle.*

Of the Segment make a Sector, then 'tis plain; from the Area of the Sector, take the Area of the Triangle, and there will remain the Area of the Segment. *See Fig. page* 164.

Or by *Ward's New Theorem* p. 406, of his *Young Mathematician's Guide.*

Viz. Let $\left\{\begin{array}{l} R = \text{the Radius.} \\ d = \text{the X between verſed Sine and Radius.} \\ C = \frac{1}{2}\text{ Chord of Seg. Baſe.} \end{array}\right.$

Theorem $\left\{\dfrac{2\frac{1}{3}RR - 1\frac{1}{3}Rd - dd}{1\frac{1}{2}R + d}\right. \times C = s$, the Area of the Segment.

15. *To Meaſure an Ellipſis.*

Multiply the Tranſverſe by the Conjugate, and that Product by the Area of Unity .785398 gives the Area of the Ellipſis in ſquare Inches, or according to what your Dimenſions were taken in.

16. *To Meaſure a Parabola.*

Note, Every Parabola is $\frac{2}{3}$ of it's Circumſcribing Parallelogram.

Multiply the greateſt Ordinate, or Baſe into the Abſciſſa, or Heighth, and that Product by 2 and divide by 3, gives the Superficial Content.

17. *To Meaſure a Pelicoides. See the Figure in page* 159.

Here you may obſerve that this Figure is a Complication of ſeveral Figures, as the Oblong, Square, Quadrant, Semicircle, Triangle, *&c.* which Figures meaſured ſeparately, and their Contents added together, gives the Content of the Pelicoides. And here for the Information of the Young Student, I ſhall give the Dimenſions of ſeveral Parts of it. Thus, Let the

Side of the Square D E be = 21
Side of the Oblong F E = 42
Semidiameter of the Circle = 21
Chord B C = 29.698
Circumference B C D = 65.97339
Quadrant A D = 32.986695
Perpendicular F G = 14.849

According to theſe Dimenſions the Superficial Content of the Parallelogram B D E F is 882 = to the Content of the Square A B C D. And the Content of the Semicircle B C D = 692.72.

18. *To Meaſure a Lune. See the Figure in page* 160.

Firſt find the Area of every one of the Semicircles, and the Area of the Triangle B A C, then the Area of the

Semicircle B A C — Area Triangle B A C = to the two Segments in that Semicircle. And the Semicircle A F C — the Segment = to the Lune F. Alſo the Semicircle B E A — that Segment = to the Lune E. If we put the Sides of the Triangle 24, 32 and 40, the Area of that △ will be 384 = to the Area of the two Lunes E and F. And the Area of the Semicircle B A C = to the other two Semicircles, becauſe they are made upon the three Sides of the Triangle A B C = to the Ratio 3, 4, 5, by the 47th of the 1ſt of *Euclid.*

19. *To Meaſure a Lune in a Quadrant. See the Figure, page* 160.

Firſt, Meaſure the Triangle, and alſo the Quadrant, and Laſtly, The Semicircle from the Area of the Quadrant, take the Area of the Triangle A, and there remains the Area of the Segment B, then from the Area of the Semicircle take the Area of the Segment B, and there remains the Area of the Lune C. F E = D E = 30.

Area of	Quadrant - -	706.858
	Triangle - -	449.999
	Segment - - -	256.859
	Semicircle - . -	706.644
	Segment ſubtract B -	256.859
	Lune C - -	449.785

20. *To Meaſure a Cycloid, or Trochoid.*

Find the Area of the Circle inſcribed in the Cycloid, which multiply by 3, and the Product is the Area of the Cycloid.

CHAP. IV.

To Meaſure all manner of Solids.

AS a Superficies has Length and Breadth, ſo has a Solid Length, Breadth, and Depth.

1. *To meaſure a Cube.*

Multiply the Side into itſelf, and that Product by the Depth, gives the Content.

2. *To meaſure a Parallelogram.*

Multiply the Length, Breadth, and Depth, one into another, gives the ſolid Content.

3. *To meaſure a Triangular Priſm.*

Find the Area of the Triangular Baſe, which multiply by the Length, gives the ſolid Content.

4. *To meaſure a Pyramid.*

If the Baſe be a Triangle, Square, Pentagon, or any other Polygon, find it's Area accordingly, and multiply it by $\frac{1}{3}$ of the Heighth gives the ſolid Content.

5. *To meaſure the Fruſtum of a Pyramid, cut parallel to it's Baſe.*

Find the Area of each End, and multiply them together, the Squaae Root of the Product is a mean Area, which add to the other two Areas, and multiply that Sum, by $\frac{1}{3}$ of the Length gives the Content.

6. *To Meaſure a Cone.* See the Figure in *page* 129.

Find the Area of the Baſe, and Multiply it by $\frac{1}{3}$ of the Height gives the Solid Content.

7. *To Meaſure the Fruſtum of a Cone cut parallel to the Baſe.*

To three times the Rect-angle of the two Diameters, add the Square of their Difference. Multiply that Sum by $\frac{1}{3}$ of the Length, gives the Solid Content. Or as in the Fruſtum of the Pyramid.

8. *To*

8. *To find the Diameter in any Part of the Fruſtum.*

Say, As the whole Length, is to the whole Difference of the two Diameters: So is any Length from the greater End, To a fourth Number; which ſubtract from the Diameter at the greater End gives the Diameter ſought. Or you may work by the Diſtance from the leſſer End, and add what comes out to the Diameter at the leſſer End, gives the ſame thing.

Or Thirdly, divide the Difference of the two Diameters by the Length, and the Quotient is a common Multiplier; by which multiply any Length from either End, the Product added to the leſſer, or ſubtracted from the greater Diameter gives the Diameter ſought.

9. *To meaſure a Cylinder.*

Find the Area of the End as a Circle, and multiply it by the Length, gives the Solid Content. If the Dimenſion were taken in Inches, divide the ſolid Content in Inches by 1728, (the cubic Inches in a Foot) and the Quotient are Feet.

But more expeditiouſly, divide the Square of the Diameter by 2200.15872 (that is 1728 multiplied by 1.273241 See *page* 733) or multiply it by .00045451 the Quotient or Product is the Area in Feet, which multiply by the Length in Inches gives the Content in Feet.

10. *To meaſure a Globe.*

Here are four ſeveral Ways to do this.

1. Multiply the Diameter by the Circumference, gives the ſuperficial Content, and that by $\frac{1}{6}$ of the Globe's Diameter gives the Solidity.

2. Multiply the Cube of the Globe's Diameter by 11, and divide by 21, gives the Solidity near the Truth.

3. Multiply the Cube of the Globe's Diameter by .523598 (that is the ſolid Content of a Globe whoſe Diameter is Unity) or it is $\frac{2}{3}$ of the Area of Unity .785398, and the Product is the ſolid Content. Or,

4. Divide the Cube of the Globe's Diameter by it's proper Diviſors gives the ſolid Content.

See my *Royal Gauger*, page 106.

10. *To meaſure the Segment of a Globe.*

A Piece being cut off leſs than half the Globe is called a Segment.

Multiply the triple Height of the Segment by the Square of half the Chord, to which add the Cube of the verſed Sine (or Segment's Height) this Sum multiply by .523598, and this gives the ſolid Content.

11. *To meaſure a Fruſtum of a Globe.*

A Fruſtum is more than half the Globe; and it is meaſured as has been taught in the Segment; the Contents of which two being added together will give the Content of the Globe of which they are Part.

Note, If the Diameter of a Globe be 1, then the Circumference and ſuperficial Content are equal, *viz.* 3.141592, and if the Diameter be 6, then the Solidity and ſuperficial Content are equal, *viz.* 113.097312.

12. *To meaſure a Spheriod.*

Multiply the Square of the Conjugate by the Tranſverſe or Length, and that Product by .523598 gives the Solidity.

14. *To meaſure a Parabolic Conoid.*

A Parabolic Conoid is equal to $\frac{1}{2}$ of it's circumſcribing Cylinder.

Multiply the Square of the Diameter of the Baſe by the Height, and that Product by .392698 (that is $\frac{3}{4}$ of .523598) and that Product is the Content.

14. *To meaſure a Hyperbolic Conoid.*

A Hyperbolic Conoid is $\frac{5}{12}$ of it's circumſcribing Cylinder. Therefore to find a Diviſor for Feet, ſay, as 5 : 12 or as 1 : 2.4 :: 2200.158 : 5820.3792.

Multiply the Square of the Diameter at the Baſe by the Height, and divide by 5820.3792 gives the ſolid Content in Feet.

15. *To meaſure a Parabolic Spindle.*

This is $\frac{8}{15}$ of it's circumſcribing Cylinder.

As 8 : 15 :: 1 : 1.875. And,

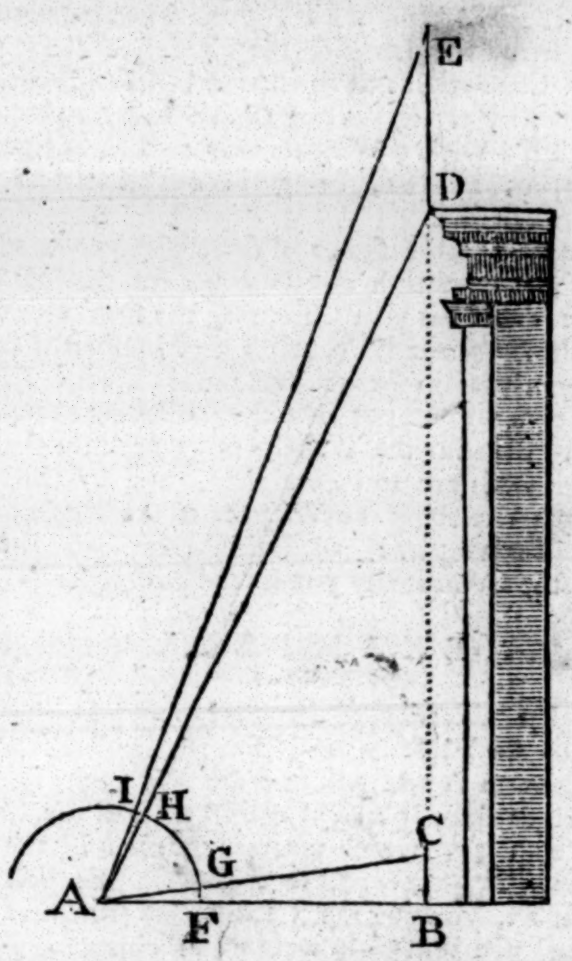
As 1 : 1.875 : : 2200.15872 : 4125.2976.

Multiply the Square of the conjugate Diameter by the Tranſverſe, and divide by 4125.2976 gives the Content in Feet.

16. I ſhall conclude this Part with two uſeful and pleaſant Propoſitions, the one in Statuary, and the other in Staticks; that is, 1. how to give the exact Height of a Statue placed on a Building, that the ſame ſhall appear equal to the common Height of a Man ſtanding on the Ground.

EXAMPLE,

Suppoſe ſtanding at A, B C be a Statue 6 Foot high, and is to be raiſed to D 200 Foot (more or leſs) high, how big muſt the Statue be made, to appear 6 Foot when it is placed at D.



Solution.

Open the Compaſſes to any convenient Radius, ſet one Foot in A, draw the Arch F I, and draw AC, and A D, now here is no more to be done but to make the Angle I A H = Angle G A F, becauſe all Objects that we ſee, appear under a certain Angle, in proportion to the Diſtance of the Object from the Eye: Therefore take F G and ſet from H to I, and draw A I E ſo will D E appear to an Eye at A of the ſame height with B C: Then from the ſame Scale that B C is 6 Foot, meaſure D E, and what you find that height on the Scale, ſo many Feet you muſt make the Statue to appear 6 Foot when elevated 200 Feet, *&c.* above the Level of the Eye.

2. For a Concluſion of this Chapter, I ſhall here ſhew how any quantity of any ſort of Goods under 121 Pounds may be Weigh'd with 5 Weights only: The Weights are theſe 1, 3, 9, 27, 81, being in a geometrical Proportion.

The following table ſhews at firſt Sight how to uſe the Weights, ſo as to weigh any Number of Pounds under 121. The Table has nine Columns, the firſt, fourth and ſeventh Columns under N, contains the Number of Pounds of any Commodity to be weighed; the Column A, contains the Weights to be put into the Weighing Scale; the Column B, contains the Weights to be put into the Scale where the Goods are to be weighed, and by this means of changing and placing the Weights as the Table directs, you may always have an Equilibrium, or the true Weight of the Commodity you intend to weigh.

EXAMPLE,

Suppoſe I would Way 58 Pounds?

Look into the Table under N for 58, and right againſt it in the Column A you will find the Weights 81, 3, 1, which together make 85; right againſt theſe in the Column B is the Weight 27, which you are to place in the other Scale. Now 85—27=58, ſo that if in the Scale B to the Weight 27, you put any Commodity 'till there be an Equilibrium, you will truly weigh 58 Pounds.

Secondly, If you would weigh 100 Pound, you muſt have 81, 27, 1 = 109, and 9 in the other Scale, for 109—9=100, *&c.*

The

The Weighing Table.

N	A	B	N	A	B	N	A	B
1	1.		41	81.	27.9.3.1	81	81.	
2	3.	1.	42	81.	27 9.3	82	81.1	
3	3.		43	81.1.	27.9.3	83	81.3	1
4	3.1		44	81.	27.9.1	84	81.3	
5	9.	3.1	45	81.	27.9	85	81.3.1	
6	9.	3.	46	81.1	27.9	86	81.9	3 1.
7	9.1.	3.	47	81.3	27.9.1	87	81.9	3
8	9.	1.	48	81.3	27.9	88	81.9.1	3
9	9.		49	81.3.1	27 9	89	81.9	1
10	9. 1.		50	81.	27.3.1	90	81.9	
11	9.3	1.	51	81.	27.3	91	81.9.1,	
12	9.3		52	81.1	27.3	92	81.9.3	1
13	9.3,1		53	81.	27.1	93	81.9.3	
14	27.	9.3 1	54	81.	27.	94	81.9.3.1	
15	27.	9.3	55	81.1	27.	95	81.27	9.3.1
16	27.1	9.3	56	81 3	27.1	96	81.27	9.3
17	27.	9.1	57	81.3	27.	97	81.27.1	9.3
18	27.	9.	58	81.3.1	27.	98	81.27	9.1
19	27.1	9.	59	81.9	27.3.1	99	81.27	9.
20	27.3	9.1	60	81.9	27.3	100	81.27 1	9.
21	27.3	9.	61	81.9.1	27.3	101	81 27 3	9 1
22	27.3 1	9.	62	81.9	27.1	102	81.27.3	9
23	27.	3.1	63	81.9	27.	103	81 27.3.1	9
24	27.	3.	64	81.9.1	27.	104	81 27	3 1
25	27.1	3.	65	81.9.3	27 1	105	81.27.	3
26	27.	1.	66	81.9.3	27	106	81.27.1	3
27	27.		67	81.9 3 1	27	107	81.27.	1
28	27.1		68	81.	9.3.1	108	81.27.	
29	17.3	1.	69	81.	9.3	109	81.27.1	
30	27.3		70	81.1	9.3	110	81.27 3	1
31	27.3 1		71	81.	9.1	111	81 27.3	
32	27.9	3.1	72	81.	9.	112	81.27.3.1	
33	27.9	3.	73	81.1.	9.	113	81.27.9	3 1
34	27.9 1	3.	74	81.3	9.1	114	81.27.9	3
35	27.9	1.	75	81.3	9.	115	81.27.9 1	3
36	27.9		76	81.3.1	9.	116	81.9 27	1
37	27.9 1		77	81.	3.1	117	81.9 27	
38	27.9 3	1.	78	81.	3.	118	81.27.9.1	
39	27.9 3		79	81.1	3.	119	81 27 9.3	1
40	27:9.3.1		80	81.	1.	120	81.27.9.3	
						121	81.27.9.31	

CHAP V.

Of the SECTOR.

THE Sector being an Inſtrument of ſuch univerſal Uſe, that to paſs it by with Silence were a Crime too great to be pardoned: Many learned Men have wrote upon the Uſe of the Sector, as *Gunter*, *Forſter*, *Stone*, *Cunn*, &c. to which I refer the inquiſitive Reader for general Satisfaction, and ſhall content myſelf only to ſhew the Uſe of ſuch Lines as are now put upon the Sector, in laying down or meaſuring Angles, in Multiplying and Dividing, and making Proportions.

Upon one Face of the Sector, there is generally a Line of Chords to 60° iſſuing from the Center to the End of each Leg, with a Line of 10 equal Parts decimally divided, iſſuing from the Center, and extends itſelf to the End of each Leg; by which Means this Line is divided into 100 equal Parts, and Numbered with 1, 2, 3, 4, *&c.* to 10, and marked at the End with Lin for Line, ſo 'tis known by the Name of the Line of Lines.

Between the Line of Chords and the Line of Lines, is placed a ſmall Line of Secants, beginning at two braſs Centers at (or near) 25 of the Line of Lines and ending near the Ends of the Sector, where is wrote *Se.* ſignifying Secant. There is alſo a Line of Latitudes anſwering a Line of Chords of the ſame Radius upon the Foot Sector, with a line of Regular Polygons, and ſometimes a Meridian Line for projecting Mercators Chart.

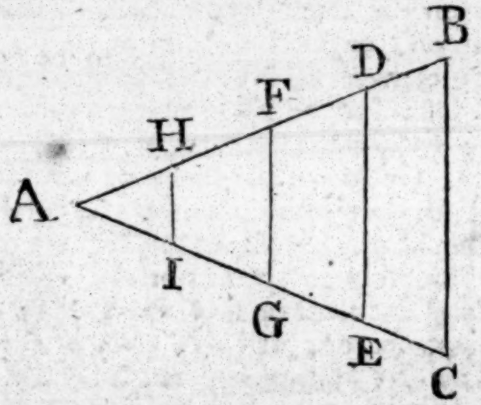
On the other Face of the Sector, is a Line of Sines iſſuing from the Center upon each Leg, numbered with 10, 20, 30, *&c.* to 90 at the End of the Sector; alſo a Line of Tangents to 45°, Numbered with 10, 20, 30, 40, 45° at the End.

Between the Lines of Sines and Tangents, on each Leg is a ſmall Line of Tangents beginning with two braſs

Centers, and N°. with 45, 50. 60, 70, 75 near the End, marked with *Tan.* for Tangents.

On this Face is alſo a Line of Inclination, fitted to a Scale of ſix Hours, theſe with the Chords and Line of Latitude on the other Face, are uſeful in Dialling; how theſe Lines are made, with their Uſe, I have fully explain'd in my *Mechanic Dialling*, which See.

There is likewiſe a Line of Numbers, commonly called *Gunter*'s Line, with the artificial Sines and Tangents, (for the firſt mentioned Lines, are natural Sines and Tangents) with a Line of Inch Meaſure, or a Foot divided into 12 equal Parts decimally divided; how the Line of Numbers is made, ſee my *Royal Gauger*, where you will meet with ample Satisfaction.



The Figure of the Sector is well repreſented by the Iſoſceles Triangle A, B. C. Here the Point A is the Joynt or Center of the Sector to which all the ſectoral Lines are drawn, and they are of an equal Length as A B, A C, and thoſe that bear the ſame Name, are equal and alike divided and numbered. And therefore in all the Operations wrought by the ſectoral Lines, the Points B C, D E, F G, H I, repreſenting the ſame Diviſions, are equally diſtant from the Center A, the ſame is true of any other two Points in the ſectoral Lines A B, A C, *&c.* when they be equal Parts, Chord, Sines, Tangents or Secants. And therefore as A C : C B : : A E : E D, and as A E is to E D : ſo is A G to G F and ſo is A I to I H, *&c. See Prob* IX. *page* 145 *Euclid* 2, *and* 4 *of the* VIth *Book.*

Note, The Lengths A B, A C, A D, A E, A F, A G, A I, are called Laterals, or Crurals, becauſe they lye or are meaſured in the Legs of the Sector,

And the Lines B C, D E, F G, H I, are called Parallels; becauſe all the Proportions wrought by the Sectorals, the two Extreams between the Legs, will be two ſuch Parallel Lines.

If A C on the Line of Lines meaſure 60 equal Parts, and ſo A B likewiſe; then is A C and A B ſaid to be 60 Crural Parts, and B C 45 parallel Parts: And ſo of the Sines, Tangents, and Secants.

Here is one Thing more to be obſerved before we come to the Uſe of the Lines, and that is, ſometimes a Number falls ſo near the Center of the Sector, that it cannot well be taken off, or meaſured; to remedy this, multiply it by any convenient Number at Pleaſure, and take that Product inſtead of the Number itſelf,

As ſuppoſe B C were 5 = Parts, now that, with other in proportion, are to be laid down by the Sector, if I take B C in my Compaſſes and apply it parallel to 5, 5 on the Line of Lines, it will be too long for the Sector, therefore I multiply it by 12 (or any other Number) and the Product is 60, then take the Line B C and apply it parallel to 60, 60, and now all other Lines depending will be in a juſt Proportion, the ſame as if I had laid down the Number 5.

Again, Suppoſe I have a Line whoſe Length exceeds 100, or the whole length of the Line; to remedy this Inconveniency, Divide your given Line by 2, 3, 4, (or any other convenient Number at pleaſure) and apply the Quotient of the given Line, inſtead of the Line itſelf, and by this Means you will bring the Line upon the Sector, which otherways it would not. Suppoſe I have a Line of 118' 120'' to lay down by the Sector, I divide it by 2, and it gives 59' 6'', then take half of the given Line, and apply it parallel to 59' 6'', and now the

Sector ſtands true all one as if I had had a Line of $118'12''$ equal Parts, your own Reaſon with a little Practice will ſoon make Maſter.

2. *The Uſe of the Sector.*

1. To open the Sector that the Line of Lines may ſtand to a Right, or to an Angle of 90 Degrees. Take the Whole Lateral Length of the Line of Lines 10, in your Compaſſes, and ſet one Foot in 6 and the other in 8, in the Line of Lines, and then they will be opened to a right Angle, becauſe $\square\, 8 + \square\, 6 = 100. = \square\, 10.$ by 47 of the 1ſt of *Euclid,* for now you have a right angled Triangle of 6, 8, and 10 Sides = 3, 4, 5. *See page* 137.

2. The Line of Lines may be opened to a right Angle, if the Lateral Sine of 90° be apply'd over parallel, between 45 and 45 on the Sines, or if the Lateral Sine of 45 be apply'd parallel over the Sine 30, 30.

3. The Line of Chords may be opened to any particular Angle, by taking out the Lateral Chord of the Angle required, and applying it over the Parallel of 60°, 60, and then thoſe Lines will ſtand open at the Angle required.

EXAMPLE.

Suppoſe I would open the Line of Chords to an Angle of 30° ?

Take the Lateral Chord of 30, and apply it parallel over 60, 60, and then the Line of Chords do ſtand open at an Angle of 30 Degrees.

4. If the Sector be opened to any Angle at venture, to find the Quantity of the Angle the Lines of Chords then make.

This is but the Converſe of the laſt, for take the parallel Chord of 60, and Meaſure it on the Lateral Chord, and that gives you the Angle ſought. Obſerve the ſame of the Line of Sines, by conſidering that the Sine is half the Chord of the double Arch *See the Figure, page* 126, where A G is the Chord of the Arch A I G, and A H = H G, it's half, being the Sine of the Arch I G.

Suppoſe it were required to open the Sector in the Line of Sines to an Angle of 40° ?

Take out the Lateral Chord of 40, and apply it parallel over 60, 60, ſo ſhall the Line of Sines be opened to an Angle of 40 Degrees.

Or if the Semiradius, as the Sector now ſtands to an Angle of 40, (that is, divide the Lateral Chord of the given Angle into two equal Parts) be applied over, parallel between 30, 30, on the Sines, it will open the Lines of Sines to the given Angle. Thus, In the laſt Example where it was required to open the Lines of Sines to an Angle of 40, divide the Lateral Chord of 40 into two equal Parts, and lay that Extent parallel over 30, 30, on the Sines, and then will the Lines of Sines be opened to an Angle of 40.

Note, It is one Thing to open the Edge of the Sector to an Angle, and another thing to open the Lines on the Sector to the ſame Angle: For when the Sector is cloſe ſhut, the Edges of it make no Angle, but the Lines of Lines, Sines, Tangents, and Secannts, make an Angle of about 6 Degrees.

5. If you would examine the Lines of Chords, Sines, Tangents and Secants, whether they be truly made, project them from a Circle of the ſame Radius. *See Difinitions.*

Open the Sectoral Lines (Sines) not the Legs of the Sector, and take the Sine 10, 10 in your Compaſſes on each Leg in a ſtraight Line, carry this Extent to the Line of Chord, ſet one Foot in the Center, and the other Foot will exactly reach to the Chord 20°; becauſe every Sine is half the Chord of the double Arch. as has been noted above; the ſame obſerve of any other Degree.

6. To lay down an Angle of any Quantity of Degrees.

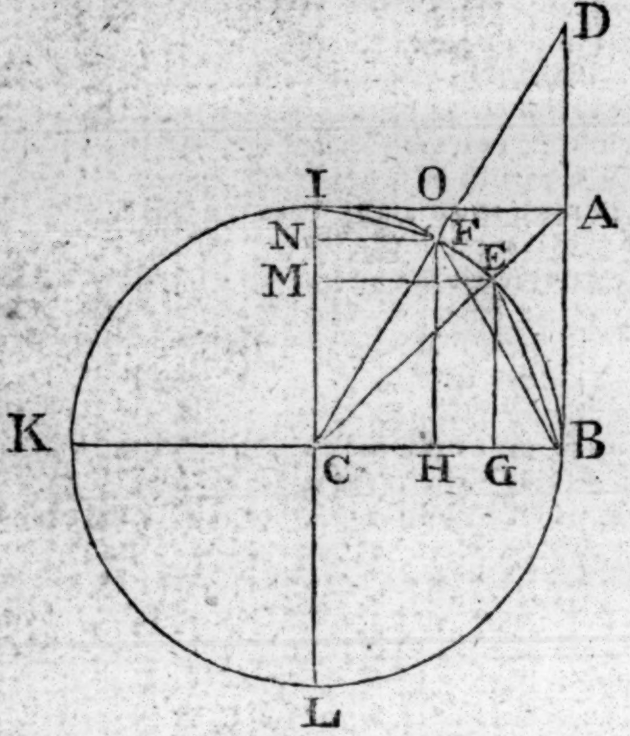
This may be perform'd, either by the Line of Chords, Sines, Tangents, or Secants, due Regard being had to each particular Line.

1. By the Chords, in the annexed Figure, take the Radius C B in your Compaſſes, and therewith ſweep the Circle K I B L, make this Radius a Parallel of 60 on the Sectoral Chords, then take the parallel Chord of 45 from the Sector and ſet it from B to E, and draw C E A, and B A, ſo ſhall the Angle A C B be = 45°, of which B E is the Chord B A is the Tangent, G E the Sine, and C A the Secant of the Arch BE, which meaſures the

Angle A C B. Again take the Radius of the Circle C B, and make it a parallel Radius upon the Sectoral Chord 60, 60, alſo draw B F, and C F D, ſo ſhall the Angle D C B, be 60 Degrees, which is meaſur'd by the Arch B E F, of which B F is the Chord, B D the Tangent, F H the Sine, C D the Secant, and H B the Verſed Sine of Arch B E F, or Angle D C B. 2. And now as the Sector ſtands opened to the Radius C B, take the Tangent B A in your Compaſſes, and apply it parallel on the Sectoral Tangent, and you will find it juſt reach to 45°. 3. Take the Sine E G in your Compaſſes and apply it parallel on the Sectoral Sines, and you will find it reach juſt to 45. The Chord B E will be a Parallel of 45 on the Sectoral Chords. 4. But for the Secant C A, becauſe it is part of the Radius C E, the Sector muſt always be opened to two Braſs Centers which lye near the Joint of the Sector, and that made equal to the Radius of the Circle, and then you will find C A taken and made a Parallel on the Sectoral Secants to be 45°. And thus you will find all the Parts of the Triangle D C B meaſured on their reſpective Lines as above directed to be 60° = to the Angle D C B. So that you ſee the Meaſuring an Angle is only the Converſe of laying down an Angle, as has been ſhewn in both.

And here you are to Note, That the Radius, or Sine of 90 = C I is always equal to the Chord of 60 = B F, and = Tangent 45° B A. That is, the Sine of 90, the Chord of 60, and Tangent of 45 are always equal in the ſame Circle as above has been proved. And that the Radius is a Mean Proportional between the Tangent of an Arch, and the Tangent Complement of the ſame Arch: As ſuppoſe in Fig. 1, page 142, you make A B = B D the Tangent of the Arch B E F in this Figure above, and B C = I O the Tangent Complement of the ſame Arch, then will the Geometrical Mean B D in Fig. 1, page 142, be = C I the Radius of this Circle, and ſo of any other.

Q. E. D.



A Synopſis of the Scheme above.

Of the Arch B E F	B F	is the	Chord.
	F H		Sine.
	B D		Tangent.
	C D		Secant.
	H B		Verſed Sine.

Of the Arch I F	I F	is the	Chord.
	N F		Sine.
	I O		Tangent.
	C O		Secant.
	I N		Verſed Sine,

or Difference between Radius and Co-Sine.

7. *To multiply on the Sector.*

The RULE *on the Line of Lines.*

As 1 is to either Factor laterally, and ſet to 10, 10 parallel, So is the other Factor taken off parallel with your Compaſſes, To the Product laterally.

EXAMPLE.

What is the Product of 4 by 7?

Operation.

Take 7 in your Compaſſes from the Center of the Sector laterally, and lay it parallel over 10, 10, as the Sector now ſtands, take off 4, 4, parallel, and carry this Extent of the Compaſſes laterally, and the other Point will reach to 28, the Product ſought.

Or take off 4 laterally, and lay it parallel on 10, 10, then take off 7 parallel, and apply it laterally, will give 28 as before. This being ſo plain, more Examples are needleſs.

8. *To divide on the Sector.*

RULE *on the Line of Lines.*

As the Dividend laterally, Is to the Diviſor parallel, So is the Parallel 10, 10, To the Quotient.

EXAMPLE.

What's the Quotient of 28 divided by 4?

Operation.

Take 28 the Dividend off laterally in your Compaſſes from the Center of the Sector, and apply it parallel to the Diviſor 4, 4, then take off 10, 10 parallel, and it will reach from the Center of the Sector laterally to 7, the Quotient ſought.

Note, If the lateral Diſtance be too long for the opening of the Sector, then take one Half, one Fourth, *&c.* or multiply them by 2, 3, 4, *&c.* of it at Pleaſure, then Work as above. Suppoſe I would divide 120 by 4? Now 120 falls ſo near the Center of the Sector, the Diviſions are ſcarce perceptable, therefore to have the Number to fall upon the Line, I multiply both Dividend and Diviſor by 2, and then they are 240 and 8, with theſe Work as above is taught, and the Quotient will be 30, the ſame as if you had divided 120 by 4.

9. *The Single Rule of Three on the Sector.*

RULE *on the Line of Lines.*

Firſt obſerve, that the Queſtion muſt be Stated as has been taught in *Chap.* iv. *page* 34. Then,

As the lateral Middle Term, Is to the Parallel firſt Term,
So is the Parallel third Term, To the lateral fourth Term.

EXAMPLE.

Suppoſe 7 Yards of Cloth coſt 22*s.* what will 35 Yards coſt at that Rate?

Operation.

Take 22 laterally and lay it parallel over 7, 7, then take the Parallel 35 and apply it laterally will give 110 *s.* and ſo much doth the 35 Yards come to at the given Price.

Theſe Things I mention more for Curioſity than Uſe, which is only to ſhew what may be done by this uſeful Inſtrument; but thoſe that are curious in Proportions, I refer them to my *Royal Gauger* for the Uſe of the Sliding Rule.

Here is yet one Beauty of this Line of Lines, *viz.* that it is a Scale of equal Parts of all Lengths; and is therefore preferable to any Scale where the Diviſions are only one particular Length.

EXAMPLE.

EXAMPLE.

Suppoſe I would form a Scale of 1 Foot 6 Inches, that may repreſent 124.28 Yards.

Here I cannot open the Sector wide enough to receive $1\frac{1}{2}$ Foot; therefore I multiply both by any Number, ſuppoſe 4, and there will be produced 6 Feet and 497.12 Yards, then I divide all by 6, and I have 1 Foot and 82.85 Yards; therefore opening the Sector 'till one Foot reaches from 82.85 to 82.85 parallel on the other Leg, it will be a Scale of 100 Yards of the Bigneſs required. The Lines of Polygons are very near the inner Edge of the Sector, at the End ſtand the Figure 6, and the other ſmall punch'd Holes in it are number'd with 7, 8, 9, 10, 11, 12 towards the Center.

To inſcribe a regular Polygon of any Number of Sides in the given Circle.

Open the Sector to the Radius of the given Circle, and as the Sector now ſtands take the Side of the given Polygon, *viz.* if 'tis a Pentagon take 5 off parallel, if a Hexagon take 6, if a Heptagon take 7, *&c.* and lay that Extent of your Compaſſes upon the given Circle, and it will inſcribe your Polygon required. In page 175, I have given a Table of regular Polygons, with the Quantity of the Angle at the Center of each; by which you may lay down any of them, for there be ſome Sectors that have not the Polygons on them; therefore, as before, ſet the Sectoral Chord of 60 to the Radius of the given, in which you would inſcribe the Polygon, and with your Compaſſes take off the parallel Chord of the Angle at the Center of the given Polygon, (as in the Table, page 175) and 'tis done.

In page 142 we have promiſed to ſhew how that Figure is laid down by the Sector, which is thus; take the given Line A B = 54 in your Compaſſes and lay it off parallel to 54, 54 on the Line of Lines on the Sector, then take the Line A E in your Compaſſes, and apply it parallel on the Line of Lines on the Sector, and you will find it to be 33.5, and E B you will find by the ſame way of working to be 20.5, After this Manner are the Lines in any Figure either meaſured or laid down. As for the other Sectoral Lines, their Uſe ſhall be ſhewn in the next Chapter when we come to treat of *Trigonometry*,

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CHAP. VI.

Of Plain Trigonometry.

DEFINITION.

TRIGONOMETRY is the Art of Meaſuring Triangles, or of calculating the Sides or finding the Angles of any Triangle ſought; and this is either Plain, or Spherical: of which I ſhall ſpeak diſtinctly; and firſt of the Plain.

The Art of Trigonometry doth much depend on the Knowledge of the Lines in the Figure page 192, which the Learner muſt carefully obſerve to underſtand well, and then he may proceed.

Firſt then, in that Figure page 192, it is plain, that as the Co-Sine is to the Sine, ſo is Radius to the Tangent. That is, As N F : H F : : C B : B D.

2. As Radius is to the Sine, ſo is the Secant to the Tangent. That is, C F : F H : : C D : B D.

3. As the Sine is to Radius, ſo is Radius to the Co-Secant. That is, As F H : F C : : F C : C O.

4. As the Tangent is to Radius, ſo is Radius to the Co-Tangent. That is, As B D : C B : : C B : I O.

Therefore the Rectangle between the Tangent and Co-Tangent of any Arch is equal to the Square of the Radius.

5. Every Triangle hath ſix Parts, of which three are Sides, and three Angles; and of theſe if we have three given we can find the reſt, (except in the Caſe where the three Angles of a plain Triangle are only given.) For from thence the Sides cannot be found becauſe two Triangles may be equi-anglar, and yet have the Sides by no means of the ſame Length. We can find the reſt, I ſay, if ſuppo-

ſing the Radius divided into any Number of equal Parts, we can but diſcover how many ſuch Parts, any Sine, Tangent, or Secant of any Arch or Angle doth contain. Now this is readily done to our Hands, in the Tables of Sines, Tangents, and Secants, which we have with prodigious Induſtry, in Books, (of which *Sherwin*'s is the beſt) ready calculated (to our Hands) for this Purpoſe.

6. When therefore any Triangle is given to be reſolved, the firſt Thing we have to do, is to conſider that there is in the Table of Logarithms, Sines, Tangents, and Secants, a Triangle exactly Similar, and equal to that we are required to ſolve, and whoſe Sides are to one another in the very ſame Proportion of thoſe of the Triangles propoſed.

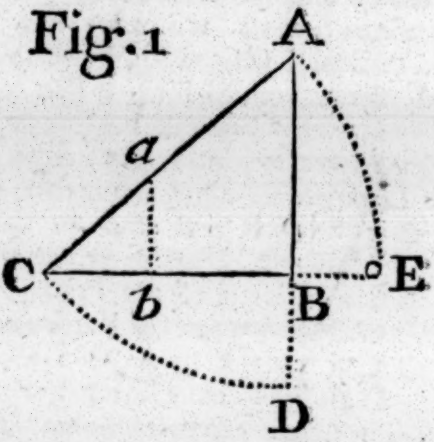
7. We muſt underſtand whatever Ratio one Side of the Triangle given, hath to the other Side about the ſame Angle, conſidered as Lengths eſtimated or number'd by any known Meaſure, as ſuppoſe Inches, Yards, Miles, Leagues, *&c.* the very ſame hath the two Sides about the ſame Angle in the Triangles in the Tables of Logarithms, *&c.* which two Things well underſtood, do lead us into the Whole Myſtery of Trigonometrical Calculation.

8. There is one general *Axiom* which ſolves all the Caſes in plain Trigonometry, which is this.

In a right-angled plain Triangle, if any of the Sides be put, or ſuppoſed the Radius, Whole Sine or Semidiameter of a Circle, then the other two ſhall be as Sines, Tangents, or Secants, as by them repreſented.

And the ſame Proportion that the Side put for Radius, has to Radius, the ſame hath the other Sides to the Sine Tangent, or Secant, by them repreſented.

Fig. 1

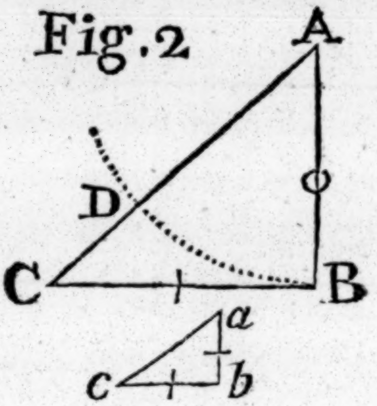


1. If Hypotheneuſe be made the Radius of a Circle, every Side is proportionable to the Sine of it's oppoſite Angle. For ſetting one Poot of the Compaſſes in A, extend the other to C, and draw the Arch C D, (which you may ſuppoſe to be a perfect Circle of which A C is the Semidiameter, or Radius). Now 'tis plain from the Figure page 192, that C B, the Baſe, is the Sine of the Arch C D, and the Arch C D meaſures the Angle C A B, conſequently the Baſe C B is the Sine of the Angle at A. Again, make C the Center of a Circle, and with the Radius C A draw the Arch A E, here the Arch A E meaſures the Angle A C B, and becauſe A B is the Sine of the Arch A E, therefore A B the *Cathetus*, or Perpendicular is alſo the Sine of the Angle at the Baſe, *viz.* Angle C.

For hence it is evident that every Side is the Sine of it's oppoſite Angle, and ſo will always hold, as Radius or Sine of the Angle B is to A C, the Side made Radius, ſo is the Sine of the Angle A to C B the Baſe, and ſo is the Sine of the Angle C to A B the Perpendicular, *& Contra.*

Secondly, Set one Foot of your Compaſſes in A, and extend the other to B, now is A B made Radius, or Semidiameter of a Circle, with which draw the Arch B D, which is ſufficient for our preſent Purpoſe, as well as if we had drawn the Whole Circle; now the Arch B D meaſures the Angle at A, and 'tis plain C B is the Tangent of the Arch D B, therefore C B is alſo the Tangent of the Angle at A, and C A is the Secant of the ſame Angle, now it will hold as Radius, or Sine of 90° is to AB (which is now made Radius) ſo is the Tangent of the Angle A to C B, and ſo is the Secant of the Angle A to C A. Or as A B is to Radius, ſo is C B to the Tangent of the Angle A, and ſo is C A to the Secant of the Angle A.

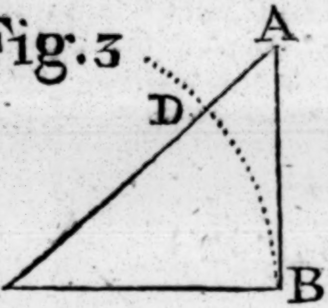
Fig. 2



Thirdly, Set one Foot of the Compaſſes in C, and extend the other to B, and with that Extent draw a Circle, or the Arch D B will do, now the Arch B D meaſures the Angle at C, and B A is the Tangent of the ſame Arch, conſequently B A is the Tangent of the Angle C. Now it will hold, As Radius, Sine of 90, is to C B (now Radius) ſo is the Tangent of the Angle C to the Perpendicular B A; and ſo is the Secant of the Angle C to the Hypothenuſe C A. Or as the Baſe C B is to Radius, ſo is the Perpendicular B A to the Tangent of the Angle at the ſe, *viz.* the Angle A C B.

Note, When we expreſs an Angle by three Letters, the middle Letter always denotes the Angle.

Fig. 3



And as in the Rule of three in common Numbers, ſo here you muſt obſerve that the Firſt and Third Terms be Things of one Name, and likewiſe the Second and Fourth Terms muſt be of one Name; that is, if the Firſt be a Side, the Third Term muſt be a Side, and the Second and Fourth Terms will be Angles; but if the Firſt Term be an Angle, the Second and Fourth will be Sides: And when you ſeek an Angle begin with a Side, but when you ſeek a Side begin with an Angle. For the Sum of the Logarithms of the Second and Third Terms, take the Logarithm of the Firſt Term, and the Remainder is the Logarithm of the Fourth Term, or Anſwer. But when Radius comes not in the Proportion, (as it will not in oblique Triangles) take Complement Arithmetical of the Logarithm of the Firſt Term, and add them all into one Sum (always rejecting Radius) gives the Logarithm of the Fourth Term, or Anſwer. The Complement Arithmetical is found by ſubtracting each Figure of the Logarithm ſeverally from 9, but the Unites Place from 10, and the Remainder is the Complement Arithmetical of the Logarithm ſought.

EXAMPLE.

What's Complement Arithmetical of the Sine of 20° 20′?

See the Work.

From - - - - - - .99999990
Take the Logar. Sine of 20° 20' - .95409314

Co. Arith. of - 20° 20' 0.4590686

What's the Co. Arith. of the Logarithm of 60?

From - - - - - - .9.9999990
Take the Logarithm of 60 - .1.7781512

Co. Arith. of the Logarithm 6 is = 8.2218488

Note, The Complement Arithmetical of a Tangent, or Tangent Complement, is the Tangent Complement of that Tangent, or the Tangent of that Tangent Complement.

EXAMPLE.

What's the Com. Arit. of the Logarithmetical Tangent of 20° 20'?

See the Work.

Logarithmetical Tangent 20° 20' is = 9.5688735
Tangent Compl. or Co. Arith. = 10.4311265
And Co Ar. of the Tangent of 69° 40' = 9.5688735

The like obſerve of any other.

If you work with Tables wherein there is no Logarithmetical Secants, that defect may readily be ſupplied by ſubtracting the Co-Sine out of the double of the Radius, the Remainder is the Secant of that Arch, or Angle, whoſe Co-Sine you made Uſe of in the Work.

EXAMPLE.

What's the Logarithmetical Secant of 20° 20'?

Operation.

Operation.

Double of the Radius = 20.0000000
Co-ſine of 20° 20′ = 9.9720579
Secant of 20 20 = 10.9279421

By what goeth before, it appears that the Secant and Co-Sine of any Arch, are the Complement one of the other to the double Radius of artificial Numbers, ſo that if one be given, the other is alſo known.

The verſed Sine may be ſound by adding .3010300 unto the double of the Sine of half the Arch, and rejecting Radius, gives the verſed Sine ſought.

EXAMPLE.

What's the verſed Sine of 20° 20′ ?

Firſt, ſet down - - - .3010300
Half of 20° 20′ = 10° 10′ it's Sine 9.2467746
The ſame again - - 9.2467746
The verſed Sine of 20° 20′ = 8.7945792

To find the Logarithm of any Number under 101000.

The largeſt and beſt Tables of Logarithms now in being, are thoſe known by the Name of *Sherwin*'s Tables; which I adviſe all Students in the Mathematics to make uſe of, as being the largeſt and moſt correct.

Firſt then, It is to be obſerved, that the Characteriſtic or Index is always leſs by one than the Number of Places of the abſolute Number, and that all Numbers with their Logarithms under 1000 are found in each Column anſwering each Number, and ſo you will find the Logarithm

of	1	to be	0.0000000
	2		0.3010300
	3		0.4771212
	4		0.6020600

And of	to be
10	1.0000000
20	1.3010300
30	1.4771212
40	1.6020600

Here you ſee that the Logarithm of 1 and of 10 is the ſame, only changing the Index as was noted above.

To find the Logarithm of any Number, from 1000, *to* 101000.

If your given Number has only four Places, the Logarithms is in the next Column under 0, by placing it's proper Index 3. If the Number has five Places, as ſuppoſe 11814, find 1181 in the firſt Column, and 4 at the Top; and in the common Angle is .0723970, to which prefix it's Index 4, and the Logarithm of 11814 will be 4.0723970; after this Manner may the Logarithm of a decimal or mixt Number be found, as is made manifeſt by theſe Examples.

Numb.	Logar.
1181.4	3.0723970
118.14	2.0723970
11.814	1.0723970
1.1814	0.0723970
.11814	1.3723970
·011814	2.0723970

If you would know the Logarithm of $27\frac{1}{10}$, ſet it down as whole thus, 27.1, and look for the Logarithm thereof, all one as if it were whole, only put one for the Index and 'tis done; ſo I find the Logarithm of 27.1 to be 1.4329693.

If you would have the Logarithm of a vulgar mixt Number, as ſuppoſe $27\frac{1}{2}$?

Reduce it into an improper Fraction, and ſubtract the Logarithm of the Denominator, from the Logarithm of the Numerator 55, and the Remainder is the Logarithm ſought.

Operation.

Numerator - -	55	Logarithm	1.7403627
Denominator -	2	Logarithm	0.3010300
Logarithm of $27\frac{1}{2}$ is	=		1.4393327

After the ſame Manner I find the Logarithm

$$\text{of } \left\{ \begin{array}{c} \frac{1}{4} \\ \frac{1}{2} \\ \frac{3}{4} \end{array} \right\} \text{ to be } \left\{ \begin{array}{l} 0.6020600 \\ 0.3010300 \\ 0.1249387 \end{array} \right.$$

See my Compleat Syſtem of Aſtronomy, page 287, for further Satisfaction.

In finding a Sine or a Tangent, if your Degree be under 45 you muſt ſeek it on the Head of the Table, and the Minute in the firſt Column on the left, down to 60; but when your Degree is more than 45, you muſt have it at the Bottom of the Table, and the Minutes on the right Hand aſcending to 60'.

Let it be required to find the Logarithm, Sine and Tangent of 29° 45'?

Sine = 9.6956712, Tangent = 9.7570520, with their Differences 2210 and 2932; by Help of theſe Differences we find the Seconds anſwering any Quantity of Seconds under 60, thus,

Suppoſe I wanted the Sine of 29°. 45'. 38'', you muſt always ſay,

If 60' : 2210 *Diff.* :: 38''

38

17680
6630

---

60) 83980 ($1399\frac{2}{3}$

23

59

58

40

To the Sine of 29° 45' - - - - 9.6956712
Add the proportional Part $1399\frac{2}{3}$
Logarithm Sine of 29° 45' 38"=$9.6958111\frac{2}{3}$

The like obſerve for Tangents to any Quantity of Seconds.

But if you have a Logarithm, Sine, Tangent or Secant, and want the Arch or Angle anſwering thereto, it is but the Converſe of the former Operation; for ſuppoſe the Sine were 9.6958111, and the Angle anſwering required?

See the Work.

Given Sine = - - - - - - 9.6958111
Neareſt leſs Sine 29° 45' = 9.6956712
Difference = 1399

Now ſay, If 2210 *Diff.* : 60' : 1399 *Diff.*
60

2210) 83940 (38"
693

1764
1768 *Ferè*

By which I find the Arch anſwering is 29° 45' 38". And here it will not be amiſs to inform the Reader that inſtead of ſaying Sine Complement or Tangent Complement, we ſay for ſhortneſs, Co-Sine, or Co-Tangent, *&c.* and the Word Complement ſignifies a filling up, or what any Arch or Angle wants of 90, or of 180°, *&c.*

Here are ſeven Caſes, which we ſhall briefly illuſtrate as follows; in which Things given are marked with a Daſh of the Pen thus ×, and Things required with a Cypher thus ∋. And further

Note, That when the Angles are required, a required Side cannot be made Radius. And in the Solution of the following Caſes we ſhall perform the ſame.

1. By Conſtruction.
2. By Logarithms, *&c.*

3. By *Gunter*'s Scale, or Slider.
4. By the Sector.
5. By the Natural Sines, Tangents, and Secants.
6. By Natural Numbers.

Caſe I.

Given, The Angles, and Baſe, to find the Cathetus, or Perpendicular, and the Hypothenuſe.

Let the Angle at the Perpendicular be 48° 30', then the < at the Baſe will be it's Complement to 90, *viz.* 41° 30', and the Baſe 56 Feet, Yards, Miles, or any other Meaſure at Pleaſure. I demand the Perpendicular and the Hypothenuſe?

The Hypothenuſe made Radius. Conſtruction, Fig. I. page 198.

Open the Sector to any convenient Diſtance, on the Line of Lines, and take off 56 and lay it from B to C, take the Chord of 60 (of any Radius whatſoever) ſet one Foot of the Compaſſes in C, and ſweep the Arch A E, as the Sector now ſtands take off from the Chords 41° 30' the Quantity of the given Angle at the Baſe, and ſet it from E to A, draw C A and B A, and the Triangle is compleated; then meaſure C A and B A ſeverally upon the Line of Lines on the Sector (being firſt ſet the Baſe 56) and you will find A B to meaſure 50 *ferè*, and A C 74.8 Feet, &c.

Analogy for the Hypothenuſe, C A.

As S < A	48°	30'	9.8744561
To C B		56	1.7411880
So Radius	90	00	10.0000000
To A C		74.77	1.8737319

For the Perpendicular B A.

As Radius	90°	00'	10.0000000
To A C		74.77	1.8737319
So S < C	41	30	9.8212646
To B A		49.545	1.6949965

Secondly, In *Caſe* I, the ſame Things given to find the required, A B made Radius. *See* Fig. II. *page* 199.

Analogy for the Perpendicular.

As Tang. < A	48°	30'	10.0531916
To B C	-	56	1.7481880
So Radius	90	00	10.0000000
To A B		49.545	1.6949964

For the Hypothenuſe A C.

As T < A	48°	30'	Co. Ar.	9.9468084
To C B		56		1.7481880
So Sec. < A	48	30		10.1787354
To C A	74.77			1.8737318

Or you may ſay thus, A B being found.

As Radius	90°	00'	10.0000000
To A B		49.545	1.6949964
So Sec < A	48	30	10.1787354
To A C		74 77	1·8737318

3. *To Work the ſame upon* Gunter'*s Scale.*

Extend the Compaſſes in the Line of Sines, from 48° 30', to 46 in the Line of Numbers, and the ſame Extent will reach from Radius or Sine of 90°, to 74.8 in the Line of Numbers. For the Perpendicular A B, extend the Compaſſes from the Sine of 90 to 74.8 in the Line of Numbers, and the ſame Extent will reach from the Sine 41° 30' to 49.5 in the Line of Numbers.

There are Sliding *Gunter*'s made, which worketh far better than with Compaſſes.

4. *By*

4. *By the Sector.*

This moſt uſeful Inſtrument is fully deſcribed in Chap. V. with it's Uſe in many Caſes; I ſhall here compleat it's Uſe in the Solution of Triangles.

Caſe I.

To find the Hypothenuſe A C *Analogy.*

As S A : C B :: R : A C.

Take the Baſe 56 in your Compaſſes, and make it a lateral Diſtance in the Line of Lines, carry this Extent to the Sines, and make it a Parallel of 48° 30' the Quantity of the Angle at the Perpendicular, then take off the Parallel Line of 90°, and that Extent will reach laterally in the Line of Lines, from the Center of the Sector, to 74.8, the Hypothenuſe A C, as required.

Secondly, *For the Perpendicular* A B *Analogy.*

As R : A C :: S. C : B A.

Take the lateral Hypothenuſe A C 74.8 on the Line of Lines, and make it a parallel Radius on the Sines 90, 90, then take the parallel Sine of 41° 31, the Angle at the Baſe, and it will reach latterally from the Center in the Line of Lines to 49.54, the Perpendicular ſought.

Secondly, *The Perpendicular* A B *made Radius, for the Perpendicular* A B *Analogy.*

As T. A : C B :: R : A B.

Take the Baſe C B = 56 laterally on the Line of Lines, and make it a Parallel on the ſmall Tangents of 48° 30', then the Parallel of the ſmall Tangent 45° will reach from the Center of the Sector laterally to 49.54, the Perpendicular ſought.

5. *By the natural Sines, Tangents, and Secants.*

In *Sherwin*'s Tables you have the Natural Sines and Tangents; anſwering to every Degree and Minute of the Quadrant: The whole Sine, or Semi-diameter of the Circle = Radius being = 10000000, which are no other than the natural Numbers of the artificial Sines and Tangents: And this Way of working was uſed before the Logarithmetic Canon was invented, but it being troubleſome, and now quite out of Uſe, I offer it here only for Curioſity, and not for Practice.

Caſe I. *Scheme* 1. *page* 198.

$$\text{Given}\left\{\begin{array}{l} CB\ 56 \\ <CAB\ 48^\circ\ 30' \\ <ACB\ 41\ \ 30 \end{array}\right\}\text{To find}\left\{\begin{array}{l} AC \\ AB \end{array}\right.$$

Firſt, *The Hypothenuſe* A C, *Analogy.*

As S A : C B : R :: A C.

Seek in the Tables for the natural Sine of the Angle C A B 48° 30', and you will find it to be 7489557, which muſt be the firſt Term in the Rule of Three; the Baſe = 56 the ſecond Term, and Radius the third Term; which being placed in due Order will ſtand thus.

	Sine A.		C B		*Radius.*
As	7489557	:	56	::	10000000.

```
                 10000000
                 --------
        7489557)560000000(74.77=A C
                 52426899
                 --------
                  35731010
                  29958228
                  --------
                   57727820
                   52426899
                   --------
                    53009210
                    52426899
                    --------
                      582311
```

The

The Analogy for the Perpendicular A B.

As S A : C B : : S C : A B. Or,
As R : A C : : S C : A B.

Operation.

```
      S. A        C B        S. C.
As 7489557   :   56   : :   6626201
                                 56
                           --------
                           39757206
                          33131005
                          ---------
                 7489557)371067256(49.54=B A.
                         29958228
                         --------
                          71484967
                          67406013
                          --------
                           40789630
                           37447785
                           --------
                            33418450
                            29958228
                            --------
                             3460222
```

Case I.

The Perpendicular made Radius. Analogy for the Hypothenuſe A C.

As T A : C B : : Sec, A : A C.

Operation.

T. A. : C. B. : : Sec. A.
As 11302944 : 56 : : 15091605
56

90549630
75458025

11302944)845129880(74.77=A C.
79120608

53923800
45211776

87120240
79120608

79996320
79120608

875712

Analogy for the Perpendicular A B.

As T. A : C B : : R : A B.

Operation.

T. A. : C B. : : R.
As 11302944 : 56 : : 10000000
10000000

11302944)560000000(49.545 fereé
45211776
107882240
101726496
61557440
56514720
50427200
45211776
52154240
56514720 +

In this way of Working, you may add Cyphers to the Remainder, and then the Quotient will be a mix'd Decimal, as you ſee in the Works above. And thus you ſee how exactly the Caſes may be ſolved by Natural Sines, Tangents, and Secants, though not ſo expeditiouſly as by the Artificial, as you will ſoon perceive if you compare the one Method with the other.

6. *The Solution of right angled plain Triangles by Natural Numbers.*

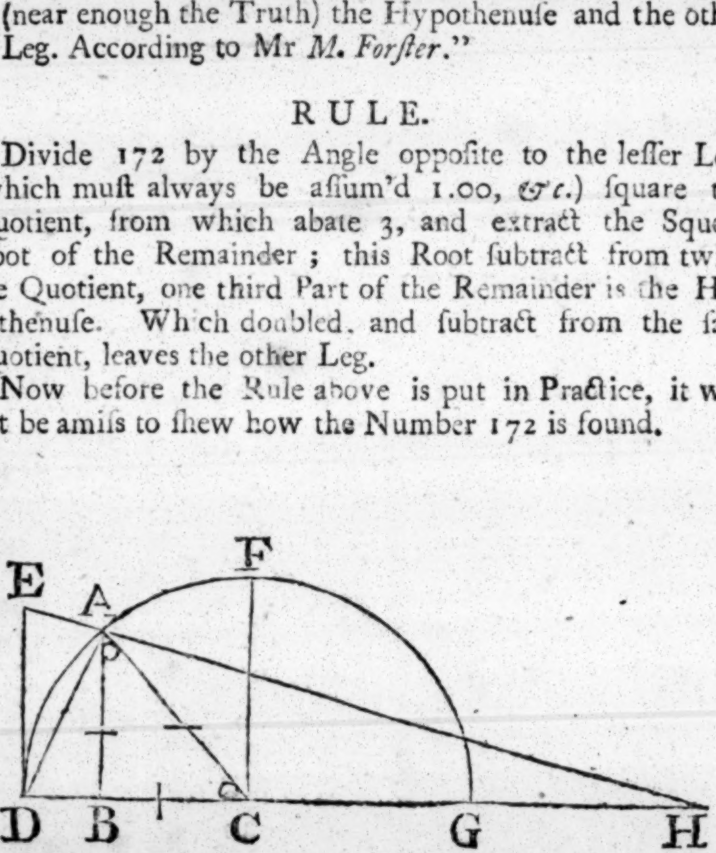
Before we proceed to the Calculation of right angled plain Triangles by Natural Arithmetic, it will be neceſſary to premiſe as chiefly preparatory thereunto;

" That having the Angles in any right angled plain " Triangle, and aſſuming the Leg oppoſite to the leſſer " Angle to be 1.00, *&c.* This following Rule will give " (near enough the Truth) the Hypothenuſe and the other " Leg. According to Mr *M. Forſter.*"

RULE.

Divide 172 by the Angle oppoſite to the leſſer Leg, (which muſt always be aſſum'd 1.00, *&c.*) ſquare the Quotient, from which abate 3, and extract the Square Root of the Remainder; this Root ſubtract from twice the Quotient, one third Part of the Remainder is the Hypothenuſe. Which doubled, and ſubtract from the ſaid Quotient, leaves the other Leg.

Now before the Rule above is put in Practice, it will not be amiſs to ſhew how the Number 172 is found.



Let

Conſtruction.

Let the equal Leg'd and right angled plain Triangle A B C be given. Then with the Radius C A = Hypothenuſe deſcribe the Semicircle D F G, and make G H = C G : Draw D E parallel to B A, as a Tangent to the Angle A C B, and draw E A H; then is B A the Sine, D A the Chord, and D E the Tangent of the Angle A C B. D H is the Triple of the Radius A C or longeſt Side of the Triangle A B C.

Demonſtration.

Becauſe D E is parallel to B A, the Triangles H B A, H D E as ſtanding on the ſame Baſe are Similar, and therefore proportional. Then,

As H B : B A : : H D : D E = D A becauſe D E is the Tangent, and D A is the Chord of the ſame Arch.

The Chord of the Angle A C B, muſt be reckoned an Arch of a Circle whoſe Circumference is 360°, it's half is = D G, being the Chord of 180°, but we muſt find the Diameter in ſuch Parts as the Circumference is 360°. Thus,

As 3.141592 : 1 : : 360 : 114.59158 = D G whoſe half is = 57.25979 = D C, whoſe Triple is 171.88737 = D H, now becauſe of the great Fraction, the 171 is made 172 Whole, whoſe half is 86 called a general Number made Uſe of in finding of Angles, as will be ſhewn by and by. And D C = C G = G H = to the Hypothenuſe C A. Now we ſuppoſe the Leg B C, to be the longeſt of the Two, and therefore it will be, as twice the Hypothenuſe C H + B : H D 172 thrice the Radius; ſo is the ſhorteſt Leg B A to D E = < B A C. In ſhort, as $\frac{1}{2}$ B C + A C : A B : : $\frac{1}{2}$ D H = 86 : to the Angle oppoſite to the ſhorteſt Side.

Now by this general Number 86 and the 47th of the Firſt of *Euclid*, are all the Caſes in right angled plain Triangles ſolved, as will more plain appear by what follows.

Caſe 1. Given, The Angles and Baſe, to find the Hypothenuſe and Perpendicular.

EXAMPLE.

EXAMPLE.

Figure 1, page 198. C A B 48° 30′ A C B 41° 30′, C B, 56, I demand A C and B A ? Firſt, therefore when any right angled plain Triangle is propoſed for Solution by this Method, having the Angle given, you muſt firſt aſſume another right angled plain Triangle as *a b* C, and the Leg oppoſite to the leſſer Angle as *a b*, to be 1.00, *&c.* where by finding the Hypothenuſe and other Leg, (the greater Leg) by the Rule here given ; which done, theſe two Triangles A B C and *a b* C are Proportional. *See Theorem* 9.

That is, (obſerving what Side is given in the propoſed Triangle) As any one Side in the aſſum'd Triangle *a b* C, is to it's correſponding Side in the propoſed Triangle ABC, ſo is any other Side in the aſſumed Triangle to it's correſponding Side in the propoſed Triangle. And this Method will be exact enough for common Uſe, if you work it to two or three Decimal Places, ever remembering to reduce the Minutes of the Angles to Decimals, which is done by Inſpection in the following Table.

A Table to Convert Minutes and Seconds into Decimals, & Contra. *One Degree the Integer.*

Sec.	Decim.	Min.	Decim.
15	.0042	29	·4833
30	.0083	30	·5
45	.0125	31	·5167
Min.		32	·5333
1	.0167	33	·55
2	·0333	34	·5667
3	.05	35	·5833
4	.0667	36	.6
5	.0833	37	.6167
6	.1	38	.6333
7	.1167	39	.65
8	.1333	40	.6667
9	.15	41	.6833
10	.1667	42	·7
11	.1833	43	·7167
12	.2	44	·7333
13	.2167	45	·75
14	.2333	46	·7667
15	.25	47	·7833
16	.2667	48	.8
17	.2833	49	.8167
18	·30	50	.8333
19	·3167	51	.85
20	·3333	52	.8667
21	·35	53	.8833
22	·3667	54	·9
23	·3833	55	·9167
24	·4	56	·9333
25	·4167	57	·95
26	·4333	58	·9667
27	·45	59	·9833
28	·4667	60	1.

Now follows the Operation of Case 1.

1. *For the Hypothenuse* A C.

Angle C = 41.5) 172.000 (4.14
. . . 2
1660 ———
8.28
600
415
1850
1660
190

QUOTIENT.

4.14
4.14
1656
414
1656
Square = 17.1396
Subtract 3. 8.28 from
14.1396 (3.76 Subtract.
. . . 3)4.52(1.51 + =* C a.
9 15
67) 513 2
469
746) 4496
4476
20

---

* The + ſhew the Quotient 1.51 is too much.

C*a* = 151
× by 2

Subtract 3 02
From 4.14

C *b* = 1.12

C *b* CB. C *a.*
Now for C A. As 1.12 : 56 : : 1.51
56

906
755

1.12).84.56(75.49 = C A.
· ·
784

6 16
5 60

560
448

1120
1008

112

Note, The Side opposite to the leſſer Angle is always called 1.

Secondly, For the Leg A B.

C *b* C B *a b.*
As 1.12 : 56 : : 1.
1

1.12) 56.00(50 = A B.
· ·
56 0

0

After this Manner are the 2d and 3d Caſes ſolved; but the 4th, 5th and 6th Caſes muſt be wrought by Natural Arithmetic, thus; For the Sides, the Square of the Hypothenuſe, is equal to the Square of the Sum of the Squares of the other two Legs. *See Theorem* 14.

And to find an Angle, you take half the longer of the two leſſer Sides, *i. e.* Baſe and Perpendicular, and add it to the Hypothenuſe, then ſay, As that Sum, is to the ſhorteſt Side, ſo is the general Number 86, to the Quantity of the Angle oppoſite to the ſhorteſt Side: As we will ſhew by and by, when we come to the Fourth Caſe.

Caſe 2. *Given, the Angles and Hypothenuſe, to find the Baſe and Perpendicular.*

N. B. The Datas are the ſame in all the following Caſes as was aſſum'd in the Firſt Caſe.

Analogy. A C *made Radius.*

As R : A C :: S A : C B. Baſe.
90 : 74.77 :: 48° 30′ : 56 = C B.

As R : A C :: S C : A B Perpendicular.
90 : 74.77 :: 41° 30′ : 49.545 = B A.

Caſe 3. *Given, the Angles and Perpendicular, to find the Hypothenuſe and Baſe.*

Analogy. A B *made Radius.*

As R : A B :: T A : C B.
90 : 49.54 :: 48 30 : 56 = Baſe.
As R : A B :: Sec. A : A C.
90 : 49.54 :: 48 30 : 74.77 Hypothenuſe.

Caſe 4. *Given, the Baſe and Perpendicular, to find the Angles and Hypothenuſe.*

Analogy. A B *made Radius.*

As A B : R :: C B : T. < A.
49.54 : 90 :: 56 : 48° 30′.

As R : A B :: Sec. A : A C.
90 : 49.54 :: 48 30 : 74.77.

By Natural Arithmetic.

1. *Firſt for the Hypothenuſe* A C.

C B = 56	A B = 49.54
56	49.54
336	19816
280	24770
	44586
□ C B = 136	19816
□ A B = 2454.2116	
Sum = 5590.2116	2454.2116

5590.2116(74.77 *ferè.* = A C.
49
144) 690
576
1487) 11421
10409
14947) 101216
104629+

For the Angles. ½ *Longeſt of the two leſſer Sides* = 28

½ C. B = 28
A C = 74,77
Sum = - - - 102.77

Sum. A B

Now ſay, **As** 102.77 : 49.54 : : 86.

86

29724

39632

102.77)4260.4400(41°.45 = <C.

41108

14964

10277

46870

41108

57620

51385

6235

<B = 90° 0'

<C = 41 27

<A = 48 33

.45 = 27'. *See Table, page* 314.

Caſe 5. *Given, the Baſe and Hypothenuſe to find the Angles and Perpendicular. Analogy, Baſe made Radius.*

As C B : R : : C A : Sec. C.

56 : 90 : : 74.77 : 41° 30' = C.

As R : C B : : T C : A B.

90 : 56 : : 41° 30' : 49.54 = A B.

Caſe 6. ***Given, the Hypothenuſe, and Perpendicular, to find the Angles and Baſe. Analogy, Hypothenuſe made Radius.***

As A C : R :: A B : S. C.
74.77 : 90 :: 49.54 : 41° 30′ = < C.
As R : A C :: S. A : C B =
90 : 74.77 :: 48 30 : 56 = Baſe.

Caſe 7. *Given, all the Sides, to find the Angles. Analogy, Hypothenuſe made Radius.*

As A C : R :: C B : S A whoſe Complement to a Quadrant = 90° Angle B = < C 41° 30′. Or,
As A C : R :: A B : C.

2. *For the ſame Angles,* A B *made Radius.*

As A B : R :: C B : T. A.

3. C B *made Radius.*

As C B : R :: A B : T. C.

Note. When you work upon *Gunter*, a Proportion that has Secants, this may be ſupplied by the Co-Sines, for by reaſon the Secant begins not 'till the Sines ends (See the Scheme in *page* 192) ſo becauſe the Secants are not placed upon *Gunter*, the Line of Sines ſupplies that Defect, *viz.* by their Co-Sines, thus,

Sines - - - - - - - 90, 80, 70, 60, 50, 40, 30, 20, 10,
Co-Sines, or Secants, 0, 10, 20, 30, 40, 50, 60, 70, 80,

So in *Caſe* 4, where A B is made Radius to find A C, extend the Compaſſes from the Radius, or Sine 90°, to the Sine of 41° 30′, Co-Sine of Angle A, and the ſame Extent will reach from 49.5 the Side A B, to 74.8 in the Line of Numbers, and ſuch is the Hypothenuſe A C, as was required.

Before I cloſe this Part, I ſhall add ſome uſeful Problems, as a Touch-ſtone to try the Learner's Ability.

PROBLEM I.

Given, the Angles in a right angled plain Triangle, with the Sum of the Sides, to find the Sides ſeverally.

EXAMPLE.

In the Triangle A B C, Fig. 1. *page* 198 the Sum of the Sides have already been found to be 180.31; and < A 48° 30′ < C 41° 30′, I demand the Sides ſeverally.

For a Solution, you may ſuppoſe any of the Sides what Length you pleaſe, as ſuppoſe the Baſe to be 40, now with this falſe Baſe 40, and the true Angles lay down the Triangle, and work with it as if it were the true Baſe thus.

A C *made Radius.*

As S. A : C B :: R : A C.
48 30 : 40 :: 90 : 53.408.

Secondly, For A B.

As S. A Co-Ar. : C B :: S C : A B.
48° 30′ : 40 :: 41° 30′ : 35.389.

Now we have found the other Sides in proportion to the gueſſed Baſe.

A C = 53.408
C B = 40
A B = 35.389
Sum 128.797

Now with this feigned Sum of the Sides find the true Baſe thus,

If the feign'd Z Sides - - - 128 793 C A. 7.8901179
Give for their Baſe - - - - 40 - - - - 1.6020600
What true Z Sides give 180.31 - - - - 2.2560198
Anſwer, true Baſe C B 56.001 - - - - 1.7481977

Now with this true Baſe and the Angles, conſtruct a new Triangle, and find the other two Sides, as has been taught in *Caſe* 1, by which you will find A C 74.77, and A B 49.54, the Sum of which three Sides is equal to 180.31, the given Sum, which proves the Work to be right.

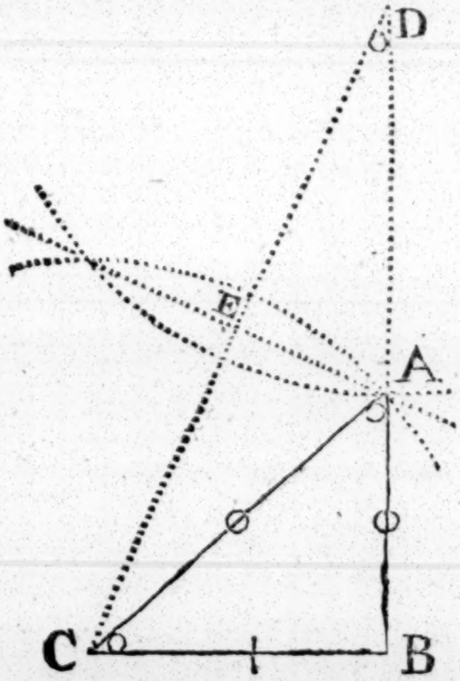
PROBLEM II.

Given, the Baſe, and the Sum of the Hypothenuſe and Cathetus, or Perpendicular, to find the Angles.

N. B. This is grounded upon Theorem 4, where it is proved that the Angle at the Center is double to the Angle at the Circumference.

EXAMPLE.

Let the Baſe C B be 56, and the Sum of the other two Sides 124.31. What are the Angles at A and B? and the Sides A C and A B ſeverally?



Conſtruction.

Draw the Baſe C B 56 of any convenient Length, take it in your Compaſſes and make it a Parallel upon the Line of Lines on the Sector, erect an occult Perpendicular, as B D, drawn at Pleaſure: Now becauſe the Sum of the Hypothenuſe and Perpendicular amounts to 124.31, this exceeding the Bounds of the Sector, take half that Number, *viz.* 62.155 in your Compaſſes from the Line of Lines, (as the Sector was juſt now ſet to the Baſe 56) and lay it twice upon the occult Line B D, and it will terminate at D; draw B D, and C D, Biſect C D in E by *Prob.* I. and draw E A at right Angles to C D, then doth the Point A determine the true Triangles; to compleat which, draw C A, and 'tis done.

Now by Conſtruction the Angle C D B is juſt half the Angle C A B.

For the Angle C A B, D B *made Radius.*

As D B	124.31		2.0945061	
To Radius	90	00	10.0000000	
So C B	56		1.7481880	
To T < C D B	24	15	9.6536819	
Doubled=CAB	48	30	Comp =< A C B	41° 30'

Now by working as in *Caſe* 1. I find A C to be 74.77, and A B 49.54 which two Sides added together, make 124.31 as was at firſt propoſed, which proves the Work to be right. The < A C D = A D C, becauſe A C=A D.

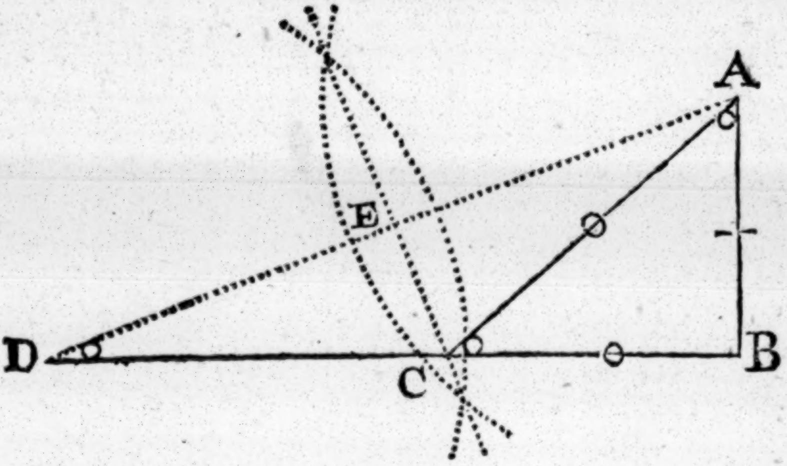
PROBLEM III.

Given, the Perpendicular and Sum of the Hypothenuſe and Baſe, to find the Angles.

EXAMPLE.

Let the Perpendicular A B = 49.54, and the Sum of the Hypothenuſe and Baſe be 130.77; I demand the Angles? and the Sides A C and C B ſeverally?

Conſtruction



Conſtruction.

Draw an occult Line at Pleaſure, and thereon make A B = 49.54, open the Sector to this Diſtance upon the Line of Lines, here let the Sector ſtand; then draw at right Angles to A B, an occult Baſe, and take from the Sector a Parallel of 65.385 on the Line of Lines, (that is the half of 130.77 the Sum of the two unknown Sides) and ſet it twice from B, it will give D, draw B D and A D; Biſect D A in E, and draw E C, and it will cut D B in C, draw A C, and 'tis done.

By Conſtruction, the Angle A D C is half the Angle A C B.

For the Angle A C B, D B *made Radius.*

As D B	130.77	2.1165081
To Radius	90 00	10.0000000
So B A	49.54	1.6949560
To T.<A D B	20° 45′	9.5784479
Doubled = A C B	41° 30′	Comp.=ACB 48° 30′

Now by working as in Caſe 1, I find A C=74.77 and C B=56, which two Sides being added together, make 130.77 as was at firſt propoſed.

PROBLEM

PROBLEM IV.

How by the Proportion of a Man's Shadow unto his Height, or other Shadow to it's Gnomon, ſet perpendicular to the Horizon, to find the Hour of the Day.

1. It is by this Problem that the Table is calculated to find the Hour of the Day by a Staff divided into 10 equal Parts, which Table is printed in many Books, but will only ſerve for one Latitude, *viz.* that for which it was calculated. In order then to make ſuch a Table for any Latitude, you muſt have the Sun's Altitude at every Hour and Quarter of the Day, for every Day in the Year, for your given Latitude, and then it will hold, As Radius, To the Length of the Staff in Parts, as ſuppoſe 10, or 100, or any other Number, So is the Co-Tangent of the Sun's Altitude, To the Length of the Shadow of the Staff in the ſame Parts. Which is inſerted into a Table right againſt the Day of the Month, and in a Column noted at the Top with the given Hour; this is done for every Hour and Quarter of the Day, and for every 5th Day of the Month for your given Place. Thus the Table being made, ſet up your Staff perpendicular on plain level Ground, and mark where the Shadow of the Top of the Staff falls, meaſure the Diſtance from the Place where the Staff ſtood and the mark juſt made, which note in Staff's Length and Parts, then look in your Table for the ſame Number; and at the Top of the Table is the Hour of the Day when the Obſervation was made.

2. By the Altitude of a walking Cane only, *&c.* ſet perpendicular to the Horizon, to find the Height of any Tree, Caſtle, Tower, or Steeple, the Sun ſhining.

The Solution of this, is from ſimilar Triangles; as ſuppoſe your Cane, or Stick be divided into any Number of equal Parts, as 10, 36, *&c.* and you walking along ſeeing an Object caſt a Shadow (the Sun ſhining) ſet up your Stick, and with it meaſure the Length of it's Shadow which we will ſuppoſe to be 40 Parts, that is one Stick's Length and 4 Parts more; with it alſo meaſure the Length of the Shadow of the Object, which let be $13\frac{1}{2}$ Sticks Length, that is 504 Inches; now it will hold, (*See Fig.* II. *page* 199)

As *c b* the Length of the Shadow of the Stick, in the little Triangle, is to *b a* the Heighth of the Stick; So is C B in the great Triangle, the Length of the Shadow of the Tower, to B A the Height thereof.

Operation.

c b. *b a*. CB. B A.
As 40 : 36 :: 504 : 453.6 = 37.75 Yards,

and ſuch is the Height of the Object. And thus by the Help of a Cane or Stick, and plain Triangle, we can tell the Height and Diſtance of any Place.

II. *Of Oblique Angled plain Triangles.*

OBLIQUE Triangles, are ſuch as have in them no right Angle, but are either acute or obtuſe, or both. See the *Geometrical Definitions*.

An Oblique angled plain Triangle has always all the Angles acute, or elſe two acute, and one obtuſe, whereof the Sum of any two of them is always a Complement of the Third to 180°, or a Semi-circle, with which you muſt work inſtead of the obtuſe Angle itſelf. And herein are three *Axioms*, on which the whole Fabric of this Doctrine is laid.

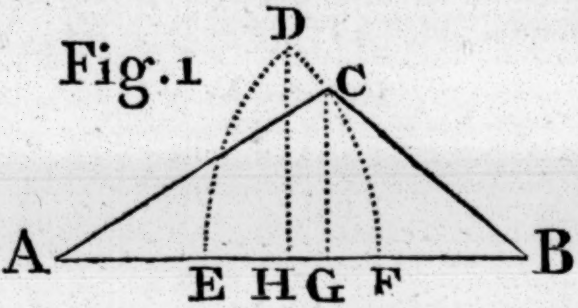
Axiom I.

The Sine of every Angle is in direct Proportion to it's oppoſite Sides, and every Side is in direct Proportion to the Sine of it's oppoſite Angle,

Conſtruction.

Draw the oblique Triangle A C B,

Fig. 1



Make B D = A C, with the Radius B D ſweep the Arch DE, with the ſame Extent ſet one Foot of the Compaſſes in A, and draw the Arch C F, let fall the Perpendiculars C G and D H upon the Side A B.

Now C G is the Sine of the Angle A, and D H, the Sine of the Angle B; the Triangles A C G and B D H are Similar and Proportional. *See Euclid* 6, 4, where 'tis demonſtrated, that the Sides of equiangular Triangles are proportional. Therefore as C G : S B : : G B : S. C G B : and C B : S C G B. The like in the Triangle A C G. And from hence it will always hold in any oblique angled Triangle, (whether plain or ſpherical) As S. A C B : A B : : S C A B : C B, and ſo is the S. A B C : A C; *& Contra.*

Axiom II.

As the Sum of two Sides including an Angle, is to their Difference; So is the Tangent of half the Sum of the other two Angles, (that is the Angles oppoſite to the given Sides) to the Tangent of half the Difference of the ſaid Angles. Which added to the half Sum, gives the greater Angle ſought; but ſubtracted from the half Sum gives the leſſer Angle.

It is by this Axiom that we find the geometric Place of a Planet.

Conſtruction.

Make C D = C B and draw B D; on which, let fall the Perpendicular C E, and it will Biſect D B in E; draw

E F parallel to **A G**, and it will Biſect **A D** in **F**, and is the Difference of the two Sides **A C** and **C B**.

Demonſtration.

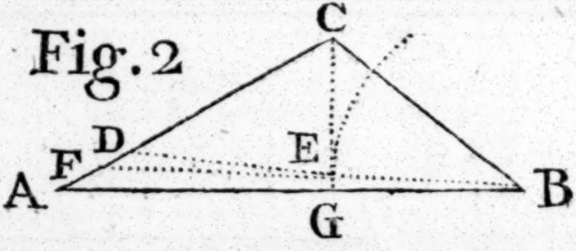
The Angle C B E is half the Sum of Angles C B A and B A C, and E B G half their Difference.

Now if B E be made the Radius of a Circle, E C will be the Tangent of $\frac{1}{2}$ the Sum, and E G of half their Difference. The Triangle C G A is cut proportional by the Line E F. For,

As C F : F A :: C E : E G.

Note, Half the Difference of any two Numbers added to, and ſubtracted from their half Sum, gives the greater and leſſer Number, that is, the Numbers themſelves. As ſuppoſe the two Numbers be 10 and 14, their Sum $= 24$, $\frac{1}{2} = 12$, their Difference $= 4$, $\frac{1}{2} = 2$, then $12 + 2 = 14$ the greater Number, and $12 - 2 = 10$ the leſſer Number.

Fig. 2



Axiom III.

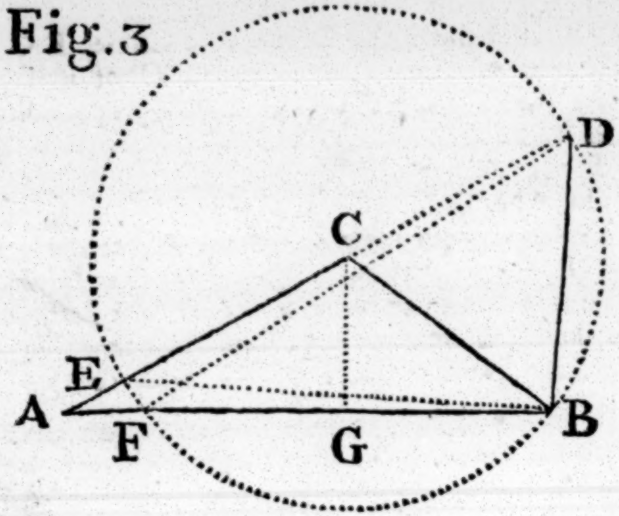
As the true Baſe, Is to the Sum of the other two Sides, So is the Difference of the ſaid Sides, To the alternate Baſe. That is, to the Difference of the Segments of the Baſe made by a Perpendicular let fall from the Angles oppoſite to the Baſe.

Conſtruction.

Lay down the Triangle A B C by Help of the Sector, (as has been already taught) by making **A B** $= 92$ on the Line of Lines, **A C** $= 60$, and **C B** $= 50$. Then on the

Center C, with the Diſtance CB, deſcribe the Circle, which will interſect both of the other Side of the Triangle at E and F; and then D F will repreſent the Sum of the Legs A C and C B, A E will repreſent their Difference, and A F will repreſent the Difference of the Segments of the Baſe made by the fall of the Perpendicular C G.

Fig. 3



Demonſtration.

I ſay AB : AD :: AE : AF.

That is, the Baſe is to the Sum of the Legs, as the Difference between the two Sides, is to the Difference of the Segment of the Baſe.

Note, If the two Legs including an Angle be equal, then the two unknown Angles are known; becauſe they are equal; whoſe Complement of the known Angle to 180, is their Sum, the half of which is each Angle ſeverally.

If the three Sides be equal, the Angles are all acute, and each 60°, and the Sum of all three = 180.

In any oblique angled plain Triangle whatſoever, where the Three Sides are given to find, an Angle. See my *Uranoſcopia, page* 55, 56.

RULE

RULE.

From the Sum of the three Sides ſubtract the Side oppoſite to the Angle required, and Note the Remainder; then to the Co. Ar. of the two Sides, including the required Angle, add the Logarithm of half the Sum of the Sides, and the Logarithm of half the Remainder; half the Sum of theſe Four Logarithms will be the Co-Sine of half the Angle required.

Here are four Caſes which we ſhall illuſtrate in the following Order.

Caſe 1. *Given, All the Angles and one Side, to find the other two Sides.*

EXAMPLE.

Let the Angle A be 30°, B 36° 52' and ACB 113° 8', the Side C B = 50, I demand the other two Sides?

1. Firſt for the Side A C. See Fig. 1, page 228.

As S < A	30°	00'	Co. Ar.	0.3010299
To C B	-	50	-	1.6989700
So S < B	36	52	-	9.7781186
To A C	59.995		-	1.7781185

2. For the Baſe A B.

As S < A	30°	00'	Co. Ar.	0.3010299
To C B	-	50	-	1.6989700
So S < C	66	52	-	9.9635957
To A B	91.959		-	1.9635956

Caſe 2. *Given, Two Sides and an Angle oppoſite to one of them, to find the Angles and the other Side.*

EXAMPLE.

Let C B be = 50, A B = 91.959 and the Angle C A B 30°, what are the other Angles, and the Side A C? See Fig. 1, page 228, by *Axiom* 1. For the Angle B.

Operation.

As C B -	50	Co. Ar.	8.3010299
To S < A	30° 00′	-	9.6989700
So A B	91.959	-	1.9635956
To S < C	66 52	-	9.9635955
From -	180 00		

Angle ACB = 113 08
Angle CAB = 30 00

Sum = 143 08
180 00

Angle ABC = 36 52

2. For the Side A C.

Operation.

As S A -	30° 00′	Co. Ar.	0.3010299
To C B -	50	-	1.6989700
So S B -	36 52	-	9.7781186
To A C -	59.995	-	1.7781185

Caſe 3. *Given, Two Sides, and the Angle included, to find the Angles and the third Side.*

EXAMPLE.

Suppoſe the Angle A B C = 36° 52′, the Baſe of A C 91.995, and C B 50, I demand the other Angles, and the Side A C.

Operation.

By Axiom 2, Fig. 2, page 229.

The Three Angles = 180° 00′
Given Angle B = 36 52

Sum of < A and C = 143 08
Half = 71 34

Secondly,

Given Sides.	A B	- -	91.995
	C B	- -	50.
Sum	=	- -	141.995
Difference	=	- - -	41.995

Analogy.

As Sum of the Sides Co. Ar.	141.995	7.8477728
To their Difference -	41.995	1.6231976
So Tangent $\frac{1}{2}$ Sum Angles	71° 34′	10.4771621
To Tangent $\frac{1}{2}$ X < < ſought	41 35	9.9481325
Sum = < A C B -	113 9	
Difference = < C A B	29 59	

Laſtly, for the Side A C by *Axiom* 1.

Analogy.

As S < A -	29° 59′	Co. Ar.	0.3012489
To C B -	50	-	1.6989700
So S < B -	36 52	-	9.7781186
To A C -	60.026	-	1.7783375

Caſe 4. *Given, The Three Sides, to find the Angles,* Fig. 3, page 230.

EXAMPLE.

Let A B be 91.959, A C 60, and C B 50, I demand the Angles?

By Axiom III.

Operation.

Two ſhorteſt Sides.	A C		60
	B C		50
Sum	=	-	110
Difference	=	-	10

Analogy.

As the Baſe A B	-	91.959	Co. Ar. 8.0364044
To the Sum of A C and C D	110.	-	2.0413927
So is the Difference	10	-	1.0000000
To the alternate Baſe A F	11.967	-	1.0777971
From the true Baſe A C =	91.959		
Remains F B =	-	79.992	

Now F G = G B, half of F B = 39.996 is the true Baſe to the right Angle plain Triangle G C B. Then F G 39.996 + A F 11.967 = A G 51.963, which is the true Baſe to the right angled plain Triangle A C G.

Now the oblique angled Triangle A C B, being reduced into two right angled plain Triangles G C B, and A C G; there are given C B = 50, and G B = 39.996 to find the Angles C B G, which by the 5th Caſe of right angled plain Triangles, will be found to be 36° 52′.

Secondly, In the Rect-angle plain Triangle A C G, there is given A C = 60, and A G = 51.963, to find the Angle of G A C; which by the ſaid 5th Caſe of right angled plain Triangles, will be found to be 30° : And the Angle A C B 113° 8′.

☞ *Note*, Becauſe C D = C B, the Angle C D B = < C B D = 56, 34 it being half of the Angle A C B, and the < B C D = 66° 52′, it being the Complement of the Angle A C B to a Semicircle, or 180°. The Angle A B D = 93° 26′. But the Angle E B D is right, as being made in a Semicircle.

Here follows two Problems Extraordinary.

Given, one Side, and the Angle adjacent, with the Sum of the other two Sides, to find the Angles, and the Sides ſeverally.

EXAMPLE.

In Fig. 3, page 230. Let A B = 91.959, and the Angle at A 30, the Sum of A C and C B = 110, I demand the Angles and Sides ſeverally?

Conſtruct the Triangle as has been already taught, make A B = 91.959, and the < B A C = 30°, draw an occult Line A D, take the Sum of the two ſhorteſt Sides, *viz.* 110, and ſet from A to D, and draw D B, now in the oblique < plain Triangle A D B, we have given A B = 91.959, A D = 110, and the included Angle at A, to find the other Angles, and the Sides A C and A B, which work by the Second Axiom, as has been taught, the < A C B = 113° 8′, the Angle A B C = 36° 52′, and the Side A C = 60, and C B = 50.

PROBLEM II.

Given, one Side and the Angle oppoſite thereto, with the Difference of the other Two Sides, to find the Two Angles and the Two Sides.

EXAMPLE.

Let A B = 91.959, and the Angle A C B = 113° 8′, the Difference of the Two unknown Sides = 10, I demand the Angles C A B, ABC, and the Side A C and C B ſeverally?

Make AB = 91.659, and the Angle ACB 113° 8′, then will C B = Radius of a Circle, which drawn, will cut A C in E, draw E B, now becauſe the Triangle E C B (by Conſtruction) is Equi-crural, the Angle C E B = to the < C B E, and it's Quantity will be known from Theorem 4, that is < E and B, will be each half the Complement of < C, to a Semicircle, and conſequently are each 33° 26′. Now in the Triangle A E B, are known AB = 91.959, AE = 10, and the < AEB, = 146° 34′ it being the Complement of the Angle C E B to 180°: Then by Axiom 1, I find the < A B E = 2° 26′ + < CBE = 33° 26′ = < ABC = 36° 52′ + < ACB = 113° 8′ = 150°. And 180 − 150 = < CAB = 30. Alſo by the ſame Axiom I find A C = 60. C B = 50.

As right angled plain Triangles are uſeful in taking of Altitudes, ſo oblique Triangles are applied to the taking of Diſtances of all Kinds, as ſhall be ſhewn when we come to treat on Surveying of Land.

CHAP.

CHAP. VII.

Of Right angled Spheric Triangles.

DEFINITION.

SPHERIC TRIGONOMETRY teaches to meaſure the Sides and Angles of Spheric Triangles.

1. A great Circle of the Sphere is that which cuts the Globe in two equal Parts, of which there are ſix principally, *viz.* the Equinoctial, Ecliptic, Meridian, Horizon, the Equinoctial, and the Solſticial Colours But you may imagine an infinite Number of great Circles at pleaſure.

2. A Spheric Triangle is made by the Interſection of three great Circles of the Sphere.

3. As (on a plain Surface) a Right Line is the neareſt Diſtance between any two Points; ſo on a Spheric Surface, an Arch of a great Circle is the neareſt Diſtance between any two Points, or Places on the Terraqueous Globe.

4 A Spheric Angle is the ſame with the Inclination of the Planes of thoſe two Circles which conſtitute the Angle; hence the oppoſite Angles at the Interſection of two Circles are equal. A C B = I C H.

5. The greateſt Angle is always oppoſite to the greateſt Side.

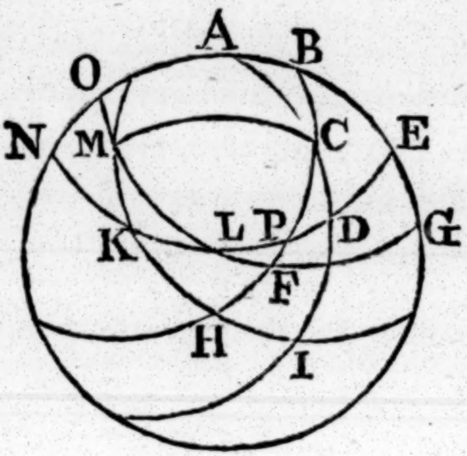
6. A Spheric Angle is meaſured by the Arch of a great Circle deſcribed on the Angular Point at the Diſtance of a Quadrant. So that with one opening of the Compaſſes you may deſcribe a Spheric Triangle, all the Angles right, and every Side a Quadrant, as in the Triangle H M C the Sides are all Quadrants, and the Angles right.

7. What we have ſaid in Plain Trigonometry, holds good here, *viz.* that the Radius is a Mean Proportional

between the Tangent of an Arch, and the Tangent Complement of the ſame Arch, *&c.* which ſee.

8. A great Circle of the Sphere paſſing through the Poles of another great Circle, cut one another at right Angles.

9. The Sides of a Spheric Triangle may be turned into Angles, and the Angles into Sides, the Complement of the greateſt Side, or greateſt Angle to a Semicircle being taken in each Converſion. For in this Scheme. Let A B C be a Spheric Triangle obtuſe angled at B, then is D E the Meaſure of the Angle D A E, becauſe A D and A E are Quadrants, by the 6 hereof; and F G is the Meaſure of the acute Angle F B G, which is the Complement of the obtuſe Angle A B C. Let H I be the Meaſure of the Angle at C: K L is equal to the Arch D E, becauſe K D and L E are Quadrants, and their common Complement is L D; L M is = to F G, becauſe L G and F M are Quadrants, and their common Complement is L F, K M is equal to the Arch H I, becauſe K I and M H are Quadrants, and their common Complement is K H. Therefore the Sides of the Triangle M K L are equal to the Angles of the Triangle A B C, taking for the greateſt Angle A B C it's Complement, *viz.* the Angle D B E. And by the like Reaſon it is demonſtrated that the Sides of the Triangle A B C are equal to the Angles of the Triangle M K L.



For the Side A C = D I, and D I is the Meaſure of the Angle H K P, which is the Complement of the obtuſe Angle M K L. The Side AB = N O; now N O is the Meaſure of the Angle K L M. And Laſtly, B C = F H which is the Meaſure of the Angle at M for A D and C I are Quadrants, ſo are A N and B O, B F and C H; and C D, A O, and C F, are the common Complements of thoſe Arches, therefore the Sides of a Spheric Triangle may be chang'd into Angles, and the Angles into Sides, *Q. E. D.*

10. The three Sides of any Spherical Triangle are leſs than two Semicircles

11. The three Angles of a Spherical Triangle, are together greater than two right Angles and therefore two Angles being known, the third is not known by conſequence as it is in Plain Triangles.

12. If a Spherical Triangle have one or more right Angles 'tis called a right angled Spherical Triangle.

13. If a Spherical Triangle have one or more of it's Sides Quadrants, 'tis called a Quadrantal Triangle.

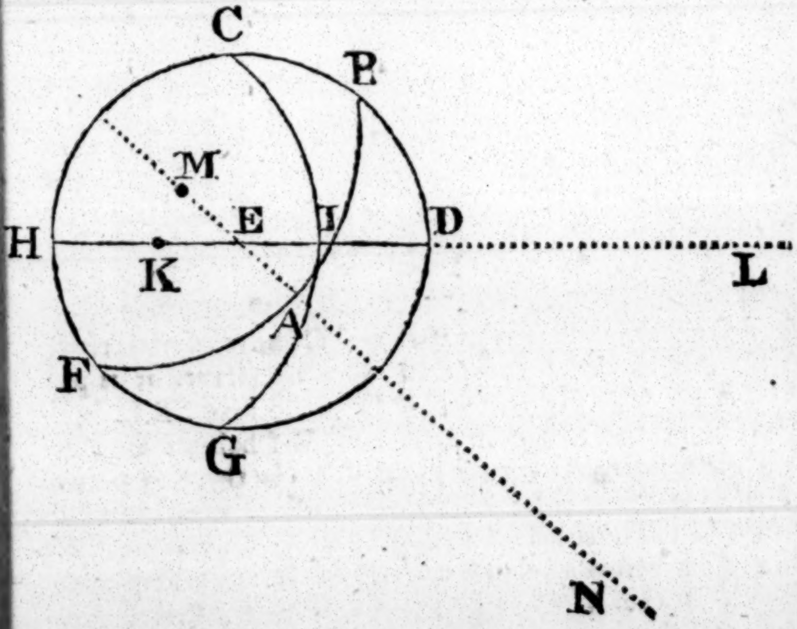
14. If it have neither a Right Angle, nor any Side a Quadrant, 'tis called an oblique Spherical Triangle.

15. If a Spherical Triangle be both right angled and quadrantal, the Sides thereof are equal to the oppoſite Angles. As in the Triangle H M C.

Note, Mention is often made of Quadrantal Sides, that is, ſuch or ſuch a Side is a Quadrant: Now how they are known to be Quadrants, the Learner is to remember that in Plain Trigonometry we have ſhewn that the Sine of 90, the Chord of 60, and the Tangent of 45° are equal to each other, and alſo to the Radius of a Circle; therefore in the Scheme above, the Primitive Circle N O A B E G is drawn upon the Center L, with the Radius L E = L G = L N = L O, conſequently all thoſe Sides are Quadrants, as being equal to the Radius of the Primitive Circle; and ſo by the ſame Rule is the Side of an oblique Circle (in this Scheme) known to be more, equal, or leſs than a Quadrant.

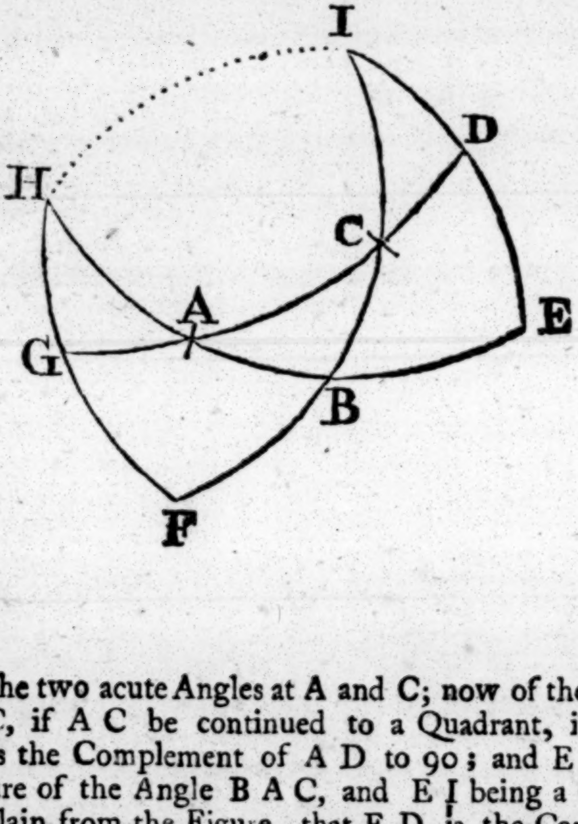
16. Every Circle, whether right or oblique hath two Poles, if it is the Primitive they both lie together in the Center thereof; if it is a right Circle as H D, they lie in the Primitive Diametrically oppoſite at C and G. If

an oblique Circle, as C A G, one Pole lyeth within the Primitive, and the other without; which how to find Geometrically, ſee my *Syſtem of Aſtronomy, Vol. I. page* 62. But by the Sector it is performed thus: Set the Sector on the Tangents 45°, 45°, to the Radius of the Primitive Circle = E D; then let the Pole of the oblique Circle C A G be ſought, take H I in your Compaſſes and meaſure it on the Tangents, you will find it to be 25°, this ſubtract from 45° = E D, there remains 20; which take from the Tangents on the Sector, and ſet from E to K, ſo is K the Pole of the oblique Circle C I G, and lyeth within the Primitive. For the other Pole add 25° that you juſt now found to 45°, and it make 70, ſet the Radius of the Circle = E D to the leſſer Tangents on 45°, and take off the Tangent of the Sum = 70, and ſet it from E to L. So is L the other Pole of the ſaid oblique Circle C I G, and falls without the Primitive Circle. Likewiſe the Poles of the oblique Circle B A F, will be found by the Rule, to fall at M, within the Primitive, and at N without the Primitive Circle. For the laying down



or meaſuring any Spheric Angle, or Side of a Spherical Triangle, with the Projection of the Sphere on any Circle. I refer my Reader for farther Satisfaction, to my *Syſtem of Aſtronomy*, *Vol. I. page* 66, *&c.*

Every right angled Shperical Triangle as A B C (in this Scheme) hath beſides the right Angle at B, five Parts, whereof thoſe three which are fartheſt from the Right Angle, are noted with their Complements, as A C, And the two acute Angles at A and C; now of the Triangle A B C, if A C be continued to a Quadrant, it is plain A C is the Complement of A D to 90; and E D is the Meaſure of the Angle B A C, and E I being a Quadrant it is plain from the Figure, that E D is the Complement of the Quadrant E I: Laſtly, F G is the Meaſure of the Angle G C F, becauſe F C and G C are Quadrants, therefore F H being a Quadrant, F G is the Complement, which is the Meaſure of the Angle at C, therefore A C and the Angles at A and C (by the Lord *Napier)* are always noted with their Complements, but the Complements,



ments of A B is A H in the Triangle A G H, and the Complement B C is C I in the Triangle C I D; ſo that the two Legs A B and B C including the Right Angle at B, are never Complements, but the Sine itſelf, or the Tangent itſelf.

In the Solution of right angled Spheric Triangles, there are always two Things given, beſides the Right Angle, to find out a fourth Term, in which the Right Angle is always ſet aſide, in being any of the Circular Parts, and conſequently doth not ſeperate the two Legs A B and B C, but are always ſaid to lye together. And of the Three Things or Parts concern'd in the Queſtion, one of them is always called the Middle Part, and the other two are called Extreams either conjunct, when they lye together, or disjunct, when they are ſeperated by any other Part.

Now in Order to know which is the Middle Part, obſerve, if the Things given and required lye together, the Middle of them Three is the Middle Part, as ſuppoſe A C and A B were given, and Angle A required; here, becauſe the Things given and required lye together, Angle A being the middlemoſt of the Three is the middle Part, or if A had been given, and A C, or A B ſought, the middle Part would have been the ſame becauſe they lye together. But when the Three Things concern'd in the Queſtion are disjoyned, that which ſtands by itſelf is the middle Parts. As ſuppoſe A B and A C were given, to find B C. Here A B and B C lye together, (for the Right Angle at B makes no ſeperation) and A C is ſeperated by the Angle A and C which have nothing to do in the Queſtion, therefore A C ſtanding alone is the middle Part. The middle Part being thus known, the Solution of right angled Spheric Triangles are perform'd by one univerſal Propoſition, *viz.*

The Sine of the middle Part with Radius is equal to the Tangents of the adjacent Extream; and with the Co-Sines of the oppoſite Extreams. That is,

As Radius, to the Tangent of one of the Extreams conjunct, ſo is the Tangent of the other Extream conjunct, to the Sine of the middle Part.

Alſo, as Radius, to Co-Sine of one of the oppoſite or disjunct Extreams, ſo is Co-Sine of the other Extream disjunct, to the Sine of the middle Part.

When the middle Part is ſought, then Radius muſt be in the firſt Place of the Proportion; but if one of the Extream Parts be ſought, then the other Extream muſt be in the Firſt Place, and Radius in the Second or Third Place, it matters not which.

Illuſtration.

In this Synopſis of the foregoing Triangle A B C, I ſhall ſet down the Quantities of each Part thereof, with their Complements, together with their Sines, Tangents, *&c.* of each Part deſign'd as a Help to the young Student for the right Underſtanding of this moſt Uſeful Part of the Mathematicks.

Sides and Angles.					Sines.	Co-Sines.
Sides.	A C =	66°	31′	00″	9.9624527	9.6004090
	C B =	51	32	00	9.8937452	9.7938317
	A B =	50	09	52	9.8852967	9.8065773
Angl.	C =	56	50	55	9.9228440	9.7378698
	A =	58	36	49	9.9312925	9.7166767

	Tangents.	Co-Tangents.
A C	10.6320437	9.6379563
C B	10.0999135	9.9003459
A B	10.0787190	9.9212810
C	10.1849771	9.8150229
A	10.2146157	9.7853843

In the right angled Spherical Triangle A B C, I have ſhewn how A B and A C are Extreams conjunct, or adjacent to the Angle at A, and that A C and Angle A and C are always Complements: Therefore by the Fundamental Axiom, or Catholick *Propoſition*. The Co-Sine of the Angle C, added to Radius is equal to the Co-Tangent of A C, added to the Tangent of C B, which is briefly expreſſed thus, *viz.* C S < C + R = C T. A C + T. C B, in Numbers, thus,

C S. < C =	56°	50'	55''	= 9.7378698
Radius	90	0	00	= 10.0000000
Sum =	-	-	-	19.7378698

C T. A C =	66	31	00	= 9.6379563	
T. C B =	51	32	00	= 10.0999135	
Sum =	-	-	-	19.7378698	The ſame as above.

Secondly. The Angle A is the Middle Part and required, therefore from the above Equation, this Analogy will ariſe, *viz.*

As R : T. A B : : C T. A C : C S : A.

Or, As R : C T. A C : : T. A B : C S. A, the Middle Part and Thing required.

Again by the Second Part of the Fundamental Axiom: Let A C and C B be given, to find A B, here the Side ſtanding alone is the Middle Part, 'tis an oppoſite Extream, and this Equation will ariſe, *viz.* C S. A C + R = C S A B + C S. C B. Now becauſe one of the Extreams disjunct, is the Middle Part, therefore the other Extream, *viz.* C B muſt come firſt in the Analogy, Radius in the Second Place, and the Middle Part in the Third Place; or Middle Part in the Second Place, and Radius in the Third, it matters not which, becauſe 'tis all one if we add the Second Term to the Third, as if we add the third Term to the Second. Thus, As CS CB : R :: CS : AC : CS CB.

Or, As CS CB : CS. AC :: R : CS A.B.

The Equation Illuſtrated.

CS. A C 66° 31' = 9.6004090
Radius - - 10.0000000
Sum - - 19.6004090

C S. A B 50° 9' 52'' = 9.8065773
C S. C B 51 32 00 = 9.7938317
Sum - - - 19.6004090 The ſame with the other above.

Note, When a Complement in the Proportion falls upon a Complement in the circular Part, you muſt take the Sine itſelf, or the Tangent itſelf, becauſe C S of C S. = S. and C T of C T = Tangent.

Illuſtration.

In the Triangle A B C, page 240. Let A B and Angle C be given, to find Angle A. This is an oppoſite or disjunct Extream, and C is the Middle Part, (becauſe it ſtands alone) and becauſe the other Extreams fall upon C S, the Angle A becomes a Sine for the Reaſon above. Therefore C S. C + R = S A + C S AB.

Analogy.

As C S A B -	50°	09'	52'' - -	9.8065773
To Radius -	90	00	00 - -	10.0000000
So C S. C -	56	50	55 - -	9.7378698
To S. A -	58	36	49 - -	9.9312925

That this Work may want nothing to make it plain to the meaneſt Capacity, I ſhall here put down the Equations and Proportions of five adjacent, and five oppoſite Extreams.

1. *The Five adjacent Extreams.*

In the Triangle A B C, page 240.

Given, $\left\{ \begin{matrix} AB \\ C \end{matrix} \right\}$ To find C B.

Equation, $S.\ CB + R = T.\ AB + CT.\ C.$
Proportion, As $R : CT.\ C :: T.\ AB : S.\ CB.$

2. Given, $\left\{ \begin{matrix} CB \\ A \end{matrix} \right\}$ To find A B.

Equation, $S.\ AB + R = T.\ CB + CT.\ A.$
Proportion, As $R : TCB :: CT.\ C : SAB.$

3. Given, $\left\{ \begin{matrix} AB \\ A \end{matrix} \right\}$ To find A C.

Equation, $CS.\ A + R = TAB + CTAC.$
Proportion, As $TAB : R :: CSA : CT.\ AC.$

4. Given, $\left\{ \begin{matrix} AC \\ A \end{matrix} \right\}$ To find C.

Equation, $CS.\ AC + R = CTA + CT.\ C.$
Proportion, As $CTA : R :: CT.\ AC : CT.\ C.$

5. Given, $\left\{ \begin{matrix} CB \\ C \end{matrix} \right\}$ To find A C.

Equation, $CSC + R = CTAC + TCB.$

Here the Reader muſt remember what has been taught before, *viz.* that the Radius being a Geometrical Mean proportional, between the Tangent of an Arch, and the Tangent Complement of the ſame Arch: Therefore in any Caſe where Tangent or C T comes in the Firſt Term of the Proportion, you may change it into the Second Place and put Radius in the Firſt Place. As for inſtance, in the Third adjacent Extream, the Proportion is there as T. A B to R: You may ſay, as Radius is to the C T A B, and ſo of any other where Tangents come in the Proportion.

Of the Five oppoſite Extreams.

1. Given, $\left\{\begin{matrix} CB \\ C \end{matrix}\right\}$ To find A.

Equation, C S A + R = S : C. + C S. C B.
Proportion, As R : S. C. :: C S C B : C S A.

2. Given. $\left\{\begin{matrix} AC \\ CB \end{matrix}\right\}$ To find A B.

Equation, C S. C A + R = C S. C B + C S. A B.
Proportion, As C S C B : R :: C S A C : C S A B.

3. Given, $\left\{\begin{matrix} AB \\ C \end{matrix}\right\}$ To find AC.

Equation, S, A B + R = S A C + S. C.
Proportion, As S, C : A B : : R : A C.

Here Co-Sines fall upon Co-Sines, therefore I take the Sines themſelves.

4. Given, $\left\{\begin{matrix} AB \\ CB \end{matrix}\right\}$ To find A C.

Equation, C S, A C + R = C S, A B + C S, C B.
Proportion, As R : C S A B :: C S C B : C S, A C.

5. Given, $\left\{\begin{matrix} AC \\ C \end{matrix}\right\}$ To find A B.

Equatoin, S, A B + R = S A C + S, C.
Proportion, As R : S A C :: S C : S A B.

Let the Young Student as he goes over theſe ten Extreams, draw Triangles upon his Slate, or on waſte Paper, and mark the Parts given and required, as expreſſed above, and this will greatly help him to the Knowledge thereof.

What we have ſpoke hitherto has been of Sines and Tangents only, but if the Reader pleaſes, he may uſe Secants alſo ; and for the doing of this let him obſerve that in Fig. page 192, are theſe Proportions, *viz.*

1. As the Tangent is to the Radius, ſo is the Radius to the Co-Tangent.

2. As the Sine is to the Radius, ſo is Radius to the Co-Secant.

3. As the Tangent of an Arch, is to the Tangent of an Arch; ſo is the Co-Tangent of the latter Arch, to the Co-Tangent of the former.

4. As the Sine of an Arch is to the Sine of an Arch, ſo is the Co-Secant of the latter Arch, to the Co-Secant of the former.

EXAMPLE.

In the firſt Caſe of the oppoſite Extream, let it be required, to work the ſame by Secants.

Given, C B 51° 32′, < C 56° 50′ 55″, *to find the Angle at* A

Firſt, You are to obſerve that for a Sine in the Proportion, you muſt always add a Co-Secant and *Contra*, and in this Caſe it will be,

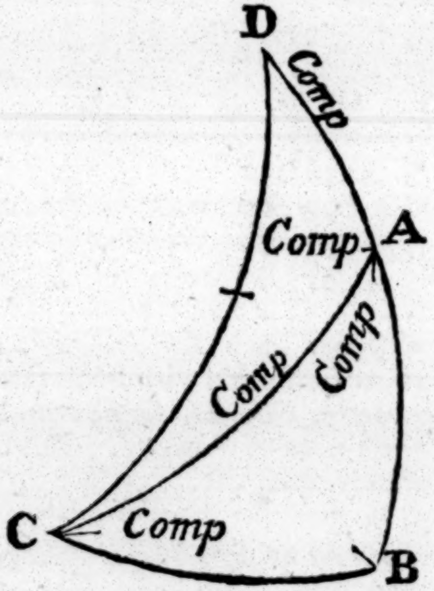
As Co-Sec. < C	56°	50′	55″	-	10.0771560
To Radius	90	00	00	-	10.0000000
So C S C B	51	32	00	-	9.7938317
To C S < A	58	36	49	-	9.7166757

And in like Manner may any Analogy, conſiſting of Sines and Tangents, be perform'd with a Mixture of Secants, (*Martin's Trigonometry, Vol. II, page* 103.) by inverting and tranſpoſing the Terms of Analogy. But theſe Things are more for Curioſity than Uſe.

2. *The Doctrine of Quadrantal Triangles.*

What a Quadrantal Triangle is, has been already ſhewn, and here is to be obſerved that the three Parts that lye fartheſt from the Quadrantal Side are Noted with their Complements. As in the right angled Spheric Triangle A B C, the three Parts, *viz.* A C, Angle A and Angle C, which lye fartheſt from the right Angle at B are Noted with their Complements; ſo likewiſe in the Quadrantal Triangle A C D, the three Parts, *viz.* A C, A D and Angle C A D being moſt remote from their Quadrantal Side C D, are Noted with their Complements, for D A is the Complement of A B to a Quadrant, and the Angle C A D is Complement of the Angle C A B to a Semicircle or 180 Degrees. And here the Univerſal Propoſition holds true, *viz.* that the Sine of the Middle Part and Radius is equal to the Tangent of the adjacent Extream, and to the Co-Sine of the oppoſite: Alſo the Middle Part is found by it ſtanding alone, as in the right Angle has been explained.

For the Learners further Improvement herein, I ſhall ſet down the Quadrantal Triangle with all it's Parts, as in the following Table.



Sides and Angles.					Sines.	Co-Sines.
C D	-	90°	00'	00''	10.0000000	
D B	-	90	00	00	10.0000000	
D A	-	38	28	00	9.7938317	9.8137452
A B	-	51	32	00	9.8137452	9.7938317
B C	-	50	9	52	9.8852967	9.8065773
A C	-	66	31	00	9.9624527	9.6004090
C A D	-	123	9	5	9.9228440	9.7378698
C D A	-	50	9	52	9.8852967	9.8065773
A C D	-	31	23	11	9.7166767	9.9312935
A C B	-	58	36	49	9.9312925	9.7166767
C A B	-	56	50	55	9.9228440	9.7378698
A B C	-	90	00	00	10.0000000	

1. In the Quadrantal Triangle A C D, let C D A be 50° 9' 52'', and A C D 31° 23' 11'', I demand the Side D A?

Equation. *s* D + R = *c t* D A + *t c.*

Analogy.

As *t* C		31°	23'	11''	-	9.7853843
To Radius	-	90	00	00	-	10.0000000
So S. D	-	50	9	59	-	9.8852967
To *c t* D A		38	28	00	-	10.0999124

2. *Another Example ſhall be of an oppoſite Extream, in the Quadrantal Triangle* C D A.

Let the Angle A D C be = 50° 9' 52'', and A D = 38° 28', I demand the Side C A?

Equation, c s. A C + R = *c s* C + *c s* D.

Analogy.

As Radius	-	90°	00'	00''	-	10.0000000
To S. A D	-	38	28	00	-	9.7938317
So C S. D	-	50	9	52	-	9.8065773
To C S, C A		66	31	00	-	9.6004090

In this Triangle C D A, the Side C A is a Middle Part becauſe it ſtands alone, being ſeperated by the Angle D A C, therefore Radius comes in the firſt Place in the Analogy. And here the Reader may obſerve that all Queſtions in Quadrantal Triangles may be ſolved in the right angled Spheric Triangle ABC. As for inſtance, in the laſt Queſtion where D and D A are given, to find A C, becauſe C D and D B are Quadrants, the Side C B is the Meaſure of the Angle D, and D A is Complement of A B to a Quadrant, therefore in the Triangle A B C, we know A B and B C, and A C is required: From which this Equation doth ariſe, $cs\ AC + R = CS\ AB + cs\ CB$.

Analogy.

As Radius	90°	00'	00''	-	10.0000000
To C S A B	51	32	00	-	9.7938317
So C S B C	50	9	52	-	9.8065773
To C S A C	66	31	00	-	9.6004090

This gives us the ſame Anſwer you ſee as when it was ſolved in the Triangle C D A. All Spheric Triangles may be projected *Stereographically*, and the Sides and Angles truly laid down and meaſured; and in this Work there are but three Kinds of Circles, *viz.*

1. The Primitive which is always drawn with the Chord of 60°.

2. A Right Circle, and this always paſſes thro' the Center of the Primitive, cutting it at right Angles, and is the ſame with the Diameter of the Primitive Circle.

3. An Oblique Circle is that which neither lyeth in the Primitive Circle, nor in the Diameter, or Right Circle, but cuts them both at Oblique Angles, and the Points of it's Interſection with the Primitive are always oppoſite, ſo that a Line drawn from one Interſection to another, will paſs through the Center of the Primitive Circle.

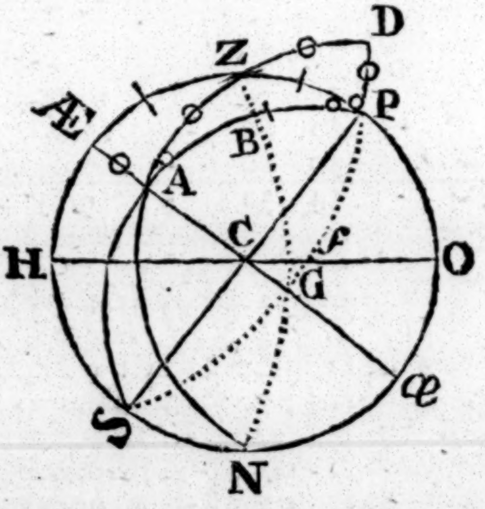
All Degrees are laid upon the Primitive Circle by the Line of Chord. And ſo any Quantity of the Primitive Circle is meaſured by the Chords alſo, as 38° 28' from Z to P in the following Scheme.

Any Quantity of a right Circle, as H O, Æ æ, SP, &c. is meaſured by the half Tangents ; and the Degree that an Oblique Circle makes with the Primitive, are laid down by the Secants.

And the ſeveral Parts of Oblique Circles, or their Angles are meaſured, by reducing of them to the Primitive Circles from their Poles.

III. *Of Oblique Angles Spheric Triangles.*

WRITERS on this Subject generally reckon twelve Caſes, the ten firſt may be reſolved by the Univerſal Propoſition ; if firſt the Oblique Triangle be reduced into two right angled Spherical Triangles, which may be done by letting fall a Perpendicular, which will ſometimes fall within, and ſometimes without the given Triangles, and to know when it will fall within, and when without, obſerve.



Let the Perpendicular fall from the End of a given Side, being adjacent to a given Angle. Whether the Perpendicular fall within or without the Triangle, it muſt always be oppoſite to a known Angle. And the Perpendicular thus let fall, either divides the Oblique angled Triangle, **A Z P** into two right angled Triangles, A Z B, B Z P, or produces two by adding of one to it **A D P**, and **Z D P**, by adding of one right angled Triangle **Z D P**, to the Oblique-angled Triangle A Z P firſt given. If in the Oblique Triangle A Z P, there be given A P, Z P, and **A Z P**, to find A Z, here the Perpendicular Z B cannot be let fall to be of any Service in this Caſe, therefore the Perpendicular muſt fall without the Triangle, as A Æ, or **P D**; for in the Rect-angle Triangle AÆZ, AZ may be found, or if the Side A Z be continu'd to D, 'till A Z D is a Quadrant, and let fall the Perpendicular D P, ſo that it may paſs through the Poles of the two Oblique Circles at F and G, the Angles ZDP, and APD will be right; then in the Rect-angled Triangle ZDP, ZD may be found, which is the Complement of A Z to a Quadrant. And thus by forming of other Triangles adjacent to the given Oblique one, may the Things required be found: As for inſtance, in the Oblique angled Spherical Triangle, let there be given A P = 90°, ZP = 38° 28', and the Angle A Z P = 150° to find A Z? Therefore in the right angled Spheric Triangle Z D P, it will be;

As *c t* Z P	-	38°	28'	-	10.0999135
To Radius	-	90	00	-	10.0000000
So *c s* P Z D		30	00	-	9.9375306
To *t*, Z D	-	34	32	-	9.8376171

Now AD = 90 — ZD = 34° 32' = A Z 55° 28' as was required. Or it may be found in the Triangle A Æ Z, for there is given Æ Z = 51° 32', and the Angle Æ Z A = 30, it being the Complement of the Angle A Z P, to a Semicircle, therefore I ſay,

As *t*, Æ Z	=	51°	32'	-	10.0999135
To Radius	=	90	00	-	10.0000000
So *c s* ÆZA	=	30	00	-	9.9375306
To *c t* A Z	=	55	28	-	9.8376171

the ſame as before. And for the Benefit of the young Student, I ſhall here ſet down all the Parts of the Triangles A Z P, and alſo of it's adjacent right ones A Æ Z and Z D P.

1. In the Triangle A Z P.

A P = 90° 00
Z P = 38 28
A Z = 55 28
AZP = 150 00
APZ = 24 19 Z P D = 65° 41'
PAZ = 18 07 = D P.

2. In the Triangle A Æ Z.

Æ Z = 51° 32'
A Z = 55 28
A Æ = 24 19
A Æ Z = 90 00
A Z Æ = 30 00
Æ A Z = 71 53

In the Triangle Z D P.

Z P = 38° 28'
Z D = 34 32
P D = 18 07
Z D P = 90 00
Z P D = 65 41
P Z D = 30 00

There are four Axioms, by which all the Caſes, commonly called 12, of Oblique Spherical Triangles are ſolved, which are theſe.

Axiom 1. As the Sine of a Side, is to the Sine of it's oppoſite Angles, ſo is the Sine of another Side, to the Sine of it's oppoſite Angle. Or, as the Sine of an Angle is to the Sine of it's oppoſite Side, ſo is the Sine of another Angle, to the Sine of it's oppoſite Side.

Axiom 2. As the Sine of half the Sum of two Sides including an Angle, is to the Sine of half the Difference, ſo is the Co-Tangent of half the contain'd Angle, to the Tangent of half the Difference of the other two Angles.

2. As the Co-Sine of half the Sum of two Sides, including an Angle, is to the Co-Sine of half their Difference, ſo is the Co-Tangent of half the included Angle, to the Tangent of half the Sum of the other two Angles.

Axiom 3. Firſt ſay, As the Sine of half the Sum of two Angles, is to the Sine of half the Difference, ſo is the Tangent of half their interjacent Sides, to the Tangent of half the Difference of the other two Sides. This reſerve, and ſay again,

Secondly, As the Co-Sine of half the Sum of two Angles is to the Co-Sine of half their Difference, ſo is the Tangent of half the interjacent Side, to the Tangent of half the Sum of the other two Sides, which added to the firſt Arch juſt now found, gives the Anſwer. This ſhall be more fully explain'd in the next Chapter.

Axiom 4. As the Rect-angle of the Sines of the two containing Sides, is to the Square of Radius, ſo is the Rect-angle of the Sines of half the Sum of the three Sides, and of the Difference of the Sides oppoſite thereto, to the Square of the Co-Sine of half the contain'd Angle ſought.

Theſe four Axioms comprehend the whole Buſineſs of Spherics; yet by Help of the Catholick Propoſition in Rect-angular Triangles, and the *Homogeneal* in the Oblique, the four Axioms will be much ſhortned and made plain and eaſy to the meaneſt Capacity.

When the Perpendicular is let fall in, or without the Oblique Triangle, proceed to find what you are ſeeking, in one of the right angled Triangles, which being done, the next Thing is to proceed to a ſecond Analogy, thereby to find the Anſwer to the Queſtion, and this is eaſilieſt done by the two *Homogeneal* Parts, or Things of the ſame Kind in each Triangle, by comparing them with the Vertical Angles, or with the Baſe, for they are proportional. As ſuppoſe in the Triangle A Z P, the Perpendicular Z B reduces it into two right angled Triangles, A B Z, and P B Z, I ſay, that in the Sines and Tangents of the Circular Parts, it will be, as A B : A : : B P : P. for by the Catholick Propoſition,

$$\text{In}\left\{\begin{matrix}A\,B\,Z\\ Z\,B\,P\end{matrix}\right\}\text{As}\left\{\begin{matrix}t\,B\,Z : R :: S\,A\,B : c\,t\,A\\ t\,B\,Z : R :: S\,B\,P : c\,t\,P.\end{matrix}\right.$$

Therefore, As $S\,A\,B : c\,t\,A :: S\,B\,P : c\,t\,P$.

Thus you ſee that having found according to the Condition of the Queſtion a Vertical Angle, or Baſe in the firſt Operation, you muſt next proceed to order your ſecond Proportion aright among the *Homogeneal* Terms; as Baſe, or Hypothenuſe, *&c.* compare the Perpendicular with each Homogeneal Term, whether it be given, or required in each Triangle, to diſtinguiſh between the Middle Part and the Extream, and to fit the artificial Sines and Tangents to them accordingly. Then reject the Perpendicular and Radius in each Compariſon; the Middle Part and Extreams in one Triangle ſhall be proportional to the Middle Part and Extreams in the other.

EXAMPLE.

In the Triangle A Z P, let there be given the Angle A P Z = 24° 19′, A P = 90°, and PZ = 38° 28′, to find the Angle Z A P?

Solution.

Let fall the Perpendicular Z B, ſo that it may paſs through the Pole of the Oblique Circle P B S, at G, and then it will be at right Angles to A P, and reduce the Oblique Triangle A Z P into two right angled Triangles A B Z, and P B Z; Now in the Triangle P B Z, we have known B P Z, and Z P, as above, to find B P, the Segment of the Baſe A P, and Angle B Z P, of the three Parts, B P, P and Z P; P is the Middle Part, becauſe it is an adjacent Extream, therefore $c\,s\,P + R = t\,B\,P + c\,t\,Z\,P$.

Analogy, For B P.

As *c t* Z P	=	38°	28'	·	10.0999135
To Radius	=	90	00	-	10.0000000
So *c s* P	=	24	19	-	9.9596535
To *t* B P	=	35	54	-	9.8597400
From A P	=	90	00		
Remain A B	=	54	06		

Secondly, For the ſecond Analogy we are to conſider where the *Homogeneal* Parts lye in each Rect-angle Triangle, therefore I make Uſe of the Perpendicular Z B in both Triangles, then if Z B, B P and P be the three Terms, it is an adjacent Extream, and B P is the Middle Part, and conſequently is a Sine, and the Angle P is *c t*.

Again in the Triangle A B Z, I make Uſe of the Perpendicular Z B, A B and Angle A, here the three Things made Uſe of lye together, and A B is the Middle Part, Angle A (which I am ſeeking) is an Extream conjunct, and is *c t*. Therefore I ſay the Homogeneal Parts are P, P B, B A and < A, now according to the Catholic Propoſition, becauſe A B and B P are Middle Parts, they are Sines, and the Angles at A and P are Co-Tangents, and conſequently are Things of a like Nature. Theſe Things being known it will hold,

As S. B P	-	35°	54'	Co Ar.	0.2318265
To *c t* < P	-	24	19	-	10.3449888
So S. A B	-	54	06	-	9.9085073
To *c t* < A.	-	18	07	-	10.4853226

And by the ſame way of Reaſoning may the firſt Ten Caſes, (as they are called) be reſolved, whether the Perpendicular be let fall within, or without the Triangle.

A View of the firſt Ten Caſes. See Fig. page 251.

Caſe 1. *Given* < A, A P *and* Z P, *to find* < Z. This is ſolved by Axiom 1.

Caſe 2. *Given*, < A, < P, *and* Z P. *to find* A Z. This is ſolved by Axiom 1.

Caſe 3. *Given Angle* A, A Z *and* Z P, *to find Angle* Z. The Perpendicular is let fall from the obtuſe Angle A Z P, within the Triangle, and is ſolved by the Homogeneal Parts.

Caſe 4. *Given Angle* A, A Z *and* Z P, *to find* A P. The Perpendicular is let fall from Angle Z within, as in *Caſe* 3.

Caſe 5. *Given, Angle* A, A Z *and* A P, *to find Angle* P. The Perpendicular falls as in the 3d and 4th *Caſes*.

Caſe 6. *Given, Angle* A, A Z *and* A P, *to find* Z P. The Perpendicular falls as in Caſe 3, 4, and 5.

Caſe 7. *Given, Angle* A, *Angle* P, *and* A Z, *to find* A P. The Perpendicular falls as in *Caſe* 3, 4, 5, 6.

Caſe 8. *Given, Angle* A, *Angle* P, *and* A Z, *to find Angle* Z. The Perpendicular falls, as in the 3d Caſe, *&c.*

Caſe 9. *Given, Angle* A, *Angle* P, *and* Z P, *to find* A P. The Perpendicular falls without the oblique △ A Z P, *viz.* by continuing the Side A Z to D a Quadrant, then from D let fall the Perpendicular D P, to paſs thro' G the Pole of the oblique Circle S A P. Therefore the firſt Proportion will be,

As R : *t* < Z P D :: *cs* Z P : *ct* < P Z D. Subtr. from 180 = < A Z P. Then ſay by Axiom 1.

As S < A : S Z P :: S < Z : S A P.

Caſe 10. *Given, Angle* A, A Z, *and Angle* AZP, *to find* P. Continue the Side P Z to Æ and 'tis then a Quadrant, draw Æ A, therefore in the right angled Spherical Triangle A Æ Z, there are known, Æ Z A, it being the Complement of the Angle A Z P to 180, and A Z, to find Æ A, which is the Meaſure of the Angle A P Z, becauſe Æ P and A P are Quadrants: The Angle Æ A Z is alſo known, it being the Complement of the given Angle Z A P to a Quadrant, then for Æ A = Angle Æ P A, ſay by the firſt Axiom,

As Radius -	90°	0′	-	10.0000000
To S A Z -	55	28	-	9.9158200
So S. Æ Z A =	30	00	-	9.6989700
To S. ÆA = ÆPA	24	19	-	9.6147900

Caſe 11. *Given, The Three Sides, to find an Angle.*

Here the Perpendicular Z B falls within the given Triangle A Z P, from the obtuſe Angle at Z upon the longeſt Side A P, the Baſe as has been before directed. Then, in order to a Solution of this Caſe,

Firſt, Take the half Sum of the Baſe A P.

Secondly, Take the half Sum of the other two Sides, *viz.* A Z and Z P.

Thirdly, Take the half Difference of the two ſhorteſt Sides, *viz.* of A Z and Z P.

Then ſay, As the Tangent of half the Baſe or longeſt Side, *viz.* A P, is to the Tangent of half the Sum of the other two Sides, ſo is the Tangent of half the Difference of the two ſhorteſt Sides, *viz.* A Z and Z P, to the Tangent of half the Difference of the Segment of the Baſe, *i. e.* The Diſtance between the Place where the Perpendicular cuts it, and the True Middle of the Baſe, which added to the half Baſe, gives the greater Segment = A B; but ſubtracted from half the Side or Baſe A P, gives the leſſer Segment = B P; then if you would find the Angle included between the Baſe and the greater Side, ſay, as Radius to the Tangent of the greater Segment of the Baſe, ſo is Co Tangent of the greater of the two leſſer Sides, to the Co-Sine of the Angle included between the Baſe and the greater of the two leſſer Sides.

But if you would find the Angle at the End of the Baſe adjacent to the leſſer of the two other Sides, then ſay, in the Second Analogy,

As Radius, to the Tangent of the leſſer Segment, ſo is the Co-Tangent of the leaſt Side, to the Co-Sine of the Angle included between the Baſe and the leſſer Side.

EXAMPLE.

In the Triangle A Z P, Let A P = 90, A Z = 55° 28′ Z P = 38° 28′; what are the Angles?

Operation.

Operation.

Baſe A P	=	90°						
Half	=	45						
A Z	=	55°	28′		A Z	=	55°	28′
Z P	=	38	28		Z P	=	38	28
Sum	=	93	56		Diff.	=	17	00
½	=	46	58		½	=	8	30

Now ſay,

As *t*. ½ Sum Baſe	45°	00′	Co. Arith.	9.9999999
To *t*. ½ Sum Sides	46	58	-	10.0298376.
So *t*. ½ Diff. Sides	8	30	-	9.1744988
To *t*. ½ Diff. Seg.	9	6	-	9.2043363
½ Baſe =	45	00		
Sum, greater Seg.	54	06	= A B.	
Diff. = leſſer Seg.	35	54	= B P.	

Secondly, For the Angle Z A P.

As Radius	=	90°	00′	10.0000000
To *t*. A B	=	54	06	10.1403339
So C *t*. A Z	=	55	28	9.8376755
To C S. < A	=	18	06	9.9780094

Laſtly, For the Angle A P Z.

As Radius	=	90	00	10.0000000
To *t*. B P	=	35	54	9.8596661
So C *t*. Z P	=	38	28	10.0999135
To C S. < P	=	24	20	9.9595796

Another way to work this Caſe.

Take half the Difference of the two Sides, containing the required Angles, and add to it half the Side oppoſite to the Angle required, alſo ſubtract it from the ſame, Noting the Sum and Difference: Then ſet down the two

Sides containing the required Angle, and to them the Co-Arith. of their Artificial Sines, and under thoſe the Artificial Sines of the aforeſaid Sum and Difference, the half Sum of theſe Logarithmetic Sines, ſhall be the Artificial Sine of half the Angle required.

Or in the firſt Part of the Work, which is better, from the half Sum of the three Sides, ſubtract the two Sides including the Angle required, theſe Differences will be the ſame with the Differences gained by the firſt Part of the Rule above. With which Work as has been taught, and the true Anſwer will come out, as will appear in working the laſt Example overagain.

The Work ſtands thus,

In the Triangle A Z P. Given, the three Sides, to find the Angle Z A P?

Sides including < Z A P	A P =	90°	00′
	A Z =	55	28
Difference	=	34	32
Half	=	17	16
Z P = 38° 28′ Half	=	19	14
Sum	- - - -	36	30
Difference	- - - - -	1	58

Or, *Secondly*, Thus,

A P	90°	00′				
A Z	55	28				
Z P	38	28				
Sum =	183	56				
Half =	91	58		Again	91°	58′
A P =	90	00		A Z =	55	28
Differ. =	1	58		Differ. =	36	30

S. A P	=	90° 00'	Co Arith.	9.9999999
S. A Z	=	55 28	Co-Arith.	0.0841799
S. Differ.	=	36 30	-	9.7743876
S. Differ.	=	1 58	-	8.5355228
Sum	=		-	18.3940902
Half S	=	9 3½	-	9.1970451

Doubled = Angle Z A P = 18° 7'.

Secondly, For the Angle APZ the Work ſtands thus,

A P	=	90° 00'				
A Z	=	55 28				
Z P	=	38 28				
Sum	=	183 56				
Half	=	91 58		Again		91° 58'
A P	=	90 00		Z P	=	38 28
Diff.	=	1 58		Diff.	=	53 30

S. A P	=	90° 00'	Co-Ar.	9.9999999
S. Z P	=	38 28	Co-Ar.	0.2061683
S. Differ.	=	53 30		9.9051787
S. Differ.	=	1 58		8.5355228
Sum	= - - - - - -			18.6468697
Half	= S.	12 9½	-	9.3233448
Double	=	24 19 = < APZ. as before.		

Caſe 12. *Given, the three Angles to find a Side.*

This is juſt the Converſe of the 11th Caſe, and may be reſolved in the ſame Manner, for if either of the Angles adjacent to the Side required be turn'd into a Side, (that is only taking it's Complement to 180) the Work is the very ſame as in the 11th Caſe, as I ſhall make more clear by Example. Let the three Angles of the △ AZP be as before, *viz.* A 18° 7', Z 150°, and P 24° 19', I demand the Side A Z ? Firſt I ſhall turn the Angles adjacent to the required Side, *viz.* A and Z into Sides, by taking their Complements to 180°.

Now ſee the Work.

Angle	A	=	161°	53′	Angle Z	=	30°	00′
	Z	=	150	00	Angle A	=	18	7
X		=	11	53	X	=	11	53
Half		=	5	56	Half	=	5	56½
Half Angle oppoſite to A Z					-	=	12	9½
					Sum	=	18	6
					Diff.	=	6	13

Or find the Requiſites thus, as in Caſe 11.

Angles	Z	=	30°	00′				
	P	=	24	19				
	A	=	10	7				
Sum		=	72	26				
Half		=	36	13	Again		36°	13′
Angle Z		=	30	00	Angle A	=	18	7
X		=	6	13	X	=	18	6

Here you ſee the Requiſites are the ſame with the other Method above.

S. Angle Z	=	30°	00′	Co-Ar.	0.3010299
S. Angle A	=	18	07	Co-Ar.	0.5073054
S. X	=	18	06	-	9.4923083
S. X	=	6	13	-	9.0345825
Sum	=	-	-	-	19.3352261
Half Sum = S.		27	43½		9.6676130
Doubled	=	55	27	= A Z, as was required.	

Laſtly, Let the Angles be as before, and the Side A P required?

See

See the Work.

Angle Z turn'd into a Side =	30°	00'			
Angle P = - -	24	19			
Angle A = - -	18	7			
Sum - -	72	26			
Half - - -	36	13	Again	36°	13'
A = - -	18	7	Angle P	24	19
X = - -	18	6	X =	11	54

S. Angle P =	24°	19'	Co-Ar.	0.3853353
S. Angle A =	18	7	Co-Ar.	0.5073054
S. X =	11	54	-	9.3142975
S. X =	18	6	-	9.4923083
Sum - - - -				19.6992465
Half Sum = S.	45	00		9.8496232
Doubled =	90	00 = A P as was required.		

Note, If to the Co-Arith. of the Sines of the two Angles adjacent to the Side required, you add the Sine of half the Sum of all the Angles, and the Sine of the Difference between half Sum of the Angles, and Angle oppoſite to the Side required, half the Sum of theſe Logarithmic Sines will be the Co-Sine of half the Side required.

I ſhall conclude Spherics with the Solution of two extraordinary Caſes.

Caſe 1. *Given, Two Sides with the Angles oppoſite them, to find the other Side.*

The Celebrated Mr *Edmund Gunter* gives this Rule.

As the Sine of half the Difference of the given Angles, to the Sine of half the Sum of thoſe Angles, ſo the Tangent of half the Difference of the Sides, to the Tangent of half the Side required.

EXAMPLE.

EXAMPLE.

In the Oblique Spherical Triangle AZP, page 251, let there be given ZP = 38° 28', AP = 90°, Angle A = 18° 7' Angle Z = 150°, to find the Side AZ?

Operation.

< Z =	150°	00'	-	150°	00'.	A P	90°	00'
< A =	18	07	-	18	07.	Z P	38	28
Sum =	168	07	X =	131	53	X =	51	32
Half =	84	03½	Half =	65	56½	½	25	46

Now ſay,

As S. ½ X << -	65°	56'	30''	Co-Ar.	0.0394669
To S. ½ their Sum	84	3	30	-	9.9976606
So T. ½ X Sides	25	46	00	-	9.6836783
To T. ½ Side required	27	44	4	-	9.7208063
Doubled = A Z =	55	28	8 as was required.		

Caſe 2. *Given, Two Sides with the Angles oppoſite to them, to find the other Angle.*

RULE.

As the Sine of half the Difference of the given Sides, to the Sine of half the Sum of thoſe Sides, ſo the Tangent of half the Difference of the given Angles, to the Co-Tangent of half the Angles required.

EXAMPLE.

In the Oblique Triangle A Z P, let A P be 90°, Z P 38° 28', the < A Z P 150° and that A 18° 7', I demand the Angle A P Z?

Operation.

A P =	90°	00′		90°	00′	Z =	150°	00′
Z P =	38	28		38	28	A =	18	07
X =	51	32	Sum	128	28	X	131	53
Half =	25	46	Half	64	14	Half	65	56½

Now ſay,

As S. ½ X Sides	-	25°	46′		Co-Ar.	0.3618031
To S. ½ their Sum	-	64	14		-	9.9545184
So *t.* ½ X	-	65	56½		-	10.3502280
To *ct.* of ½ < P	-	12	9	40″		.10.6665495
Doubled = < P	-	24	19	20		

Caſe 3. *In the Triangle* A Z P. Let A Z = 55° 28′ Z P = 38° 28′ and the Angle included, *viz.* A Z P = 150°, I demand the Side A P ?

The Perpendicular here is Æ A, by which means the Oblique angled Triangle A Z P is reduced into two right angled Spherical Triangles, *viz.* AÆZ, and A Æ P, both right angled at Æ ; now in the Triangle are given, A Z = 55° 28′, and the Angle Æ Z A = 30 ; it being the Complement of the Angle A Z P to a Semicircle, to find Æ Z ? *By Tranſpoſition I ſay,*

As Radius	-	90°	00′	-	10.0000000
To *t.* A Z	-	55	28	-	10.1623245
So *cs.* < Æ Z A		30	00	-	9.9375306
To *t.* Æ Z		51	32	-	10.0998551
Add Z P =		38	28		
Sum = Æ P	=	90	00		

Now 'tis plain from the Projection that A P = Æ P, and conſequently A P is ſound without any further Calculation to be a Quadrant alſo.

But *Secondly*, If in this Caſe, we ſuppoſe A P < P, and Z P to be given, to find A Z, the Perpendicular Z B will fall within the Triangle, then proceed to find the

Segment of the Baſe P B, and then the Side A Z will come out as has been taught above.

The Verſed Sine of an Arch is the Segment of the Diameter cut by a right Line falling perpendicular upon it, from equal Parts in the Circumference, and number'd in the Diameter with 10, 20, 30, 40, *&c.* to 180. Therefore the Natural Sine of any Arch taken from Radius will leave the Arithmetical Complement, or Verſed Sine of that Arch's Complement; thus, the Natural Sine of 40° 6427876 ſubtracted from Radius = 10.000000 the Remainder = 3572124 is the verſed Sine of 50, alſo the Natural Sine of 60° = 8660254 taken from Radius 10.000000 leaves 1339746 the verſed Sine of 30°, *&c.* And for Arches above 90 we need no Natural verſed Sines, becauſe the Natural Sine of any Arch exceſs above 90, added to the Radius, is equal to the verſed Sine of the ſaid Arch; thus the Natural Sine of 40° = 6427876 added to Radius 10 000000 = 1642787 = to the verſed Sine of 130°.

Alſo the Natural Sine of 13° = 2249511
Added to Radius = 90 = 10000000
Verſed Sine of - 103 = 12249511

Again, The Natural Sine of 1° 32' = 267585
Add Radius = 90 00 = 1000000
Verſed Sine of 91 32 = 10.267585

The Excellent Uſe of theſe verſed Sines will appear by the following Examples, for in all doubtful Caſes where the Thing ſought is more than a Quadrant, here it is ſeen at one View, for if the Sum of the Logarithms exceed Radius, then the Side or Angle enquired after, is greater than a Quadrant, but if the Sum of the Logarithms be leſs than Radius, the Thing ſought is alſo leſs than 90, or a Quadrant.

Theſe verſed Sines are of great Uſe to calculate the Diſtance of Places on the Earth, according to the Arch of a great Circle, by having given their Latitudes, or Longitudes. Or the Diſtance of two Stars may be exactly found, or the Latitude of a Place may be had by the Altitude of the two Stars, or Sun, by knowning the Azimuth, or Difference of right Aſcenſions.

EXAMPLE.

In the Triangle A Z P, page 251, there are known, A Z = 55° 28′, Z P = 38° 28′ and the included Angle A Z P = 150 to find the Side A P?

RULE.

As the Cube of Radius, is to the Rect-angle of the Sines of the comprehended Sides, ſo is the Square of the Sine of half the contain'd Angle, to half the Difference of the verſed Sines of the third Side, and of the Arch of the Difference between the two including Sides.

Which is thus, ſet down the Logarithm Sines of the two given Sides, and of the Double of half the given Angle, the Sum of all three Logarithms ſhall be the Logarithm Sine of an Arch, then take out of the Tables the Natural Sine of this Arch, and double it, to which add the Natural verſed Sine of the Difference of the two given Sides, this Sum is the Natural verſed Sine of the Side ſought.

Operation.

The	A Z =	55°	28′	L. Sine ·	9.9158200
Sides.	Z P =	38	28	L. Sine ·	9.7938317
½ given < double L. Line		75°		-	19.9698876
Log. Sine of		28	34		9.6795393
Natural Sine of		28	34	Double =	.9563620
X given Sides		17	00	V. Sine =	.0436952
The Side A P		90	00	V. Sine =	10.000572

EXAMPLE II.

Given A P = 90, Z P = 38° 28′, and the Angle A P Z = 24° 19′, required A Z?

Operation.

The Sides.	AZ = 90°	00	Log. S.	10.0000000
	ZP = 38	28	Log. S.	9.7938317
Half Given <	12°	9′½	doub.Log.S. =	18.6469740
Logarithm Sine of	1	35	=	8.4408057
Natural Sine of	1	35	= Doubl. =	552618
X given Sides	51	32	= V. Sine =	3779407
The Side AZ =	55	28	= V. Sine =	4332025

EXAMPLE III.

Given, AP = 90°; AZ = 55° 28, and the Angle ZAP = 18° 6′, required ZP?

Operation.

Sides	AP = 90°	00′	Log. S.	=	10.0000000
	ZA = 55	28	Log. S.	=	9.9158200
Half given <	9	3½	doubled	=	18.3934392
Log. Sine of	1	10		=	8.3092592
Natural Sine of	1	10	double	=	407216
X given Sides =	34	32	V. Sine	=	1762035
The Side ZP =	38	28	V. Sine	=	2162035

If you would find the Verſed Sine of an Arch or Angle, See page 202.

CHAP.

CHAP VIII.

Of ASTRONOMY.

DEFINITION.

ASTRONOMY is deriv'd from two *Greek* Words, *viz. Aſter*, a Star, and *Nomos* a Law, or Rule. It is a Science which by infalliable Demonſtration, teaches us the Motions, Diſtances, and Magnitudes of the Heavenly Bodies; their Revolutions, *Anomalies*, *Aplelions*, *Eccenericilies*, *Elongations* and *Parallaxes* of the Planets, Eclipſes of the *Luminaries*, *Occultations* of the Primary *Planets* and fixed Stars by the Moon. As alſo the Riſing, Culminating, Setting, and Amplitudes, and in ſhort whatever belongs to the right Underſtanding of the true Syſtem of the World; (*See my* Uranoſcopia.) Of which there are Eight ſeveral Sorts, (but only one true), of which I ſhall firſt give a ſhort View.

1. *Plato* the Divine *Athenian* Philoſopher, travelled into *Egypt* and there Learned *Aſtronomy*, about 420 Years before *Chriſt*, invented a Syſtem, in which he makes the Earth the Center of the World immoveable, ſurrounded by the Air; next above that he imagines the Element of Fire; then the Sphere of the Moon; next above the Moon, the Sun; then *Mercury*; then the Orb of *Venus*; next above her, *Mars*; then *Jupiter*, then *Saturn*, and above *Saturn*, the Fixed Stars.

2. *Anno Dom* 135, Flouriſhed the Great *Ptolony*; there is a Syſtem goes by his Name, which has the Earth placed in the Center of the Univerſe immoveable: This he ſuppoſes is ſurrounded by Air, and the Air to be ſurrounded by the Element of Fire, next above this the Sphere of the Moon, next above the Moon is the Orb of *Mercury*, then *Venus*,

then the Sun ; next above the Sun the Orb of *Mars* ; then *Jupiter*, and the higheſt of all the Sphere of *Saturn* ; and he here ſuppoſes all the Orbs to be ſolid, and the Fixed Stars ſurrounding them all.

3. *Porphyrius* a Follower of *Plato*. Flouriſhed 375 Years after *Chriſt*, agreeing with *Plato* in all Things, only in this, he placed *Venus* next to the Sun, and *Mercury* between the Orbs of *Venus* and *Mars*, theſe three Syſtems are all compoſed of Concentric Circles.

4. The *Egyptian* Syſtem was embraced by *Vitruvius*, *Martianus*, *Capella*, *Macrobius*, *Beda*, and *Argol*, about the Year 1638 ; this Syſtem is compoſed of Eccentric Orbs, for they make the Earth fixed, and the Center of the *Moon*, *Sun*, *Mars*, *Jupiter*, and *Saturn*, and the Fixed Stars ; but the Sun the Center of *Venus*, and *Mercury* interſecting the Sun's Orb twice, if this Syſtem were true, 'tis poſſible they would meet the Sun in his Way, what would be the Conſequence of ſuch a Salutation is well known to Aſtronomers.

5. In the Year 1572, Flouriſhed *Tycho Brahe*, a *Dean* and Lord of *Knudſthorp*, in the Iſland *Schonen* ; He contrived a Syſtem as a Mean between the *Ptolomaick*, and *Copernican*, in which he ſuppoſes the Earth fixed in the Center of the Univerſe. Concentric to which is firſt the Sphere, or Orb of the Moon, next that of the Sun's Annual Motion ; then the Sphere of the Fixed Stars, the Sun being the Center of the other Five Planets. The Orb of *Mars* interſecting the Sun's Orb twice and in oppoſition is nearer the Earth than the Sun itſelf, and *Saturn* and *Jupiter* in Oppoſition, are nearer the Earth than *Venus* is when in *Apogeon* : The Orbs of *Venus* and *Mercury* are drawn as it were two Epicycles to the Sun's Orb, both of them interſecting it twice ; now according to this it is poſſible for the Sun in his Way to meet with *Mars*, (like two Coaches in a narrow Lane, or Street of the City, when there is not Room to paſs) the Conſequence of ſuch a Joſtle, I leave to be diſcuſs'd by Aſtronomers.

6. *John Baptiſt Ricciolus*, of *Ferrara* in *Italy*, about the Year 1651, broach'd a New Syſtem, in which he makes the Earth the Center of the Moon, Sun, *Jupiter*, *Saturn*, and Fixed Stars, all moving round her and ſhe at reſt ; and the Sun moving round the Earth, he makes the Center of

Mercury, *Venus*, and *Mars* all moving round him, in which the Orb of *Mars* cuts the Sun's Orb twice, he has *Jupiter*'s four Satellites, and the two innermoſt of *Saturn*'s.

Theſe two laſt Syſtems ſuppoſes the Heavens, or Æthereial Region, to be pervius, fluid, and of a thin, liquid, and tranſparant Subſtance, like the Air we breath in, but more pure, and not conſiſting of ſolid Orbs, as the *Peripateticks*, and thoſe of the *Ptolomaick* Schools affirm.

7. *Roſlinus* publiſhed a Syſtem at *Nuremberg*, in the Year 1661, a ſtrange Piece of hotch-potch Stuff, made up of an unaccountable Quantity of Eccentric Circles, bearing no Teſtimony of Truth, and not fit to be once named. Theſe Syſtems are all exploded.

8. *Laſtly*, We come now to the moſt celebrated, and at this Day, moſt generally received, *Mundane*'s Syſtem, named from it's Reviver, *Nicholas Copernicus* a Native of *Thorn* in *Poliſh Pruſſia*: He was born *Anno* 1473, died *Anno* 1543, only ſee the firſt Sheet of his Works Printed, as ſaith *Fontinelle*. This Syſtem was firſt invented by *Pythagoras* 509 Years before *Chriſt*, (ſee *Mercus Menelius*), the Reaſon we don't give *Copernicus* place according to his time, is, becauſe we would ſpeak of that laſt which will be moſt laſting, or durable. For in this Syſtem we ſee as in a Glaſs, the Beauty of the Univerſe diſplay'd; here are neither ſolid Orbs nor Epicycles, but every Thing appears in a demonſtrable Order.

Fiſt, We find the Sun the Center of the World unmoveable, having no Circular Motion, but a Central only about it's Axis in the Space of 25¼ Days, which was diſcovered by the Teleſcope. This Syſtem is made up and adorned with Seventeen Bodies, which we ſhall ſpeak of in the following Order, and firſt of the Sun.

All Philoſophers and Aſtronomers now agree, that the Sun is a formal fiery Body, conſiſting of a true proper Elementary Fire; partly liquid, partly ſolid. The Liquid being an Ocean of Light, and moving with fiery Billows, and flaming Ebullitions, as is manifeſt to thoſe who look at it thro' a Teleſcope.

The Sun is fix'd upon the lower *Foci*, or *Umbelici*, at 50, at that End towards B, in the Ellipſes, page 153, and with this Law, that the Areas deſcribed by Lines drawn from the Earth, (which moves in the Curve A C, B D, round the Sun) to the Sun, ſhall always be equal, and in equal and proportionable Times. And from this Figure, 'tis alſo plain, that the Sun's Diameter will appear greater when he is in *Pergeon* which is a little after the Winter *Solſtice*, viz. *December* 18, and is then 32′ 44″, and leſs when ſhe is in her *Apogeon* which is a little after the Summer *Solſtice*, June 18, for then the Sun's apparent Diameter is but 31′ 38″ as is found by Obſervation; and this difference in his apparent Diameter is a manifeſt Proof that the Earth moves in an Ellipſis and not in a Circle.

The Length of the Tropical Year, or the time the Sun apparently runs thro' the 12 Signs of the *Zodiack* is 365 *d*. 5 *h*. 49′ 7″; and if to this we add the Annual Anticipation of the Equinoxes 50″ turn'd into Time = 20′ 17″ 27‴, we ſhall have for the Length of the Syderial Year 365 *d* 6 *h*. 9′ 24″ 27‴.

The Angle of the Inclination of the Planes of the Ecliptic and Equator, or the Sun's greateſt Declination, (commonly called the Obliquity of the Ecliptic,) hath always been invariably the ſame, *viz*. 23° 29′. And the Sun's *Horizontal Parallax*, (that is what the Earth's Semidiameter appears to be to an Eye at the Sun) is no more than 10″; and from hence is proved the vaſt Diſtance of the Sun from our Earth, for as all Objects are beheld under a certain Angle, (*See page* 183), therefore it muſt neceſſarily follow that the leſſer the Angle that any Object is ſeen under, the greater muſt be the Diſtance of that Object from the Eye of the Obſerver; from hence it is plain that the Earth's Semidiameter appearing under an Angle of ſo ſmall a Quantity as 10″, the Diſtance of the Sun from our Earth muſt be 82183140 *Engliſh* Miles; and this we ſetch from the Moon's *Horizontal Parallax* 57′ 30″, of Sir *Iſaac Newton*'s Theory gives the Moon's mean Diſtance from the Earth 238212, then by a Reciprocal Proportion ſay,

Miles. *Miles.*
If 57′ 30″ : 238212 :: 10″ : 82183140.

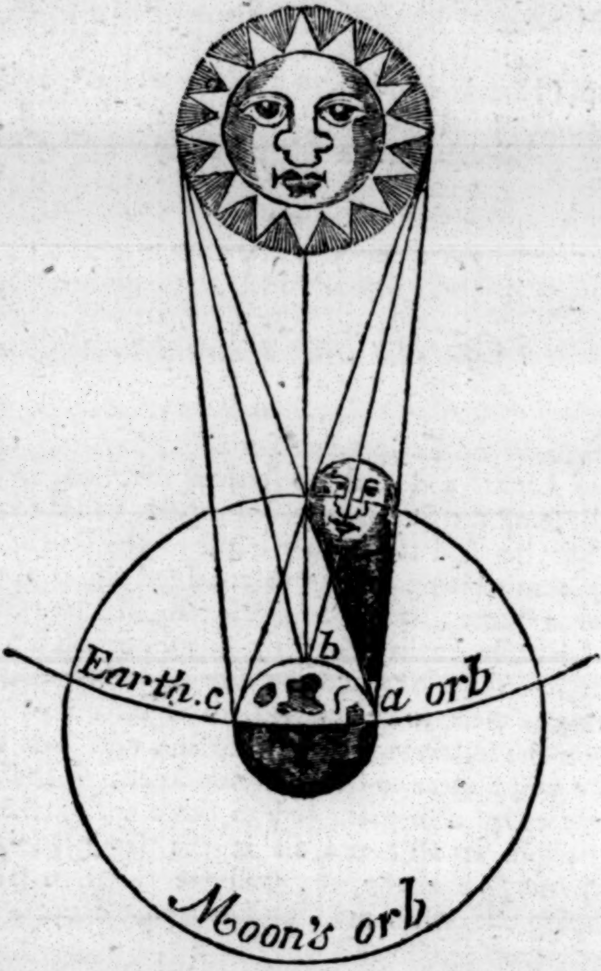
And he is 258309 times bigger than our Earth; his Apogeon this preſent Year 1739 is ♋ 8° 24′ 19″. Eccentricity

1692 Parts, ſuch as the mean Diſtance from Earth is 10000001

His ap-	Hourly Motion	0	0°	2′	27″	51‴
parent	Diurnal Motion	0	0	59	8	20
Mean,	Annual Motion	11	29	45	39	51

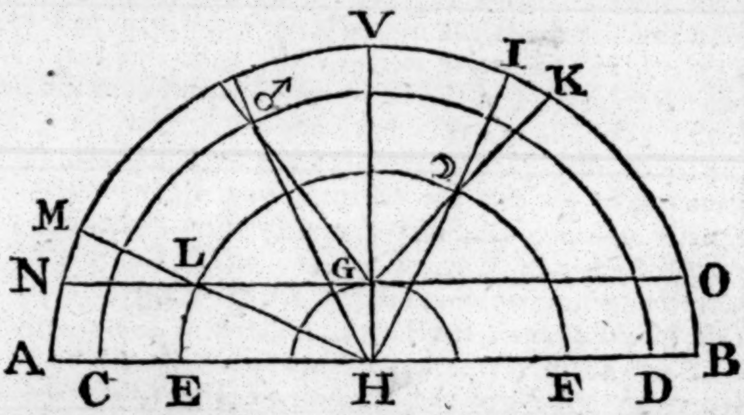
Of the Sun's Eclipſe.

THE Word Eclipſe ſignifies a Deprivation, or Want of Light, and that in either the Sun or Moon: The Eclipſe of the Sun is very improperly called ſo; for it may rather be ſaid to be an Eclipſe of the Earth, or that the People inhabiting the Earth are deprived of the Sun's Light for a Time; when at the ſame Time the Sun looſing no Light in reality, but by the Interpoſition of the Moon between the Sun and the Eye of the Spectator, the Sun ſeems to them to be eclipſed, as to thoſe People living at *(a)*, (in the following Figure) the Sun will ſeem to them at *(b)* he will appear to be totally darkned, to thoſe living to be only eclipſed inpart, and to thoſe at *(c)* there will be no Eclipſe at all; and all at the ſame Moment of Time: An Eye at the ☽, will ſee the Sun free from any Eclipſe, but will behold the Earth part dark.



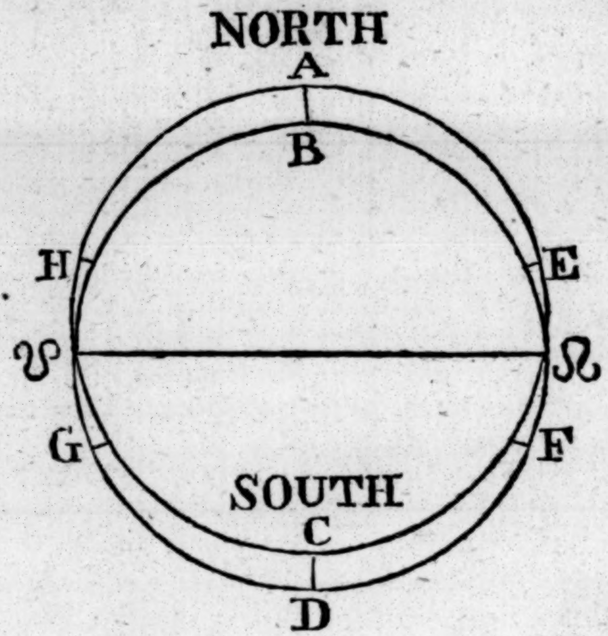
This Diverſity of *Aſpect* is cauſed by the *Parallax* of the Moon. The Word *Parallax* ſignifies Variation or Diverſity of *Aſpect*, but in *Aſtronomy* it is the Angle that is made at the Moon by two Lines, one drawn from the Earth's Center, and the other from a Superficies, interſecting the Moon's Center, and being continued amongſt the Fixed Stars, ſhe will appear there conſiderably lower than ſhe will to an Eye at the Earth's Center, and the nearer unto the Horizon that the Moon's is, the greater is her *Parallax* at that Time, and conſequently her *Horizontal* *Parallax*

Parallax, is the greateſt of all other. In this Scheme let N O be the viſible *Horizon*, A B the True ; let E ☽ F repreſent the Moon's Orb, H the Earth's Center, G the Superficies : I ſay, an Eye at H will behold the Moon amongſt the Fixed Stars at I; but to him that views her at G, the Earth's Surface, will ſee her at K; therefore the Angle G ☽ H = I ☽ K is the Angle of *Parallax:* Of which there are ſeveral Sorts, but the *Parallaxes* in Longitude and Latitude are what are chiefly uſeful in the Sun's Eclipſe, as now practiſed. The *Horizontal Parallax* is the greateſt, being equal to the Angle G L H. The *Parallax* of *Mar*'s is = Angle H ♂ G, being leſs than the Moon's *Parallax* when they have equal Altitudes, becauſe *Mars*'s diſtance from the Earth is greater than that of the Moon's ; when a Planet is in the Vertex at V, there the *Parallaxes* vaniſh.



There are Four Sorts of *Parallaxes* in the Calculation of a Solar Eclipſe, *viz.* the *Horizontal*, the *Parallax*, in Longitude, Latitude, and in Altitude; if the Luminaries at their Conjunction appear not in the Nonageſſima Degree, but betwixt it and the *Eaſtern Horizon*, then the Time of the viſible Conjunction will be accelerated, and will be before the Time of the true ; and the Time of that Difference will always be proportional to the *Parallax* of Longitude at that Time: If the Luminaries be conjoin'd in the Nonageſſima Degree, then the true and viſible Conjunction will be Coincident : Laſtly, if they be conjoin'd in the occidental Quadrant, that is between the Nonageſſima Degree and the *Weſtern Horizon*, then the viſible Conjunction will be retarded proportionably to the *Parallax* of Longitude at that Time; when we ſpeak of the Nonageſſima Degree, you are not to underſtand it to be the Culminating Point, but that Degree of the Ecliptic which is 90° diſtant from the *Horizon*, except when *Cancer* and *Capricorn* are upon the *Meridian*, and then the Culminating Point, and Nonageſſima Degree are Coincident.

Secondly, By the *Parallax* of Latitude is determined the Quantity of a Solar Eclipſe; for to all thoſe People that live in Northern Regions, if at any Conjunction of the Sun and Moon, her Southern Latitude exceeds the Sum of the Semidiameters of the Sun and Moon; I ſay by her *Parallax* in Latitude, ſhe will then be depreſſed below the Sun's Limb, and conſequently the Sun will be free from any Eclipſe to us at all: And this is the Cauſe that ſo many Conjunctions of the Sun and Moon paſs us in *England*, and we ſeeing ſo few viſible Solar Eclipſes. The ſame Reaſon hold in the Southern Parts, when at the Conjunction the Moon has North Latitude. Now ſince it is the Moon that is the Cauſe of the Sun's Eclipſe, (as we have noted above), ſhe being a dark Body of the ſame Species as our Earth, and alſo very near Spherical, as is proved by her Shadow, which is a perfect Cone, now this Conical Point reaching further than our Earth, therefore this Shadow upon the Earth, Diſk is a Conic Section, *viz.* an Ellipſis, which we ſhall ſhew by and by when we come to ſpeak of the *Penumbra*: The Reaſon that the Sun is not Eclipſed at every Conjunction or New Moon, is becauſe the Moon moves not in the Plane of the Ecliptic ☊ B ☋ D, but in an Orbit ☊ A ☋ C making an Angle therewith of 5° 17' 20'', and this Interſection of the Ecliptic and Moon's Orb, is always exactly in two oppoſite Places, *(ſee Diagram below,)* or juſt ſix Sines aſſunder,



and theſe Interſections are called the Nodes of the Moon, theſe Nodes have a Motion of 3′ 11″ every Day in *Antecedentia*, or contrary to the Succeſſion of Signs, that is from *Aries* to *Piſces*,&c.and this is the Cauſe why the Eclipſes do not always happen in the ſame Place of the Zodiack, for the ſame Eclipſes that happen now, will again happen 19 Years hence, but differ both in Time, Quantity, and Place, but the Period of Eclipſes is ſomething leſs than 19 Years, *viz.* it is only 18 y. 11 d. 7 h. 43′ 15″, in which Time the Nodes move 11 S. 18° 43′ 38″, as above. This we ſhall demonſtrate more plain from the *Diagram* above, for if at the Time of the true Conjunction of the Sun and Moon, they happen to be within 18° : 10′ : 41″ of either Node, either before or after them, that is between F E, or G H, the Sun will then be Eclipſed, otherways not, and this I call the Ecliptic Boundaries ; otherways at the viſible Conjunction, the Moon's viſible Latitude muſt be leſs than the Sum of Semidiameters of the Luminaries, or the Sun cannot be Eclipſed at that Time ; theſe are the Laws which God Almighty gave to the

Heavenly Bodies at the Creation, which Laws are conſtant, uniform, and regular without variation.

Of Eclipſes there are four Sorts, *viz* 1 *Partial.* 2. *Total without continuance.* 3. *Total with continuance.* And laſtly, *Annular.*

1. *Partial*, is when Part of the Sun's Diameter is obſcured from ſome particular Tract of the Earth, and theſe happen more frequent than any other.

2. *Total without continuance*, is when at the Time of the viſible Conjunction the true Latitude of the Moon is equal to her *Parallax* in Latitude from the Sun, that ſo the Center of one is exactly ſeen in the Center of the other, and then alſo their viſible Diameters are equal, that ſo the Sun is no ſooner hid from our Sight by the dark Body of the Moon, but very ſpeedily he is ſeen to recover his Light on the other Side.

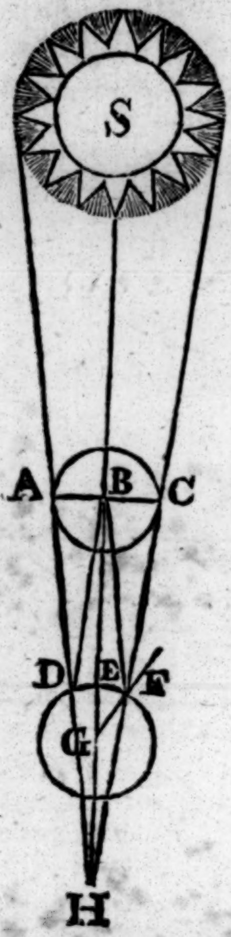
3. *Total with continuance*, is when at the viſible Conjunction the Eclipſe is central, (which is always when the North Latitude and *Parallax* are equal) the Moon is in Perigeon, and the Sun in Apogeon, then the apparent Diameter of the Moon exceeds that of the Sun, and this Exceſs can never amount to one Minute, ſo that the Total Darneſs of any Solar Eclipſe can never exceed 4' or 5' in Time, an Inſtance of this Nature we had in the Year of our Lord 1715, in which the Total Darkneſs at *London* was no more than 3' 20'', the like having not happened at *London* for 575 Years before, which was in the fifth Year of the Reign of K. *Stephen*, *Anno* 1140, on *Wedneſday*, *March* 20, it began at *London*, at 38' 37'' paſt Noon, and continued 3 h. 1' 7'', it's Total Darkneſs I make to be 3' 36'' and Digits 12° 5'.

4. *Laſtly*, *Annular*, is when the viſible Central Conjunction, happeneth the Sun being in Perigeon, and the Moon in Apogeon, here the Diameter of the Sun exceeds that of the Moon, and conſequently there will then be a Ring of Light round the Moon. An inſtance of this Nature we read was obſerved at *Rome*, by *Chriſtopher Clavius*, *April* 9, at Noon, *Anno* 1567.

That the Sun's Eclipſe always begins on the Weſt Side, and goes off, or ends on the Eaſt Side of his Body, this is a moſt manifeſt Truth becauſe the Moon (who is always the Cauſe of his Obſcurity) moving in her Annual Motion always *in Conſequentia*, or according to the Order of the Signs muſt of neceſſity firſt touch the Sun's Weſtern Limb, and laſt leave his Eaſtern. Aſtronomers have divided the Sun's Diameter into 12 equal Parts, which they call Digits, ſo that if we ſpeak of a Digits or Finger's Breadth, it is no more than $\frac{1}{12}$ Part of the Sun's Diameter. The Eclipſe of the Sun does not agree to any particular Place on the Earth, but only to that Latitude and Longitude, for which it is calculated by Reaſon of the ſudden Change of the Moon's *Parallax* above-mentioned. (*See the Scheme page 275.*)

To calculate the Total Shadow of the Moon on the Earth in the Sun's Eclipſe.

IN this Scheme let S be the Sun, B the Moon, and G the Earth, in which we are to find DEF, that is the Space that the Moon's Shadow falls upon in a Total Eclipſe of the Sun.



In the Triangle H G F, we have given the Side G F, the Earth's Semidiameter, G H the Diſtance of the Earth from the Top of the Moon's Shadow, (which how to find will be ſhewn towards the End of this Chapter) = 1.019, and the Angle G H F, equal to the Sun's Semidiameter, to find the Angle G F H.

Suppoſing the Sun in Perigeon.

See the Work.

As G F = 1 Semidiamer - -		0.0000000
To S. G H F, Sun's Semidiameter	16' 22''	7.6774984
So G H - - -	1.019	0.0081742
To S. G F H - -	16 40	7.6856726

Now the Angle $EGF = GHF + GFH = 33' 2''$ by Theorem 3, and ſuch is the Arch EF, whoſe double $= DF\ 1^\circ 6' 4'' \times 69.5 = 74$ Miles, and ſo far on the Earth will the Sun's Eclipſe appear Central and Total, when the Moon is in Perigeon.

1. To Exerciſe the young Tyro in theſe Matters, I ſhall here ſubjoin the Times of the Sun's Eclipſe, *July* 14, 1748, from my Syſtem, by the New Equation, in my *Uranoſcopia*, = Time true ☌ 13 d. 23 h. 28' 25''. ⊙ and ☽ in ♌ 2° 42' 34''.

Hence the Apparent Time at *London*, of the

	d.	h.	'	''	
Beg. 1748, *July*	13	21	4	55	PM.
Viſible ☌ -		22	39	58	
Greateſt Obſcuration		22	40	49	
End - -	14	0	19	1	
Total Duration -		3	14	6	
Digits Eclipſed -		10	26	13	

on the Upper Side of the Sun.

The Uſe that may be made of Eclipſe, is very great for determining the true Longitude of Places both at Sea and Land, for having an Eclipſe truly calculated to any particular Place, as ſuppoſe *London*, having this Calculation with you at Sea, &c. at the Beginning of the Eclipſe you muſt carefully obſerve the Time, as we will ſuppoſe it to be at 20h. 29' 47'' PM, at *London* by our Calculation it begins at 21 h. 4' 55'', the Difference is 35' 8'' which turn'd into Time = 8° 47' 00'', by which I am aſſured that I am then ſo far to the *Eaſt* of the *Meridian* of *London*.

The Calculation of this General Eclipſe ſtands thus, and in this we have nothing to do with *Parallaxes* at all, we have here given the Times and Places of the Paſſage of the *Penumbra* over the Earth's Diſk, which are gain'd by Trigonometry, (*See my* Uranoſcopia) only, in finding of which we are to obſerve, when we have gained the Time of the true Conjunction from Aſtronomical Tables: If then, I ſay, the Moon's true Latitude exceed the Sum of the Semidiameters of the *Penumbra* and Earth's Diſk, the Sun at that Time will be Eclipſed no where upon Earth, but if leſs, then it will; the Semidiameter of the *Penumbra* is ever equal to the Sum of the Semidiameters of the Luminaries, and is nothing elſe but that faint Shadow which encompaſſeth the perfect Shadow of the Moon, and is the *Penumbra*, ſo that we at *London* at the Time of this Eclipſe are not immers'd into the perfect Shadow, but in the *Penumbra :* Now this diminiſh'd Light which encompaſſes the Shadow every Way is call'd the *Penumbra.* The Eclipſe is ſeen to begin in the Supream Point of his vertical Diameter as follows.

The Times of the General Eclipſe at London.

		D.	h.	'	''
Apparent Time at *London* of the	Beg. at ☉ Riſing, 1748, *July*	13	20	26	29
	Central Eclipſe begins ☉ Riſing		21	40	34
	In the *Meridian* -		23	3	9
	Nonageſſimal Sun -		23	19	49
	Middle - - -		23	25	7
	Central Eclipſe ends at ☉ ſetting		1	9	40
	Ends at ☉ ſetting -		2	23	45
	Duration - - - - -		5	57	16

The Places where this General Eclipſe happens is thus.

	Lat.			Long.		
Sun's begin Eclipſe at Riſing	35°	9'	N.	51°	10'	W.
Riſes Centrally Eclipſed	45	23		76	17	W.
Centrally Eclipſed in *Meridian*	51	38		14	13	E.
Centrally Eclipſed in *Nonageſſ.*	48	47		20	8	E.
Sets Centrally Eclipſed	10	30		76	22	E.
Ends at ☉ Setting -	0	14	S.	53	59	E.

Sun's Lower Limb touched by ☽ Upper Limb beyond the Pole.

Sun's Upper Limb touched by ☽ Lower Limb 21° 19'N. 14° 13'E.

II. *Of Mercury,*

NEXT to the Sun in our Syſtem we find nimble heel'd *Mercury*, he is called an inferiour Planet, becauſe his Orb is circumſcribed by the Earth's Orb, and by

Reaſon of his Vicinity to the Sun, he is ſeldom ſeen with the naked Eye. For at his greateſt Diſtance from the Sun when in Aphelion is no more than 28° 21′ 8″; (*See my* Uranoſcopia, *page* 65,) and when the Earth is in Aphelion and *Mercury* is in Perihelion, his greateſt Elongation from the Sun, (or Angle at the Earth) is no more than 17° 35′ 42″. His Eccentricity is 7964, ſuch Parts as the Mean Diſtance of the Earth from the Sun is 10.00000, and his Mean Diſtance from the Sun 38262; his greateſt Diſtance from the Sun in *Engliſh* Miles are 80353439. his leaſt 52914839, his greateſt Diſtance from the Earth is 97171952 and his leaſt 37988985 of our Miles. He makes one Revolution round the Sun in 87d. 23h. 15′ 53″ his Heliocentric Diurnal Motion is 4° 5′ 32″, and hourly Motion 10′ 13″ 52‴, his Aphelion this preſent Year 1739, is ♐ 13° 17′ 37″, the Place of his Aſcending Node ♉ 15° 19′ 50″, and the deſcending Node, or ♏ 15° 19′ 50″, ſo that if his Geometric Latitude be leſs than the Sun's apparent Semidiameter, (which at a Mean Diſtance from the Sun is = 16′ 5″,) he may be ſeen with a good Teleſcope to make a black Spot on the Sun's Diſk; the Times when he may be ſeen thus, I have calculated, as here you ſee.

1707	*April*	24d.	12 h.	0′	0″	
1710	*October*	26	12	0	0	
1720	*April*	26	21	25	19	very near but did not [touch.
1723	*October*	29	6	24	17	
1730	*October*	22	5	42	25	very near the Sun.
1736	*October*	31	0	4	8	
1740	*April*	21	12	54	5	
1743	*October*	25	0	14	8	
1753	*April*	24	20	37	23	
1756	*October*	26	18	0	13	
1769	*October*	29	11	41	53	
1776	*October*	22	11	41	1	
1782	*November*	1	5	20	34	
1786	*April*	22	20	46	51	
1789	*October*	25	5	32	37	
1799	*April*	26	4	34	15	

The Inclination of his Orbit = 6° 59′ 20″.

When *Mercury* is in Conjunction with the Sun in the Upper Part of his Orbit, and near either Node as abovementioned, he will paſs behind the Sun, and in this Caſe is always direct in Motion. Every thirteen Years he is nearly in the ſame Place of the Heavens.

III. *Of Venus.*

NEXT above *Mercury*, is the glittering Planet *Venus*, who performs her Revolution round the Sun in the Space of 224 d. 16 h. 49′ 24″.

Her Eccentricity Parts				505
Mean Diſtance from the Sun ditto				72337
Aphelion this Year 1739	♏	7°	8′	12″
Inclination of her Orbit		3	23	20
Heliocentric Diurnal Motion		1	36	8
Hourly Motion			4	00⅓
Place of her North Node	♊	14	18	2

Her greateſt Elongation from the Sun, is when the Earth is in Perihelion, and *Venus* in Aphelion, for then the greateſt Side of the Triangle ſubtends the Angle at the Earth, and this Angle which is her Elongation, when at greateſt is 47° 38′ 35″ (*See my* Uranoſcopia, *page* 66,) but if ſhe be in Perihelion her greateſt Elongation can be no more than 44° 56′ 14″, and when ſhe is in either of theſe Poſitions ſhe is Direct. And every two Years, ſhe is Stationary, and Retrograde us at the Earth, and when Retrograde, is always Occidental of the Sun. She ſtrays further from the Ecliptic than any other Planet, for ſometimes ſhe will have 9° of Latitude to us at the Earth.

Her greateſt Diſtance from the Earth is = 134017363, and leaſt Diſtance 15165311 *Engliſh* Miles: She turns once round upon her Axis in 23 Hours, as has been diſcovered by the Teleſcope. She is ſuppoſed to have an Atmoſphere, which reflects ſo ſtrong and glaring a Light, that her Body is rarely ſeen clear and diſtinct, without applying a proper Aperture to the Object Glaſs of your Teleſcope, and then you may ſee her increaſe and decreaſe in Light, Horn'd, and *Gibbous* as the Moon and *Mercury* are. This Planet alſo, as well as *Mercury*, when in Conjunction with the Sun in the Lower Part of her Orbit, and near one of her Nodes, (may be ſeen, by the Help of a Teleſcope) to make a black Spot on the Sun's Diſk, like a Patch on a Lady's Face, as I have found by Calculation, and ſhall here ſet down.

1639 *Novemb.*	24 d.	3 h.	19′	0″	obſerved by *Horrox.*
1761 *May*	25	22	44	5	Retrograde.
1769 *May*	23	16	10	20	

When this *Planet* is conjoined with the Sun in the upper Part of her Orbit, ſhe is always direct in Motion, and if her Geometric Latitude be then leſs than the Sun's apparent Semi-diameter, ſhe will paſs behind the Sun. This *Planet*, (as well as *Mercury*) can never be ſeen at Midnight, but always either in the Morning or Evening; and when ſhe is ſeen in the Morning, ſhe is called the Morning Star, and when in the Evening, the Evening Star.

Since theſe two inferior *Planets*, *Venus* and *Mercury* never recedes farther from the Sun than has been abovementioned; Aſtronomers well know from hence that the Earth's Orb, circumſcribes their Orbits, and that they turn about the Sun, ſometimes direct and ſometimes Retrograde, ſhewing various Faces, and are never in oppoſition, no not ſo much as a Quadrat, or Sextile Aſpect, which they would be if the Earth were at Reſt in the Center of the Univerſe.

IV. *Of the Earth.*

THE Earth is one of the Seven *Planets*, and is plac'd in an Orb between *Venus* and *Mars*, ſhe makes one entire Revolution round the Sun in 365 Days, 5 Hours, 4 Minutes, 7 Seconds, (*ſee page* 272) this is called the

Tropical Year, but the Siderial Year is longer by 20′ 17″ 27‴, being what is increaſed by the Proceſſion of the Equinox; beſides this Annual Motion, which is always direct, and according to the Order of the Signs, ſhe has a Second Motion upon her own Axis from Weſt to Eaſt, which in reſpect to the Fixed Stars, is performed in 23 h. 56′. The Earth's Axis is inclin'd to that of the Ecliptic, in an Angle of 66° 31′, and it's Figure is that of an Oblate Spheriod, ſwelling out towards the Equitorial, and flatted and contracted towards the Poles: So that the Diameter of it at the Equator, is longer than the Axis by about 34 Miles, as Sir *Iſaac Newton* has demonſtrated; for the Polar Diameter, or Axis, is to the Equatorial one, as 689 to 692. Experiments alſo made on Pendulums, which require different Lengths to ſwing Seconds, here and at the Equator do prove the ſame Thing. Our Countryman Mr *Rich. Norwood* found, by meaſuring from the *Tower* of *London* to the Middle of the City of *York*, in the Year 1635, that the Degree of the Arch of a great Circle upon the Earth's Surface, contain'd 69½ Miles; which Experiment agrees very well with what the *French* Aſtronomers have found it to be. According to which Meaſure, we find the Earth's Circumference 25035.84, it's Diameter 7969.16, and the Heighth of the Atmoſphere 47.12 *Engliſh* Miles. Eccentricity 1692, *Horizontal Parallax* 16′ 5″ diſtance from the Sun 82183140 as was ſaid in the Sun's Theory. For the Moon undoubtedly revolves about our Earth, and the Diſtance of the Sun is to that of the Moon at a mean Rate, as 20625, is to 60 in Earth's Semidiameters, and the Moon's Periodical Time, 27 d. 7 h. 43′ 71″ the Periodical Time of the Sun, (if he moved) will be thus found, *viz.*

As the Cube of the Moon's Diſtance from the Earth, in Earth's Semidiameters = 216000, to the Cube of the Sun's diſtance 8773681640625, ſo is the Square of the Moon's Periodical Time 27, (omitting the Hours) = 729, to the Square of the Moon's Periodical Time, 29611175537 whoſe Square Root = 172079 Days = 471 Years, and ſo long would the Sun be in performing one Revolution, if he moved, and the Earth fixed in the Center of the World, as *Ptolomy*, *Tycho*, and their Followers vainly imagine. By which the Controverſy between our true Syſtem, and all the other falſe ones is abſolutely determined in favour of the *Copernican*, and the Earth's Annual and Diurnal Motions for ever unqueſtionably eſtabliſhed.

That the different Heats of our Seaſons of the Year do not depend upon the nearneſs of the Sun to the Earth, but upon the Sun's Rays falling more direct, or more oblique upon us; for in the Diſtance of 82 Million of Miles, a little approach of the Earth to, or it's Receſs from the Sun, will make no ſenſible Alteration as to Heat or Cold.

The Sun's Meridian Altitude at London,

On	*June*	10	is =	61°	57'
	March	10		38	28
	September	12		38	28
	December	10		14	59

From hence it is plain, that the Sun's Heat in Winter muſt be weakeſt, becauſe then the Angle is more acute than it is in *June*, and the Rays falling moſt oblique on us, the ſame Quantity of Rays are ſcatter'd over a greater Space of Earth, and conſequently it muſt be colder there than where the ſame Quantity of ſolar Rays poſſeſs a leſs Space or Earth; beſides the Sun's Rays paſſing through a greater Part of the Atmoſphere in the Winter than in the Summer, they muſt be more weak and faint in the firſt, than in the latter Caſe.

Of the Equation of Time.

THE Daily Revolutions of the Earth's Equator round it's Axis are exactly equal in Times to one another, and yet the Time from the Apparent Noon of one Day to that of the next, is unequal, and ſometimes greater, and ſometimes leſſer.

And there is a double Cauſe of this Inequality ; Firſt That the Plane of the Ecliptic doth not lye in the Plane of the Equinoctial, but makes an Angle with it of 23° 29' the Difference between the Sun's Place in the Ecliptic, and his right Aſcenſion in the Equinoctial turn'd into Time, by allowing 15° to an Hour, and 1° to 4' in Time, and ſo proportionably for a leſſer or greater Quantity, which in the firſt and third Quadrants of the Ecliptic is to be added, but in the ſecond and fourth is to be ſubtracted.

The ſecond Cauſe is the Earth's annual Orbit, not being a Circle, but an Ellipſis, therefore with the Earth's Mean *Anomaly*, the Ecliptic Equation is taken out of it's proper Table in my Syſtem, and reduced into Time as before, by allowing 15° to an Hour, *&c.* gives you the Second Part of the Equation of Time, which if the Mean *Anomaly* be leſs then ſix Signs it addeth, if more ſubtracteth, to or from the equal Time gives the Apparent ; now if both theſe Parts add, or both ſubtract, their Sum, otherwiſe their Difference, is the abſolute Equation of Time. Which applied to the equal Time, according to the Title of the greateſt Part, gives the Apparent Time ; but to reduce the Apparent Time to the equal, uſe the contrary Titles.

EXAMPLE.

Anno 1739, *June* 5, at Noon equal Time, I demand the true Equation of Time ?

Operation.

Sun's Place at Noon is ♊ 24° 44' 52'' and the Mean Anomaly from my Tables 11 S. 15° 56' 21''.

Sun's Place	♊ 24	44	52	gives	-	1'	54''	+
Anomaly	11 15	56	21	gives	-	1	51	−
Diff. is the true Equation of Time					=	0	3	+

So that this Day at Noon if your Clock or Watch be true, it will be 3'' ſlower than the Sun; and the ſame is to be obſerved for any Day in the Year by the following Table, which (as a Mean) is calculated for Second paſt Leap Year, and may ſerve (for common Uſe) this Age, without any ſenſible Error. Here are four Days in the Year in which the Sun and Clock are together, *viz. April* 4, *June* 5, *Auguſt* 20, and *December* 13; there are four Days in the Year, in which the Sun and Pendulum Clocks differ moſt, which are theſe, with their Difference.

January	31,	Equation	14'	49''	Clock too faſt.
May	4,		4	5	Clock too ſlow.
July	15,	-	5	57	Clock too faſt.
October	22,	-	16	13	Clock too ſlow.

See the following Table.

A Table of Equation of Time, for regulating of Pendulum Clocks with the Sun.

Deg.	*Janua.*			*Febr.*			*March.*			*April*			*May.*			*June*		
	'		''	'		''	'		''	'		''	'		''	'		''
1	8		58	14		48	10		8	0		50	4		2	0		50
2	9		21	14		47	9		51	0		34	4		4	0	Slow	37
3	9		43	14		45	9		34	0		18	4		5	0		25
4	10		6	14		42	9		17	0	+	2	4		5	0		12
5	10		26	14		38	9		0	0		13	4		4	0	−	1
6	10	Clocks too	45	14	Clocks too	33	8	Clocks too	42	0	Clocks too	28	4	Clocks too	3	0	Clocks too	11
7	11		4	14		28	8		24	0		42	4		2	0		24
8	11		22	14		22	8		5	0		56	4		0	0		37
9	11		40	14		16	7		47	1		10	3		58	0		50
10	11		57	14		9	7		29	1		23	3		56	1		3
11	12	faſt, ſubtract	14	14	faſt, ſubtract	4	7	faſt, ſubtract	11	1	ſlow, add	36	3	ſlow, add	53	1	faſt, ſubtract	16
12	12		30	13		52	6		52	1		48	3		49	1		29
13	12		44	13		43	6		33	2		0	3		44	1		42
14	12		58	13		34	6		15	2		11	3		38	1		55
15	13		11	13		24	5		56	2		21	3		32	2		7
16	13	from	23	13	from	13	5	from	37	2	to the	31	3	to the	25	2	from the	20
17	13		35	13		2	5		19	2		41	3		18	2		32
18	13		46	12		50	5		1	2		51	3		11	2		44
19	13		55	12		37	4		42	3		0	3		4	2		56
20	14		05	12		28	4		23	3		8	2		56	3		8
21	14	equal	13	12	equal	12	4	equal	4	3	equal	15	2	equal	47	3	equal	19
22	14		20	11		58	3		45	3		22	2		38	3		30
23	14		26	11		43	3		27	3		29	2		29	3		41
24	14		32	11		28	3		9	3		35	2		19	3		52
25	14		37	11		13	2		51	3		41	2		9	4		2
26	14	Time	41	10	Time.	58	2	Time,	33	3	Time.	46	1	Time.	59	4	Time.	12
27	14		44	10		42	2		15	3		50	1		48	4		22
28	14		46	10		25	1		57	3		54	1		37	4		31
29	14		47	—		—	1		40	3		57	1		26	4		40
30	14		48	—		—	1		23	4		0	1		14	4		48
31	14		49	—		—	1		6	—		—	1		3	—		—

A Table of Equation of Time for regulating of Pendulum Clocks with the Sun.

Degr.	July.			August			Septem.			October			Novem.			Decem.		
	'		"	'		"	'		"	'		"	'		"	'		"
1	4		57	4	Clocks too	34	3		54	13		27	15		34	5	Clocks too	48
2	5		5	4		24	4		15	13		41	15		26	5		19
3	5		12	4		14	4		36	13		54	15		16	4		50
4	5		19	4		3	4		57	14		9	15		5	4		21
5	5		25	3		52	5		18	14		20	14		54	3		51
6	5	Clocks too	31	3	faſt, ſubtract	40	5	Clocks too	39	14	Clocks too	32	14	Clocks too	42	3	ſlow, add.	21
7	5		36	3		27	6		00	14		43	14		30	2		51
8	5		41	3		13	6		21	14		53	14		16	2		21
9	5		45	2		59	6		42	15		3	14		1	1		51
10	5		48	2		45	7		3	15		13	13		46	1		21
11	5	faſt, ſubtract	50	2	from the	31	7	ſlow, add	24	15	ſlow, add	20	13	ſlow, add	30	0	− Too	51
12	5		52	2		16	7		44	15		31	13		13	0		21
13	5		54	2		1	8		4	15		39	12		55	0		9
14	5		56	1		45	8		24	15		45	12		37	0		39
15	5		57	1		29	8		44	15		51	12		18	1		9
16	5	from the	57	1	equal Time.	12	9	to the	4	15	to the	56	11	to the	59	1	faſt, ſubtract	39
17	5		55	0		55	9		24	16		00	11		39	2		9
18	5		53	0		38	9		44	16		4	11		18	2		38
19	5		51	0		20	10		4	16		7	10		56	3		7
20	5		49	0		2	10		23	16		9	10		33	3		36
21	5	equal	46	0	+ Too ſlow,	15	10	equal	42	16	equal	11	10	equal	10	4	from the	5
22	5		43	0		33	11		1	16		13	9		46	4		34
23	5		39	0		52	11		19	16		12	9		21	5		2
24	5		34	1		12	11		37	16		1	8		56	5		29
25	5		29	1		32	11		54	16		9	8		30	5		56
26	5	Time.	22	1	add.	52	12	Time.	11	16	Time	6	8	Time.	4	6	equal Time.	23
27	5		15	2		12	12		27	16		3	7		38	6		50
28	5		8	2		32	12		43	15		59	7		11	7		16
29	5		1	2		53	12		58	15		54	6		46	7		41
30	4		53	3		14	13		13	15		48	6		16	8		5
31	4		44	3		34	—		—	15		41	—		—	8		29

This Table ſhews you what the Sun gains or loſes of the Pendulum Clock every Day in the Second paſt Leap Year, the Pendulum Clock keeps equal Time all the Year round, though the Sun doth not do ſo, but is very unequal in it's Apparent Motion, ſometimes too faſt, at other Times too ſlow, as the Table plainly ſheweth: So that if at any Time you want to ſet your Clock or Watch by the Sun, look into this Table and ſee what the Equation is on that Day, and ſet it accordingly ſo many Minutes and Seconds more or leſs, as you ſee by the Table, the Clock is too faſt or too ſlow for the Sun.

EXAMPLE.

January 17, I ſee by this Table that the Clock is 13′ 35″ too faſt for the Sun, then looking upon a good Sun Dial, I ſet my Watch or Clock ſo much faſter than the Time is, by the Sun Dial, and then 'tis right. Again, on *Auguſt* 29, I ſee by the Table the Clock is too ſlow 2′ 53″, therefore ſet your Clock and Watch ſo much behind the Time ſhewn by the Sun Dial and 'tis right; more Examples in a Thing ſo plain are needleſs.

There is nothing more eaſy and ſimple than the Motion of the Earth, becauſe it accounts for Appearances of Day and Night, in an eaſy and Natural Manner; for as the Earth revolves from Weſt to Eaſt in 24 Hours Time nearly, it makes the Sun appear to do ſo from Eaſt to Weſt in the ſame Time; and makes it Day to thoſe Places of it's Surface, which are turned towards the Sun, and Night to ſuch who are on the oppoſite Part of the Globe.

The Twilight is that dubious half Light, which we perceive before the Sun Riſing, and after Sun Setting. 'Tis occaſioned by the Earth's Atmoſpere, and the Splendor of the *Æther* which environs the Sun; but becauſe of many accidental Variations in both the Sun and Earth's Atmoſphere, it cannot be always of one Degree of Duration or Brightneſs; yet it uſually holds in the Evenings 'till the Sun is 18 Degrees below the *Horizon*, and appears ſo long before his Riſing in the Morning. And where the Parallel of the Sun's Declination cuts the Parallel of 18°, there is ſhewn the Time in the Projection of the Sphere, when the Twilight begins and ends: For by a true Conſideration of the Sphere, it will be eaſy to determine in any Latitude, where the Parallel of Declination cuts the Parallel of 18°, and from thence to determine the Day when it ceaſes to be perfect Night, and alſo when perfect Night begins again.

EXAMPLE.

Suppoſe I would know in the Latitude of 51° 32' N. the Day when it ceaſes to be perfect Night, and alſo the Day when the Parallel of Twilight cuts the Parallel of 18°, for that is the Day when it begins to be perfect Night again?

Firſt you are to conſider that from the *Zenith* to the *Horizon* is 90° + 18 = 108 the Diſtance of the Parallel of Twilight from the *Zenith*, from this Sum = 108, ſubtract the Complement of the Latitude of the Place, and the Remainder is the Sun's Diſtance from the North Pole, or Complement of his Declination; ſeek this in the Tables of the Sun's Declination, in the Months between *March* and *June*, and where you find it is the Day that it ceaſes to be perfect Night, look again in the Months after *June*, and before *September*, and where you find the Sun's Declination, that is the Day that perfect Night begins again.

Operation.

Sun's Diſtance *à Zenith*, when Day breaks	108°	0'
Co-Lat. ſubtract - - - - -	38	28
Sun's Diſtance *à North Pole* - -	69	32
Complement = Declination North -	20	28

In the Tables of the Sun's Declination, I find this to anſwer to *May* 11, and *July* 11, between which two Days there is no perfect Night but Twilight, for all that Time the Parallel of the Sun's Declination never touches the Parallel of 18° below the *Horizon*.

EXAMPLE II.

For *Guernſey*, Latitude 49° 36' North.

Sun's Diſtance from the *Zenith* -	108°	0'
Co-Latitude ſubtract - - -	40	24
Sun's Diſtance from North Pole -	67	36
Complement = ☉ Declination North	22	24

The Days for anſwering this Declination ☉, are *May* 24, and *June* 28. The laſt and firſt Appearance of perfect Night in that Latitnde.

EXAMPLE. III.

At *Madrid*, Latitude 40° 10' North, which are the two Days when the perfect Night ceaſes, and alſo when perfect Night begins again?

Operation.

Sun's Diſtance from the *Zenith* -	108°	00'
Co-Latitude ſubtract - - - -	49	50
Remains - - - - - -	58	10

This 58° 10' being leſs than the Sun's leaſt Diſtance from the North Pole 66° 31' proves there is perfect Darkneſs at Midnight all the Month of *June:* For the Parallel of the Sun's Declination, never leaves the Parallel of the Twilight. But at *Petersbourgh* in *Ruſſia*, Latitude 60° 4' North, there is no Night but Twilight, from the Tenth of *April*, to the Eleventh of *Auguſt*.

V. *Of the Moon.*

THE Moon is a Secondary Planet, and reſpects our Earth for her Center; ſhe always accompanies our Earth, and therefore is properly called the Earth's Satellite: She performs one Revolution round the Earth in 27 d. 7 h. 43' 7'', and in the ſame Space of Time by a ſtrange Correſpondence and Harmony of the two Motions, ſhe revolves the ſame Way about her own Axis, whereby one Motion as much conventing it to, as the other turns it from the Earth, the ſame Side is always expoſed to our Sight

The Figure of the Moon's Orbit is always changing, ſo that at one Time it is a long Ellipſis, at another time more ſhort, and nearer a Circle, by which her Eccentricity at one Time is 66782 in Apogeon, and in Perigeon 43319 which are the Two Extreams, between which the Moon's Eccentricity is always found. *See my Satellite Aſtronomy, page* 16. Her mean Diurnal Motion is = 13° 10′ 35″, and her mean hourly Motion 32′ 56″; and her

Greateſt	Diſtance from the Earth.	62.518	Earth's Semidiameters.
Mean.		59.784	
Leaſt		55.985	

Which in *Engliſh* Miles are	249107.97
	238241.13
	223076.71

We have ſhewn before that all Objects are ſeen under a certain Angle; therefore the ſame Body, at different Diſtances will appear to have very different Magnitudes: It alſo follows that a ſmall Body, when near us, will appear to be equal, or even to exceed another at a great Diſtance, tho' immenſly bigger. And this is the Reaſon that the Moon, which is the leaſt of all the Planets, appears to us vaſtly bigger than any of them, nay, even to equal the Sun himſelf, which is many thouſand times greater in Magnitude.

By the Will of the great Creator, which Philoſophers call the Law of Gravity, or Gravitation, whereby all heavy Bodies have a Tendency towards the Center of the Earth, *&c.* and by which, that vaſt Machine, the Planetary Syſtem is governed, all Bodies gravitating towards each other; and this Gravity is Proportional to the Quantity of Matter at unequal Diſtances, it is univerſally as the Square of the Diſtance: All Bodies mutually attract, or tend towards each other.

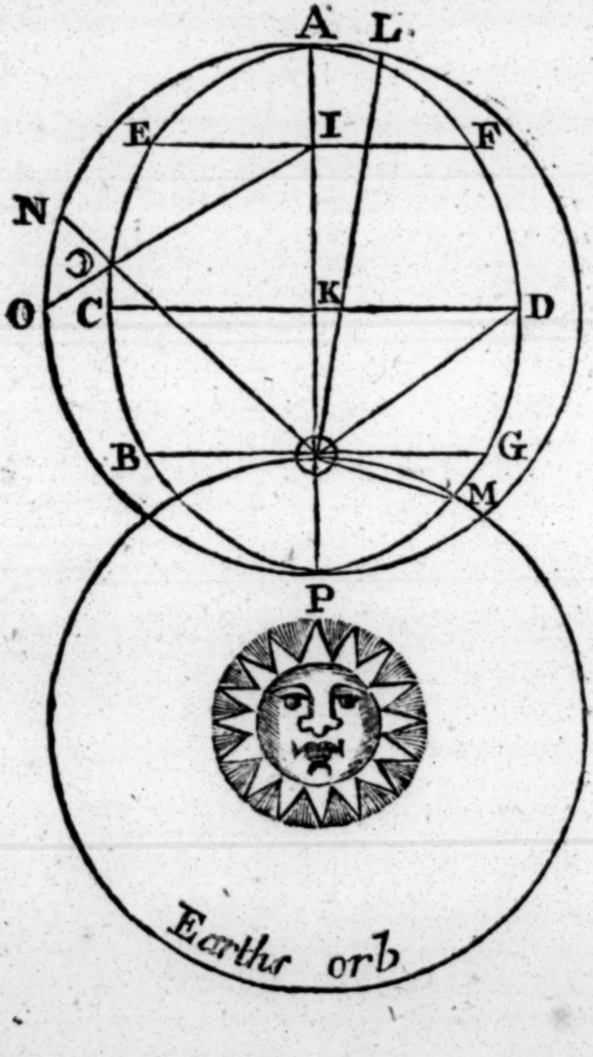
The Differenee between Gravity and Attraction is this, *viz.* Gravity is that by which one Body tends towards another. Attraction is that by which one Body tends towards another by ſome Force, though perhaps it may happen by Impulſe.

The *Centripetal* Force, is that by which a Body, (ſuppoſe the Moon, or any other Planet) is drawn or impell'd towards the Center.

Centrifugal

Contrifugal Force, is that by which a Body endeavours to fly off in a Tangent Line. Now theſe two Forces being equal, the Planets are all kept in their Orbits, they neither fly off nor drop down to their Centers: Which they would do if either of theſe Laws were taken away.

The Moon moves in an Ellipſis, ACPD, whoſe lower *Focus* is the Center of the Earth, round about which ſhe deſcribes Areas Proportional to the Times, wherein they are deſcribed, and the Earth carries the Moon round the Sun in her Annual Motion.



Let the Moon be at L, from whence in a certain Time ſhe moves to A, the Space or Ray ⊖ L, deſcribes is L ⊖ A; the Moon moving on 'till ſhe comes to M, and from the Center of the ⊖, draw the Line ⊖ M, ſo that the Elliptic Space or Area ⊖ P M may be equal to the Area L ⊖ A, then in this Caſe ſhe will move thro' the Arch P M in the ſame Compaſs of time, that ſhe did thro' the Arch L A. Which Arches are unequal, and nearly in a Reciprocal Proportion to their Diſtances from the Earth's Center. For becauſe of equal Areas, the Arch P M muſt be ſo much in Proportion greater than the Arch L A, as ⊖ A is greater than ⊖ P: This Law was firſt diſcovered by *Kepler*, and now demonſtrated by Sir *Iſaac Newton*. And for a farther Vindication of this Univerſal Law of the Heavenly Bodies, they have demonſtrated that the Squares of the Periodical Times of the Planets, are as the Cubes of their Diſtances from the Center of their Orbits, about which they perform their Motions regularly.

Of the many inequalities in the Moon's Motion, that which is called her Elliptic Equation is greateſt, which ariſes from a two-fold Motion, one perform'd on the Upper *Focus* at I, and the other on the Lower at ⊖; for ſuppoſe the Moon in her Orbit at ☽, to which if we draw the Line ⊖ ☽ and I ☽, her mean Place will be at O, and her true Place at N, ſo that the Elliptic Equation is no more than the Angle ⊖ ☽ I = N ☽ O, which how to find, ſee my *Satellite Aſtronomy*, *page* 16. And this Equation is to be ſubtracted all the Time the Moon moves from the Apogeon at A, to the Perigeon at P, that is the firſt Six Signs of Anomaly. But in the other Semicircle of her Orbit, while ſhe moves from her Perigeon at P, to her Apogeon at A, that is when her mean Anomaly is 6, 7, 8, 9, 10, or 11 Signs, this Elliptic Equation is to be added to her mean Place to gain her true. And the ſame Reaſon holds good in all the Planets.

Of the Moon's Phaſes.

NEW Moon, or which is all one, the Change of the Moon, is when the Sun and Moon are both together upon one, and the ſame Circle of Longitude, that is, when they are in the ſame Sign and Degree of the *Zodiack*, and now ſets with the Sun; and by Reaſon the Moon is always direct in Motion to us at the Earth, ſoon after the Change ſhe is ſeen in the Evening after the Sun is ſet, and puts on then a falcated Face, and her Horns point towards the Eaſt; when ſhe is two Signs, or 60° diſtant from the Sun, that is one Sixth Part of the *Zodiack*; this Diſtance is called a *Sextile Aſpect*; ſhe moving one Sign more in *Conſequentia*, 'till ſhe is 3 Signs or 90° from the Sun, this is called a Quadrate or Square Aſpect, or the firſt Quarter of the Moon, and ſhe is now Dechotomized. When by her Menſtrual Motion ſhe is four Signs diſtant from the Sun towards the Eaſt; the Face ſhe puts on now is *Gibbous*, and ſo continues 'till ſhe comes to the Full, that is Six Signs diſtant or Diametrically oppoſite to the Sun, ſhe now ſhews a full Face to us at the Earth, becauſe that Side which is towards the Sun, is alſo turned towards the Earth. (*See my Lunar Inſtrument*). The Earth being partly between the Sun and Moon; ſhe now riſeth as the Sun apparently ſets, (if the Time of the Full Moon be at Sun ſetting, elſe not ſo exactly,) we have above ſhewn, that the Moon performs one Revolution round the Earth in 27d. 7 h. 43′ 7″, which is called her Menſtrual Month, but in that Time the Sun has apparently moved 26° 55′ 46″ from the Place of the laſt Conjunction with the Sun, therefore the Moon has yet 2 d. 5 h. 0′ 59″ farther to travel before ſhe can come up with the Sun to make the next New Moon, which make in all 29 d. 12 h. 44′ 6″, as I thus prove.

Moon's Revolution.

27 d.	7 h.	43'	7''
2	1	3	2
	3	40	9
		16	29
		1	14
			5
29	12	44	6

Sun moves in that Time.

26°	55'	46''
2	00	52
	9	3
		41
		3
29	06	25

This 29d. 12h. 44' 6'' the Time between one New Moon and the next, is called the Synodical Month. And now from the Full to the laſt Quarter, the Moon puts on a *Gibbous* Face again, but on the contrary Side to what it was from the firſt Quarter to the Full; becauſe that Side of her Face that was then towards the Earth, is now turned from it (by Reaſon of the Moon's Rotation upon her Axis). The Moon moving on her Menſtrual Motion, 'till ſhe is Eight Signs diſtant from the Sun in *Conſequentia*, ſhe makes then what is called a *Trine Aſpect*, which ſhe made when ſhe was Four Signs from him, which takes it's Name from being an Equilateral Triangle, or one third Part of the *Zodiack*. One Sign farther towards the Eaſt brings her to a Quadrat again which is called the laſt Quarter, and is now again Dechotomized, or juſt half enlighten'd to us at the Earth, as ſhe was at the firſt Quarter, but now turns the contrary Side towards us. From the laſt Quarter to the Change or New Moon ſhe is again falcated, but as the Horns in the firſt Quarter were turned towards the Eaſt they now are towards the Weſt. This Law the Moon obſerves continually, only with a little Variation cauſed by her Libration, which is more Philoſophical than Aſtronomical, and ſo I ſhall paſs it by at this Time.

Aſtronomers divide the Diameter of the Sun and Moon into 12 equal Parts called Digits, which according to every Day of her Age is ſhewn by this Table, her Increaſe from Change to Full, and her Decreaſe from Full to Change.

A Table of the Digits and Decimal Parts of a Digit, to every Day of the Moon's Age. According to her Mean Motion.

Moon's Age.	Digit's Light.	Moon's Age.	Digit's Light.
0 New 0		29 12	
1	0.81	29	0.44
2	1.63	28	1.22
3 Falcated.	2.44	27 Falcated.	2,03
4 ☽	3.25	26 ☾	2.85
5	4.07	25	3.66
6	4.88	24	4.47
7	5.69	23	5.29
7 First 9	6.	22 Laſt 3	6.
8 Quarter.	6.5	22 Quarter.	6.10
9	7.32	21	6.92
10	8.14	20	7.73
11 Gibbous.	8.95	19 Gibbous.	8.51
12	9.76	18	9.36
13	10.58	17	10.17
14	11.39	16	10.98
14 Full 18	12.	15 0	11.8

This Table is ſo plain, little needs be ſaid in Explanation, only this, look in the firſt, or third Column for the Moon's Age, and in the next on the Right Hand, is the Quantity of Light that the Moon has at that Time. Examples are needleſs.

A Table of the Moon's Place, 1739.

Days	*January.*			*February.*			*March.*			*April.*		
	S.	°	'	S.	°	'	S.	°	'	S.	°	'
1	11	0	3	0	18	55	0	26	29	2	11	38
2	11	14	0	1	1	42	1	9	4	2	24	6
3	11	27	30	1	14	7	1	21	33	3	5	56
4	0	10	33	1	26	16	2	3	45	3	17	43
5	0	23	15	2	8	14	2	15	44	3	29	32
6	1	5	26	2	20	2	2	27	35	4	11	29
7	1	17	34	3	1	51	3	9	24	4	23	33
8	1	29	33	3	13	41	3	21	17	5	6	16
9	2	11	25	3	25	38	4	3	14	5	18	56
10	2	23	15	4	7	43	4	15	24	6	2	12
11	3	5	8	4	20	0	4	27	52	6	15	51
12	3	17	4	5	2	31	5	10	38	6	29	54
13	3	29	9	5	15	17	5	23	45	7	14	12
14	4	11	18	5	28	19	6	7	13	7	28	16
15	4	23	35	6	11	35	6	20	58	8	13	15
16	5	6	4	6	25	1	7	4	57	8	27	43
17	5	18	43	7	8	35	7	19	5	9	12	2
18	6	1	35	7	22	28	8	3	19	9	26	13
19	6	14	42	8	6	24	8	17	31	10	10	10
20	6	28	26	8	20	45	9	1	36	10	23	56
21	7	11	47	9	4	56	9	15	47	11	7	33
22	7	25	45	9	19	12	9	29	48	11	21	2
23	8	10	1	10	3	30	10	13	44	0	4	19
24	8	24	32	10	17	46	10	27	35	0	17	27
25	9	9	18	11	1	57	11	11	18	1	0	25
26	9	24	10	11	15	57	11	24	54	1	13	11
27	10	9	1	11	29	43	0	8	18	1	25	43
28	10	23	43	0	13	11	0	21	28	2	8	4
29	11	1	16	—	—	—	1	4	24	2	20	12
30	11	22	7	—	—	—	1	17	7	3	2	10
31	0	5	4	—	—	—	1	29	29	—	—	—

A Table of the Moon's Place 1739.

Days	May.			June.			July.			Augnſt.		
	S.	°	'	S	°	'	S.	°	'	S.	°	'
1	3	13	58	4	27	55	6	1	35	7	20	34
2	3	25	46	5	10	4	6	14	10	8	4	24
3	4	7	34	5	22	29	6	27	29	8	18	34
4	4	19	30	6	5	16	7	11	2	9	3	5
5	5	1	38	6	18	29	7	25	1	9	17	54
6	5	14	4	7	2	11	8	9	29	10	2	56
7	5	26	54	7	16	22	8	24	19	10	18	1
8	6	10	11	8	0	51	9	9	27	11	3	2
9	6	23	56	8	16	0	9	24	42	11	17	44
10	7	8	10	9	1	10	10	9	53	0	2	5
11	7	22	46	9	16	22	10	24	50	0	15	58
12	8	7	30	10	1	23	11	9	28	0	29	24
13	8	22	36	10	16	9	11	23	40	1	12	22
14	9	7	28	11	0	29	0	7	24	1	24	57
15	9	22	10	11	14	27	0	23	42	2	7	13
16	10	6	34	11	28	2	1	3	35	2	19	14
17	10	20	40	0	11	13	1	16	7	3	1	4
18	11	4	7	0	24	8	1	28	24	3	12	53
19	11	17	56	1	6	46	2	10	28	3	24	40
20	0	1	12	1	19	10	2	22	24	4	6	31
21	0	14	13	2	1	24	3	4	15	4	18	27
22	0	27	1	2	13	30	3	16	4	5	0	33
23	1	9	40	2	25	27	3	27	53	5	12	51
24	1	22	7	3	7	21	4	9	45	5	21	22
25	2	4	24	3	19	12	4	21	43	6	8	5
26	2	16	33	4	1	2	5	3	49	6	20	59
27	2	28	32	4	12	55	5	16	4	7	4	7
28	3	10	28	4	24	5[illegible]	5	28	28	7	17	2
29	3	22	8	5	6	54	6	11	7	8	0	5
30	4	47	7	5	19	8	6	23	58	8	14	42
31	4	15	5	—	—		7	7	7	8	28	4

A

A Table of the Moon's Place 1739.

Days	September.			October.			November.			December.		
	S.	°	'	S.	°	'		°	'	S.	°	'
1	9	12	53	10	21	50	0	13	46	1	19	48
2	9	27	21	11	6	4	0	27	11	2	2	27
3	10	11	54	11	20	27	1	10	32	2	14	54
4	10	26	37	0	4	36	1	23	35	2	27	10
5	11	11	19	0	18	22	2	6	23	3	9	18
6	11	25	51	1	2	4	2	19	2	3	21	19
7	0	10	7	1	15	25	3	1	11	4	3	13
8	0	24	1	1	28	26	3	13	15	4	15	4
9	1	7	30	2	11	5	3	25	11	4	26	53
10	1	20	32	2	23	23	4	6	59	5	8	49
11	2	3	13	3	5	26	4	18	46	5	20	50
12	2	15	28	3	17	19	5	0	40	6	3	8
13	2	27	30	3	29	4	5	12	43	6	15	43
14	3	9	21	4	10	50	5	25	00	6	28	43
15	3	21	5	4	22	42	6	7	40	7	12	6
16	4	2	5	5	4	45	6	20	43	7	26	10
17	4	14	44	5	17	4	7	4	13	8	10	18
18	4	26	43	5	29	41	7	18	5	8	25	1
19	5	8	59	6	12	39	8	2	18	9	9	56
20	5	21	28	6	26	1	8	16	49	9	25	1
21	6	4	15	7	9	40	9	1	28	10	10	2
22	6	17	19	7	23	37	9	16	10	10	24	54
23	7	0	37	8	7	44	10	00	46	11	9	27
24	7	14	10	8	21	53	10	15	12	11	23	39
25	7	27	49	9	6	10	10	29	25	0	7	29
26	8	11	38	9	20	22	11	13	22	0	21	9
27	8	25	31	10	4	31	11	27	6	1	4	15
28	9	9	27	10	18	31	0	10	33	1	17	5
29	9	23	30	11	2	22	0	23	52	1	29	38
30	10	7	38	11	16	19	1	6	56	2	11	58
31	—	—	—	0	0	7	—	—	—	2	23	59

The Uſe of the foregoing Tables of the Moon's Place.

This Table of the Moon's Place, is that in Longitude, which is ſufficient for my Reader's Purpoſe in this Place.

The Epacts.	The Yearly mean Motion of the Moon to be added to her true Place, any Day in the Year 1739.		
	S.	°	'
12	4	12	41
23	8	25	23
4	1	8	4
15	5	20	46
26	10	3	27
7	2	16	9
18	6	28	50
29	11	11	31
11	3	24	13
22	8	6	54
3	0	19	36
14	5	2	17
25	9	14	58
6	1	27	40
17	6	10	21
28	10	23	3
9	3	5	4
20	7	18	46
1	0	1	7

Look in this Table for the Epact for the given Year, and the Number right againſt it added to the Moon's true Place any Day in the Year 1739, gives her Mean Place the ſame Day at Noon. Remembring in Leap Year to add the Motion of the Moon in a Day, 13° 11' more to the reſt.

EXAMPLE I.

Anno 1740, *April* 9, I would know the Moon's Place at Noon?

	S.	°	'
1739, *April* 9, ☽ Place	5	18	56
Epact 12 gives	4	12	41
Leap Year 1 Day		13	11
Moon's Place	10	14	48

EXAMPLE

EXAMPLE II.

Anno 1740, *Auguſt* 10, I would know the Moon's Place at Noon.

Operation.

	S.	°	'
1739, *Aug.* 10, ☽ Place	0	2	5
Epact 12 gives - - -	4	12	41
Leap Year, 1 Day - -		13	11
Moon's Place - - -	4	28	7

EXAMPLE III.

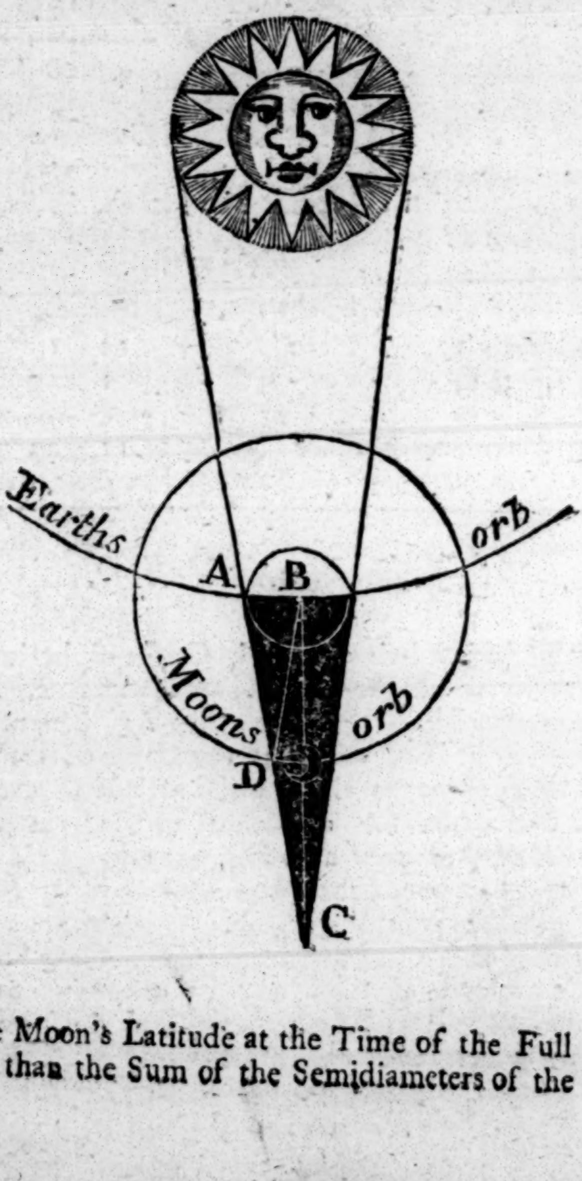
I would know the Moon's Place in the *Zodiack*, *Anno* 1734, *June* 12?

Operation.

1739, *June* 12, ☽ Place	10	1	23
1734 Epact 6, - - -	1	27	40
1734, *June* 12, ☽ Place	11	29	3

Of the Moon's Eclipſe.

THE Moon being a dark Opake Spherical Body, having no Light but what ſhe receives from the Sun, can never be eclipſed but at the Full Moon, becauſe the Earth's Shadow, which is the Cauſe of the Eclipſe, is always oppoſite to the Sun; but not at every Full Moon; but when ſhe is within 12° of either Node. *See the Scheme, in page* 277, and then her Latitude will be leſs than the Sum of the Semidiameters of the Moon and Earth's Shadow; but all the Full Moons that happen between EAH and GDF will paſs without any Eclipſe at all: The Latitude of the Moon is meaſured on a Circle of Longitude, from the Moon to the Ecliptic, and paſſing thro' it's Poles, and conſequently cutting the Ecliptic at right Angles. The Quantity of an Eclipſe depends on the Moon's Latitude, for the leſs the Latitude, the greater will the Eclipſe be, and the greater the Latitude the leſs will the Eclipſe be: So that if the Full Moon happen at the very Node, the Eclipſe will be Central and Total with continuance, I ſay continuance, becauſe the Moon's Diameter is but a ſmall Matter more than $\frac{1}{3}$ of the Earth's Shadow through which the Moon then paſſes, as you may the better perceive by this Figure.



If the Moon's Latitude at the Time of the Full Moon be more than the Sum of the Semidiameters of the Moon and Earth's Shadow, there will be no Eclipſe at that Full Moon, for if the Latitude be North, ſhe will paſs over the Shadow, if South under it. There are three Sorts of Lunar Eclipſes, *viz.* 1. *Partial.* 2 *Total without continuance.* 3 *Total with continuance.*

1. *Partial,* is when ſome Part of the Moon's Diameter or Body falls within the Earth's Shadow, and this always happens when the Sum of the Moon's Latitude and her Semidiameter is greater than the Semidiameter of the Earth's Shadow, in which the Digits eclipſed are always leſs than 12, and if her Latitude be then North, ſhe looſeth Light on the South Side of her Body, but if her Latitude be South, then ſhe is Eclipſed on the North, or Upper Side.

2. *Total without continuance.* is when the Moon's Latitude is equal to the Difference between the Semidiameter of the Moon and Earth's Shadow, for in this Caſe the Moon is no ſooner involved in the Shadow, but ſhe is out again, and therefore the Total Darkneſs is of no Duration, the Digits are juſt 12.

3. *Total with continuance,* is when the Sum of the Moon's Latitude and her Semidiameter is leſs than the Semidiameter of her Shadow: or, which is the ſameThing, when her Latitude is leſs than the Difference of her Semidiameter and Shadow, here the Digits are more than 12, and leſs than 22° 28' 12'', as I thus prove from the Table, page 62, of my *Compleat Syſtem.*

Moon's greateſt *Horizontal Parallax*	61 -	24.
Sun's add - - - - - -		10
Sum - - - - - -	61	24
Sun's leaſt Semidiameter ſubtract	15'	49''
Rem. greateſt Semidiameter ⊖ Shadow	45	45
Moon's greateſt Semidiameter, add	16	40
Sum - - - - - - - -	62	25
Latitude Moon, ſubtract - -	0	00
Reſt Parts deficient - - -	62	25

Now for the Digits eclipſed, ſay,

As Semidiamer ☽	-	16'	40''	L. L.	5563	add
To Six Digits 6°	-	00	00	-	10000	à
So Parts deficient	-	62	25	-	171	to
Sum	-	-	-	-	5734	ſub.
To the Digits eclipſed		22°	28'	12''	4266	

This is the greateſt Quantity that can be eclipſed.

4. The Eclipſes do not always happen in the ſame Place of the *Zodiack* by Reaſon of the Retrograde Motion of the Nodes, and Sun's apparent Motion, *ſee page* 299, which is 3' 11'' a Day, and they perform one Revolution in 18 Years 224 d. 4 h. 38'. So that that Eclipſe which falls this Year in ♉ 13°, will the next Year fall in ♉ 2° and 11 Days ſooner, by which the Period of Eclipſes called the *Chaldean Saros* is = 18 y. 10 d. 7 h. 43' 15'', when the ſame Eclipſe will return again. The greateſt Duration of Eclipſes happen about, or near the Apogeon of the Moon, ſhe moving then ſloweſt, tho' paſſes through a leſs Portion of the Earth's Shadow than when in Perigeon: The Moon's Apogeon moves forwards according to the Order of the Signs 6' 41'' in one Day, in a Year 1 S. 10° 39' 50'', therefore is one Revolution perform'd in about 8 Years, 309 Days, 8 Hours, and 32 Minutes. The Moon's Eclipſe always begins on the Eaſt Side and ends on the Weſt, contrary to that of the Sun; for the Moon's Eclipſe being real by reaſon of her direct Motion in Longitude always Eaſtward, the Eaſt Side of her Body muſt neceſſarily firſt touch the Earth's Shadow, and the Weſt Side laſt leave it. In Total Eclipſe of the Moon, even when ſhe is near the Center of the Shadow, her Body is frequently to be ſeen of a languid Colour, as I have often obſerved, and particularly, *Auguſt* 29, 1718, at 54' paſt 7 at Night, when ſhe was within 50' of her Southern Node, and Total for the Space of 1 h. 37' fall that Time ſhe did not look Black at all, but of a Kind of a yellow ſwarthy Colour, which could not be without being illuminated with ſome Light; (for of herſelf

ſhe has none,) and ſome will wonder from whence ariſe this Light, ſome ſuſpected that it was the Native and proper Light of the Moon herſelf, others derived it from the Planets and Stars; for the Interpoſition of the Earth intercepts all the Light of the Sun, and ſeems to bring a thick Darkneſs upon the Whole Space taken up by the Conical Shadow. But we muſt conſider that the Earth is ſurrounded with a Space of Air 49.66 Miles high, which is of a conſiderable Denſity, and has a Refracting Power whereby it turns the Rays of the Sun out of their Way, when they fall upon it, and makes them enter the Conical Shadow; which will therefore be illuminated by that ſmall Quantity of Light which falls obliquely on our Atmoſphere, and imparts to all the Bodies within it, a faint Light, the which will illuminate the Moon, even when ſhe is in the midſt of the Shadow, and will make her viſible to us.

5. The Length of the Earth's Shadow may be found at any Time by Trigonometry, thus, for the Semiangle of the Cone ACB, See Fig. p. 306, is always equal to the Sun's apparent Semidiameter, which when the Sun is in Apogeon it is 15′ 49″, at a Mean Diſtance 16′ 5″, and in Perigeon 16′ 22″, ſo that in the Triangle ABC, we have given the Angle A C B equal to the Sun's Apparent Semidiameter, and A B, one Semidiameter Earth to find B C, the Length of the Axis of the Cone of the Shadow.

Operation.

As T. < A C A	- 0° 15′ 49″	7.6627105
To one Semidiameter = A B 1	-	0.0000000
So Radius	- - - - -	10.0000000
To B C. ⊖ Semidiameter	217.41	2.3372895

217 41 × 3984.58 = 866287.5378 Miles.

At a Mean Diſtance from Earth, the Height of the Shadow is 213.775 Semidiameter of the Earth = 851803.5895 Miles. And at the leaſt Diſtance 210.014 Semidiameter of the Earth = 836817.158412 Miles

6. By the *Horizontal Parallax* of the Moon we alſo find her Diſtance from the Earth, for in the Scheme, page 275 we have given, the Angle H L G the Moon's *Ho-*

rizontal Parallax, and HG Earth's Semidiameter, to find HL her Diſtance from the Earth's Surface.

Let it be required then to find her Diſtance from the Earth's Surface when ſhe is in Apogeon? Look into the Table page 62, of Vol. II. of my *Syſtem* and you'll ſee her *Horizontal Parallax* is = 54′ 59″, and 57′ 50″, at a Mean Diſtance from the Earth, and 61′ 24″ in Perigeon.

Now for her greateſt Diſtance, ſay,

As S. < HLG *Horiz. Par.*	54′ 59″	.8.2039376
To HG 1 Semidiameter	-	0.0000000
So CS. < HLG	54 59	9.9999444
To HL ⊖ Semidiameters	62.518	1.7960068

62.518 × 3984.58 = 249107.97244 Miles, but from the ⊖ Center 249315.1706.

Secondly, For her Mean Diſtance ☽ from the Earth.

As S. HLG *Horiz. Paral.*	57′50″ Co-Ar.	1.7741252
To 1 Semidiameter ⊖	- - -	0.0000000
So C S. H L G -	57 50	9 9999386
To HL in ⊖ Semidiameter	59.43	1.7740638

59.43 × 3984.58 = 236803.5894 Miles, but from the ⊖ Center 238246.00736 Miles.

Laſtly, For the leaſt Diſtance.

As S. HLG *Horiz. Paral.*	1° 1′ 24″	8.2518576
To HG 1 Semidiameter ⊖	-	0.0000000
So CS. HLG -	1 1 24	9.9999306
To HL in ⊖ Semidiameters	55.985	1.7480730

55.985 × 3984.58 = 223076.7113 from ⊖ Surface. But from the Earth's Center 55.994 = 223112.57252 Miles.

VII. *To calculate the Diameter of the Earth's Shadow at the Diſtance of the Moon.*

IN the Scheme, page 306, let B be the Earth's Center, ☽ the Moon's Center, B C in the Cone of the Earth's Shadow, and at the Earth's greateſt Diſtance = 217.41 Semidiameters, therefore D C the Cone of the Earth's Shadow beyond the Moon, is found in the three ſeveral Stations thus,

BC = Cone ⊖ Shad.	217.41 and	213.775 and	210.014
Diſt. ☽ à ⊖ = B ☽	62.518	59.784	55.985
Shad. beyond ☽ =	154.892 =	153.991 =	154.029

As Radius	-	90° 0' 0''	10.0000000
To ☽ C =	-	154.892	2.1900236
So T. ☽ C D	-	15' 49''	7.6627105
To D ☽ = Semidiam. Shad. ⊖		.71241	9.8527441

2 D ☽ = Diameter Shadow 1.42482 × 3984.58 = 5669.4198, which is the Diameter of the Earth's Shadow at the Moon, in Apogeon of the Sun; but when the Sun is at a Mean Diſtance from the Earth, it is 5740.4250228 the Sun in Perigeon, the Diameter ⊖ Shadow at the Moon is 5840.995822.

VIII. *To calculate the Diameter of the Moon, and Height of her Shadow.*

IN the Scheme, page 306, Let B be the Earth's Center, ☽ the Moon's Center, B ☽ the Moon's Diſtance from the Earth, and the Angle D B ☽, the Moon's Apparent Semidiameter, to find D ☽.

1. *The Moon in Apogeon.*

As C S. D B ☽	0° 14' 52"	Co-Ar.	0.0000039
To B ☽	62.518	-	1.7960068
So S. D B ☽	0 14 52		7.6358290
To D ☽ .27029 =	1076.9921282		9.4318316

Hence becauſe the Cone of the Earth's and Moon's Shadow are Similar Bodies, their Heigths will be in proportion to the Semidiamer of their Baſes Baſe: For having the Baſe and Height of the Earth's Cone, and the Baſe of the Moon's, the Height of the Cone of the Moon's Shadow will be obtain'd thus,

As Semidiameter ⊖	.1	-	0.0000000
To Height of it's Shadow	217.41		2.3372895
So Semidiam. Baſe ☽ Shadow	.27029		9.4318316
To Height of it's Shadow	58.766		1.7691211

And after this Manner will the Height of the Moon's Shadow at a Mean Diſtance from the Earth be found to be 58.279 Semidiameter of the Earth. And when ſhe is in Perigeon or neareſt to the Earth, the Height of the Cone of her Shadow is 57.013 Semidiameter of the Earth, which multiply'd by 3984.58 = 2271.7285954 Miles

How to Calculate the Times and Quantities of Eclipſes, particular and general. See my Compleat Syſtem of Aſtronomy, *and my* Uranoſcopia, *where the Reader will find Satisfaction to his Deſire.*

The Uſe that may be made of Eclipſes is for diſcovering the Longitude of Places at Sea or Land, for having an Eclipſe truly calculated to any particular *Meridian*, as ſuppoſe *London*, the Eclipſe is found by Calculation to begin at 5 h. 39' 52'' P M, and being at Sea, the ſame Eclipſe is obſerved to begin at 6 h. 15' PM. the Difference is 35' 8'', which turn'd into Time = 8° 47' 00'' to the Eaſt of the *Meridian* of *London*. But if you obſerve the Time ſooner than it is by the Calculation, you are then ſo much to the Weſt of *London*.

VI. *Of Mars.*

M*ARS* is a Superiour Planet, and moves in an Orbit round the Sun, between the Earth and *Jupiter*. And make one Revolution in 321 d. 23 h. 27' 30''. His Mean Annual Motion is 6 S. 11° 17' 10'', Diurnal Motion is 31' 27'', and in an Hour he moves 1' 19''. His Eccentricity 14169.5 and his Mean Diſtance from the Earth 112022636 *Engliſh* Miles. There are two Years and fifty Days between every Conjunction with the Sun. The Inclination of his Orbit 1° 51'; *Anno* 1739, the Place of his Aſcending Node 1 S. 17° 49' 24'', and the Place of his Aphelion ♍ 1° 18' 51'', it's Annual Motion 1' 12''. This Planet has an Atmoſphere like our Earth: He is Direct, Stationary, and Retrograde to us at the Earth, and has a Rotation about his own Axis from Weſt to Eaſt in 24 h. 40'. He has no Satellite or Moon to attend him, but has Faſciæ or Spots, as appears thro' the Teleſcope.

VII. *Of the Planet Jupiter.*

JUPITER is a Primary and Superiour Planet, moving round the Sun in an Orb between *Mars* and *Saturn*; and appears thro' the Teleſcope to have always a round full Face; he is liable to be eclipſed to an Inhabitant of the Earth by all the Planets except *Saturn*: Receives all his Light from the Sun, and by the Teleſcope he is known to have Belts which ſurround his Body, with ſeveral Spots by which is diſcovered his Rotation upon his Axis, from Weſt to Eaſt in 9 h. 56': He makes one Revolution round the Sun in 11 y. 317 d. 12 h. 20' 25",

His Mean Annual Motion is	1S.	0°	20'	38"
Diurnal Motion - - - -			4	59
Hourly Motion - - - -				12
Place of his Aſcending Node this Year 1739 - -	♋	8	6	40
Aphelion - - - -	♎	10	20	36
Eccentricity - - - -				25074.5

Mean Diſtance from the Earth 337681693 *Engliſh* Miles. This Planet as well as *Mars*, and *Saturn* is much nearer the Earth in Oppoſition to the Sun, than in any other Poſition. *Jupiter* is attended with four Satellites, and were firſt diſcovered by *Gabilæus*, *Anno Domin.* 1610. Theſe Moons move round *Jupiter* in a conſtant direct Order, as here is ſet down.

Periadocal Times.					*Diurnal Motiotion*				
	d.	h.	'	"	S.	°	'	"	Magnitudes.
1. in	1	18	28	36	6	23	29	20	2
2. in	3	13	17	54	3	11	22	29	3
3. in	7	3	59	36	1	20	19	3	1
4. in	16	8	5	12	0	21	34	16	4

Their Diſtances from Jupiter *in Semidiameter of* Jupiter *are,*

1.	2.	3.	4.
5.91	9.4	15.	26.38.

By theſe Magnitudes and Diſtances from *Jupiter,* they may be eaſily diſtinguiſhed one from another, when you look at them through a Refracting Teleſcope, they will appear on the contrary Side of *Jupiter,* but a Reflecting Teleſcope will ſhew them really as they are in the Heavens. The Apſis of the Moons this Year 1739, is ♓ 21° 12', with a Yearly Motion of 36'. They always appear amongſt the Fixed Stars to lie nearly in a ſtrait Line, except their Latitudes cauſe a little Variation, (as I have often obſerved). And the Poſition of this Line has Reſpect unto the Altitude of the Nonageſſime Degree in *Jupiter*'s Orb, as being at one Time Parallel to the *Horizon,* at other Times more oblique, making Angles therewith of different Quantities. The Fourth, or outmoſt Satellite paſſes wide of the Shadow two Years in every ſix, as it did in the Year 1734, and will again in 1740, *&c.* but the other three paſs through *Jupiter*'s Shadow once in every Revolution, and are then eclipſed; and they alſo eclipſe one another. Beſides the Immerſions, they become twice inviſible to us in every Revolution except when their Latitude is too great, that is, once when they are between the Eye and *Jupiter,* and again when they are behind his Body. When *Jupiter* is our Morning Star, and Riſing before the Sun, the Spectator ſees only the Immerſions, or the Entrance of the Satellite into *Jupiter*'s Shadow; but when he is our Evening Star and Setting after the Sun, we then ſee the Emerſions out of the Shadow. Four and thirty Days every Year *Jupiter* lies hid under the Sun Beams, and conſequently can't be ſeen with the naked Eye. By obſerving the Eclipſes of *Jupiter*'s Moons from oppoſite Parts of the *Magnus Orbis,* it is found that Light is not inſtantaneous, but takes up 9 or 10' Minutes Time to travel from the Sun to us, which diſtance is more than 82 Millions of Miles: Which Space the Light paſſes thro', in ſo ſmall a Time that ſo prodigious a Velocity cannot eaſily be conceived by us, which ſo much exceeds the Velocity of the ſwifteſt Bodies we know.

By the Eclipſes of theſe Satellites, the Longitude of Places upon the Terraqueous Globe may be known after the ſame Manner as has been ſhewn in the Eclipſes of the Sun and Moon.

VIII. *Of Saturn.*

SATURN is a Primary and Superior Planet, and is the higheſt in our Syſtem; he is always obſerved with a round full Face, is of a heavy dull Lead Colour, and is liable to be eclipſed from our Sight, by all the other Planets, if their Latitudes correſpond; he makes one Revolution round the Sun in 29 y. 174 d. 6 h. 36′ 26″; his mean annual Motion is 12° 13′ 21″, his Diurnal Motion 2′ 1″ and hourly Motion 5″. Every Year and thirteen Days is conjoin'd with the Sun; he is direct Stationary and Retrograde to us at the Earth once every Year, and makes all his Aſpects with the Sun. It is uncertain whether this Planet moves round his Axis or not, by Reaſon of his great Diſtance from us, which is 575426579 *Engliſh* Miles. His Eccentricity 54376, ſuch Parts as the Mean Diſtance of the Earth from the Sun is 1000000. The Place of his Aphelion this Year 1739, is in ♐ 29° 25′ 20″, and the Place of his Aſcending Node ♋ 21° 16′ 48″, the annual Motion of the Aphelion 1′ 20″, and of the Node 18″, the Inclination of his Orbit 2° 30′ 10″, and by the Teleſcope's Obſervations is found to have five Satellites or Moons moving round him, as he moves round the Sun, and according to the lateſt and beſt Obſervations, do make their Revolution in Times, as is here ſet down.

	d.	h.	'	"	Diſtance.	
1. in	1	21	18	26	4.893	Semidiameters of of *Saturn*'s Globe.
2. in	2	17	41	10	6.268	
3. in	4	12	25	10	2.754	
4. in	15	22	41	28	20.295	
5. in	79	7	46	00	59 154	

Beſides theſe five Moons this Planet is known to be encompaſſed on every Side with a Ring, but doth no where touch his Body, the Diameter of *Saturn* to that of the Ring is as 4 to 9. *Caſſini* ſuppoſes the four interior Satellites to be moved according to the Plane of the Ring, or their Orbits to be inclined to the Orbit of *Saturn* by an Angle of 30°, but *Hugens* of 31°. When *Saturn* is in the Middle of the Sign *Gemini* and *Sagittari*, then the greateſt Axis of his broad ſpreading Ring, is found to be preciſely double to the leſſer, and lieth moſt open to our View, as it was in the Year 1737, and will be ſo again in 1751; but on the contrary, when this Planeti in *Virgo* or *Piſces*, the Ring ſeems to us to be quite ſhut, and only as it were a right Line, and no more to be ſeen and this will be ſeen in the Year 1744, and again in 1758, and 1759. But this is ſpoken to *Aſtronomers* who are furniſhed with long Glaſſes wherewith to make Obſervations of this Kind. Thoſe that would ſee the Reaſon of the *Retrogradations* of the Planets, let them conſult my Planetary Inſtruments. And here in the Satellites as well as in the Planets themſelves, this Univerſal Law of Nature alſo holds good, *viz.* that the Squares of the Periodic Times are as the Cubes of their Diſtances from the Centers of their Primary.

IX. *The Characters and Poetical Names of the Planets.*

♄ *Saturn, Chronus, Phænon, Falciſer.*
♃ *Jupiter, Phaëton, Zeus, Jove.*
♂ *Mars, Aris, Pyrois, Mavors, Gradivus.*

☉ *Sol, Titon, Itios, Phœbus, Apollo, Phaon, Oſyris Deiſpiter.*
♀ *Venus, Cytheria, Aphrodite, Erycina.*
☿ *Mercury, Hermes, Stilbone, Cyllenius, Archus.*
☽ *Moon, Lucina, Cynthia, Diana, Phœbe, Proſepina, Noctiluca, Latona.*

The Reaſon of which Names may be ſeen in Myſtagogus Poeticus.

⊖ *Terra,* the Earth,
☊ *Dragon's Head.*
☋ *Dragon's Tail.*

X. *The Characters and Names of the Twelve Signs, with the Numbers of Stars they contain.*

♈	*Aries*	21.
♉	*Taurus*	43.
♊	*Gemini*	30.
♋	*Cancer*	15.
♌	*Leo*	40.
♍	*Virgo*	39.
♎	*Libra*	18.
♏	*Scorpio*	10.
♐	*Saggitarius*	14.
♑	*Capricorn*	28.
♒	*Aquarius*	41.
♓	*Piſces*	36.

XI. *The Characters and Names of the Five Aſpects.*

☌ *Conjunction,* is the ſame Sign and Degree.
⚹ *Sextile,* two Signs, or 60 Degrees aſſunder.
□ *Square,* three Signs, or 90 Degrees aſſunder.

△ *Trine*, four Signs, or 120 Degrees aſſunder.
☍ *Oppoſition*, ſix Signs, or 180 Degrees aſſunder.

XII. *Of the Fixed Stars.*

FIXED Stars are ſuch as do not, like the Planets or Erratic Stars, change their Poſitions of Diſtances in Reſpect of one another. And becauſe they have no ſenſible Parallax ariſing from the annual Motion of the Earth, they are juſtly eſteemed to be of ſuch an immenſe Diſtance, that the Earth's annual Orbit is but as a Point in compariſon of it.

Light takes up more Time in Travelling from the Stars to us, than we in making a *Weſt India Voyage*. That Sound would not arrive to us from thence in 50000 Years, nor a Cannon Ball in a much longer Time, this is eaſily computed by allowing 10′ for the Journey of Light from the Sun hither, and that it moves 968 Feet in a Second of Time.

Where-ever the Spectator is placed in our Syſtem, whether in the Sun, in the Earth, or even in *Saturn*, will ſee the ſame Face of the Heavens, and the ſame Stars, with the ſame Magnitudes, and the ſame Figure of the Conſtellations, and the Heavens which ſurrounds and involves them all, will have the ſame Face.

The Antients divided the Viſible Firmament into 48 Images, 12 of which filled the *Zodiack*, the Names of which we have given in page 318. In the Northern Region there are 21 Images, *viz.* the *Leſſer Bear*, the *Great Bear*, the *Dragon*, *Cephus*, *Bootes*, the *Northern Crown*, *Hercules*, the *Harp*, the *Swan*, *Caſſiopia*, *Perſeus*, *Andromeda*, the *Triangle*, *Ariga*, *Pegaſus*, *Equilus*, the *Dolphin*, the *Arrow*, the *Eagle*, *Serpentarius*, and the *Serpent*. Afterwards they added to them two others, *viz.* that of *Antinous*, which was made of the Stars that are not included within any Image, and are near the *Eagle*; and the Conſtellation called *Berenice*'s *Hair*, theſe Stars lie near the *Lyon*'s *Tail*.

On the South Side there are 15 *Aſtreriſms*, which were known to the Antients, *viz.* the *Whale*, the *River Eridinus*, the *Hair*, *Orion*, the *Great Dog*, the *Leſſer Dog*, the *Ship Argo*, *Hydra*, the *Cup*, the *Crow*, the *Centaur*, the *Wolf*, the *Altar*, the *Southern Crown*, and the *Southern Fiſh*. To theſe are added 12 more Conſtellations, which are not ſeen by us who inhabit the Northern Regions, becauſe of the Convexity of our Earth, but in the Southern Parts they are very conſpicuous. Theſe are the *Phœnix*, the *Crane*, the *Peacock*, the *Indian*, the Bird of *Paradiſe*, the *Southern Triangle*, the *Fly*, the *Chameleon*, the *Flying Fiſh*, the *Toucan*, or *American Gooſe*, *Hydrus*, or *Water Serpent*, *Xiphas*, or the *Sword Fiſh*.

Aſtronomers have added two in the Northern Hemiſphere, *viz.* *Charles*'s *Heart*, and *Sobieski*'s *Shield*.

The *Galaxy*, *Via Lectea*; Or *Milkey Way*, is a broad white Tract encompaſſing the whole Heavens, and extending itſelf in the Sign of *Capricorn* from the Equinoctial to the Tropic of *Cancer* with a double Path, and the reſt of it is a ſingle one. Some of the Antients, as *Ariſtole*, imagined that this Path conſiſted only of a certain Exhalation hanging in the Air, but by the Teleſcope's Obſervations of this Age it has been diſcovered to conſiſt of an innumerable Quantity of Fixed Stars, different in Situation and Magnitude; from the confuſed Mixture of whoſe Light it's white Colour is ſuppoſed to be occaſioned. (With the Teleſcope I have counted 23 Stars in the Compaſs of the Pleiades.) It's greateſt Declination North is about 65°, and South 69°, it croſſes the Equinoctial from South to North in ♑ 5°, and from North to South in ♋ 5°. It's breadth, where broadeſt, which is about *Aquila*, is 25°, in other Places it doth not exceed 10 Degrees.

Aſtronomers have divided the Stars into ſix ſeveral Sizes or Magnitudes, of which the greateſt or brighteſt of them are called Stars of the firſt Magnitude, as *Capella*, *Acturus*, *Regulus*, *Sirius*, &c and the next to them in brightneſs are called the Stars of the ſecond Magnitude, next to them in brightneſs are Stars of the third Magnitude and ſo forth, 'till we come to Stars of the ſixth Magnitude, which comprehend the ſmalleſt Stars that can be diſcern'd with the naked Eye.

The

The Number of Fixed Stars that their Places have been Rectified by ſeveral Aſtronomers are theſe.

Hypparchus, 140 before Chriſt had		1022
Pliny		1600
Ptolomy 135 Years after Chriſt		1026
1437 *Uligh Beighi*		1017
1460 *Regiomontanus.*	Ditto	-
1536 *Nich. Copernicus*	Ditto	
1572 *Tycho Brahe*		777
1620 *John Kepler*		1163
1603 Dr *John Bayerus*		1725
Prince of *Heſs*		400
1635 *Brachius*		1672
1651 *John Ricciolus*		1468
1670 *John Hevelius*		1888
1676 Dr *Edmund Halley*		373
1690 *John Flamſtead*		3000
1726 This Author		777

The Fixed Stars move upon the Poles of the Ecliptic, by which they have a progreſſive Motion of 50'' a Year, that is one Degree in 72 Years, and one Revolution in 25920 Years; by which Motion they are always kept parallel to the Ecliptic, and ſo never alter their Latitudes; but Dr *Halley* tells us the *Philoſophical Tranſactions*, Numb 355, that four of the Fixed Stars have altered their Latitudes ſince *Ptolomy*'s Time, *viz.* the *Bull*'s *Eye* 20' more South, *Sinius* 22, *Arcturus* 33', and the Bright Shoulder of *Orion*, almoſt a Degree more North.

But theſe may be look'd upon as bad Obſervations made in *Ptolomy*'s Time, for want of good Inſtruments, rather than attributing the Difference of the Latitude to any unequality in the Stars Motions.

The Fixed Stars are known from the Planets by their Scintillation, or Sparkling; for the Planets have no ſuch Vibration, twinkling or glimmering of Light: But generally all the Fixed Stars more or leſs, and at ſome Times more than others: The Cauſe of their Scintillation is variouſly diſcourſed of, both by Philoſophers and Aſtronomers. *Ariſtotle*, among the Antients, aſſigns the

Cauſe thereof to their remoteneſs from our Sight, by which they are weakly, and as it were by a trembling wearineſs reached. Some again will have the Cauſe of this Twinkling to proceed from Refraction; others aſſign the Cauſe to ariſe from the unequal Superficies of the Fluctuating Air or Medium, as Stones in the Bottom of a River by the Rapid Motion of the Water, ſeem to have a kind of termulus Motion, which is only in the criſped and uneven Undulation of the Stream. *Gaſſendus* more probably conceives this *Scintillation* of the Fixed Stars to proceed from that Native and Primigenial Light they are indued with, like that of the Sun's ſparkling, caſting forth ſuch quick darted Rays as our weaker Light cannot behold without that trembling Paſſion.

XIII. *Of Comets,*

COMETS are what are commonly called Blazing Stars. The Antients, eſpecially *Ariſtotle*, and his Followers ſuppoſed them to be Meteors or *Exhalations*, ſet on Fire in the Middle Region of the Air. The Modern Aſtronomers have found that they are within our Planetary Syſtem, and that their Orbits are very Eccentric, being ſometimes at a moderate Diſtance from the Sun, at other Times they aſcend to vaſt Diſtances, far beyond the Orbit of *Saturn*, and ſo become inviſible. The great Comet that appeared here in 1680, 1681, was ſeen before in our Hemiſphere, *Anno Domin.* 1106, once before, about the Year 532, and alſo 44 Years before our Saviour's Birth: And therefore Aſtronomers conclude the Time of it's Periodical Revolution to be 575 Years. The Time of the Revolution of another Comet which appeared in the Year 1682, will appear to us again in 1758, is 75½ Years in making one Revolution, another probably may be ſeen here again *An. Dom.* 1789, makes it's Ellipſis round the Sun in 129 Years. And that theſe Periods are all that are known to Aſtronomers, is what we are to be ſatisfied with 'till further Diſcoveries are made.

I might here give the Projection of the Sphere, and apply Spherical Trigonometry in the Solution of all the Practical Problems of the Sun, Moon, and Stars, but this being already handled in my *Compleat Syſtem of Aſtronomy*, I ſhall forbear it here, and proceed to the Art of Dialling.

CHAP IX.

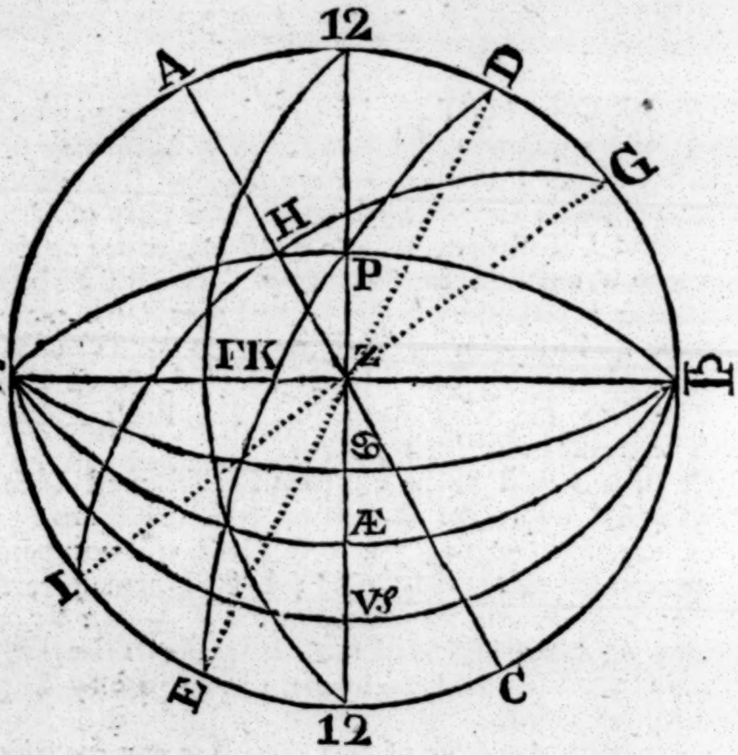
Of DIALLING.

HOROLOGY, or Dialling is a very curious Art; and requires a full Knowledge in Aſtronomy to be a perfect Maſter; in my *Mechanic Dialling* I have ſhewn how to make all Sorts of Dials without the Help of Trigonometrical Calculation, which Work is intended only for him who is unſkill'd in Aſtronomy: Here it ſhall be my Buſineſs to explain the calculative Part even to the meaneſt Capacity. Firſt then of Dials there are divers Sorts, as 1. The Horizontal. 2. The North and South Erect. 3. The Eaſt and Weſt Erect. 4. The Erect Decliner. 5. The Recliner, or Incliner, &c.

He that would be a compleat Dialliſt muſt be fully acquainted with the Projection of the Sphere on any Circle, but becauſe my narrow Limits will not permit me to treat ſo largely as I would; I ſhall here give my Reader a View of the Fundamental Diagram, for expreſſing all Sorts of Planes, and how the Triangles lie in which the Hour diſtances upon the Plane, are by Trigonometry calculated.

Firſt, With a Sweep of 60° opened to any convenient Radius, draw the Primitive Circle ♈ 12 ♎ 12, which ſhall repreſent the Horizon of the Place, and Z the Zenith. 2. Quarter it, and draw ♈ Z ♎ for the Prime Vertical, and 12 Z 12 for the Solſtitial Colure. 3. Take the Tangent of 19° 14′, it being ½ the Co-Latitude of *London*, and ſet it from Z to P, ſo ſhall P repreſent the North Pole, thro' the three Points ♈ P ♎ (or with the Secant of 51° 32′) draw ♈ P ♎ for the Equinoctial Colure, or Six o'-

Clock Hour. 4. With the Secant of 61° 57′, the Sun's greateſt Meridian Altitude at *London* draw ♈ ♋ ♎. 5. With the Secant of 38° 28′, being the Height of the Equinoctial at *London*, draw ♈ Æ ♎. 6. With the Secant of 14° 59′ the Sun's leaſt Meridian Altitude at *London*, draw ♈ ♑ ♎: A Z C, 12 F 12, D P E, and G H I are Planes of different Poſitions drawn at Pleaſure: Thus is the Scheme finiſhed ſufficient for our preſent Purpoſe. *See p.* 250.



Then doth the Primitive Circle repreſent a Horizontal Plane. 2. A Vertical Plane, parallel to the Prime Vertical Circle paſſing through the Zenith and Eaſt and Weſt Points of the Horizon, is the Erect, Direct North and South Plane, and is repreſented by ♈ Z ♎ the Prime Vertical Circle. 3. A Meridian Plane, parallel to the Meridian, and paſſing through the Zenith and Pole, ſuch are the Eaſt and

Weſt direct Planes, and repreſented by 12 P Z 12, the Solſticial Colure. 4. A Polar Plane, parallel to the Earth's Axis, or Hour of Six is here repreſented by ♈ P ♎, it paſſes through the Poles, and through the Eaſt and Weſt Points, cuts the Equinoctial and Meridian at right Angles, but inclines to the Horizon with an Angle = 12 ♈ P = 51° 32′, reclining from the Zenith 38 28 = < P ♈ Z. This is called a direct Incliner. 5. The Equinoctial Plane, is parallel to the Equinoctial paſſing through the Eaſt and Weſt Points being right to the Meridian, but Inclining to the South Horizon (in North Latitudes) with an < = 38° 28′ the Co-Latitude = Æ ♈ 12 repreſented by ♈ Æ ♎ the Equinoctial, and conſequently and properly is called a Direct Incliner, becauſe the upper Face looks to the North, and the under to the South. 6. A Direct inclining Plane, parallel to any great Circle which paſſes through the Eaſt and Weſt Points of the Horizon, being right to the Meridian, but not paſſing thro' the Poles of the World, nor parallel to the Equator, is repreſented by ♈ ♋ ♎, or by ♈ ♑ ♎ being the two Poſitions of the Ecliptic, when the very Beginning of *Cancer* and *Capricorn* are upon the Meridian. 7. A Vertical declining Plane, parallel to any great Circle which paſſeth through the Zenith, being right to the Horizon, but inclining to the Meridian with an Angle of 30 Degrees. 8. A Polar declining Plane, parallel to ſome great Circle which paſſeth through the Pole, being right angled with the Equinoctial, but inclining to the Meridian with an Angle of 24° = 12 P D, and this Plane is repreſented by D P E. 9. A Meridian Plane inclining to the Horizon, parallel to any great Circle which paſſeth through the Points of North and South, and cuts the Prime Vertical at right Angles, this Plane is repreſented by 12 F 12. 10. A Declining inclining Plane, parallel to any great Circle, which paſſeth neither through the North, South, Eaſt, or Weſt, nor through the Pole, nor Zenith, but lieth oblique to them all, it is repreſented by G H I making an Angle with the Horizon of 45°, and Declining from the Meridian 51½° = 12 G. Each of theſe Planes except the Horizontal have two Faces, whereon Hour Lines may be drawn.

I. To make an Horizontal Dial.

THIS Plane is repreſented in the Fundamental Diagram by the Primitive Circle A ♈ C ♎, which is beſt made of Braſs to ſtand the Weather. Now if we project the Sphere Stereographically upon the Plane of the Horizon for the given Latitude, and where the projected Meridians or Hour Lines cut the Plane of the Horizon, draw Lines from the Center or Zenith to the Border of the Dial, and they ſhall be the true Hour Lines, and the Stile or Gnomon muſt ſtand upon the Meridian, or 12 o'Clock Hour Line, and to make an Angle at the Center of the Hour Lines, equal to the Latitude of the Place for which the Dial is made.

But to do this by Trigonometry, let A B C D repreſent a Horizontal Plane, on which I am to draw a Dial; upon which draw as many Concentric Circles for the Boundaries of the Dial as you fancy will ſerve your Turn, and draw the Diameter A B for the Meridian, or 12 o'Clock Hour Line: Take the Radius of the Circle in your Compaſs and ſet the Sector on the Line of Tangent, and take off the Tangent 19° 14' that being half the Complement of the given Latitude, and ſet it from Z the Zenith, to P the North Pole. Through the Pole P, at right Angles to the Meridian A B draw the Line 6 P 6 for the Six o'Clock Hour Line. Now for the other Hour Lines, their Diſtance from the Twelve o'Clock or Meridian, we muſt have recourſe to the Fundamental Diagram in page 324, then ſuppoſe the oblique Circle D P E to repreſent the One or 11 o'Clock Hour Circle, then the Angle 12 P D = 15°, in which Triangle right Angle at 12, there are given P 12 = 51° 32' the Latitude of the Place = Elevation of the Pole above the Horizon, with the Angle 12 P D = 15, to find the Side 12 D the diſtance of One and Eleven o'Clock from the Meridian upon the Horizon.

Equation. $S P 12 + R = C T < P + T 12 D.$

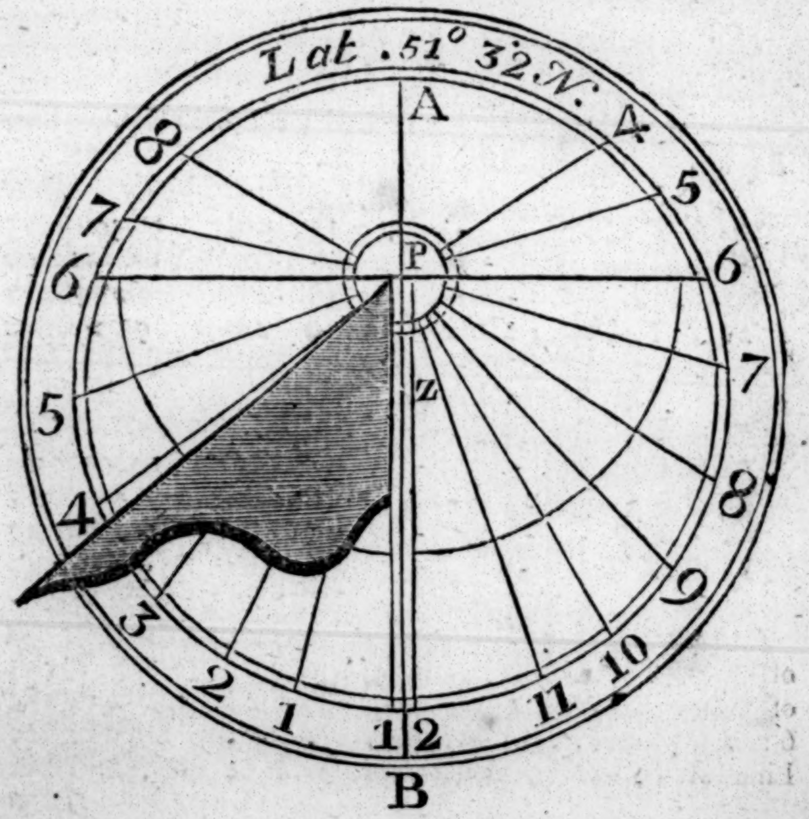
Analogy.

As C T. < P	- -	15°	0'	10.5719475
To Radius	- -	90	0	10.0000000
So S. P 12 = Pole's Elevation		51	32	9.8937452
To *t* 12 D, 1 Hour on the Plane		11	51	9.3217977

Or by Tranſpoſition, ſee p. 245.

As Radius	- -	90°	0'	10.0000000
To T < P		15	00	9.4280525
So S. P 12 = Latitude		51	32	9.8937452
To T. 12 D	-	11	51	9.3217977

And thus by making the Angle at the Pole 30, 45, 60, 75°, and working as above you will have the diſtance of every Hour upon the Plane, from the Meridian Line AB. Then by allowing for the thickneſs of the Stile, draw two Quadrants on each Side the Twelve o'Clock Hour Line, and ſet the Sector to that Radius, and take the Chords of the ſeveral Hour Diſtances, and ſet them off in each Quadrant, and thoſe Marks draw Lines from the Center at P to the Margin of the Dial, and they ſhall be the true Hour Lines required. The Stile or Gnomon muſt Make an Angle at P = the Latitude of the Place for which the Dial is made, which being ſet at right Angles over the Twelve o'Clock Hour Line, the Dial is finiſhed.



II. *Of an Erect Direct North and South Dial.*

THE Hour Lines of theſe Dials are calculated by the Triangle KPZ, right angled at Z. In the Scheme, page 324, there are given, the Angle at P 15, 30, 45, 60, 75° the Equinoctial Diſtances, and ZP the Diſtance between the Zenith and Pole, equal to the Complement of the Latitude of the Place, to find KZ the Hour Diſtances upon the Plane.

Equation.

$$S.\,PZ + R = T\,KZ + CT < P.$$

Analogy.

For the Hour Lines of 11 and 1 o'Clock, for the Lat. *London.*

As CT < P	-	15° 00'	10.5719475
To Radius	-	90 00	10.0000000
So S PZ	- -	38 28	9.7938317
To T. KZ = 1 Hour		9 28	9.2218842

Or by Tranſpoſition.

As Radius	- -	90° 00'	10.0000000
To C S. Latitude	-	51 32	9.7938317
So T. < P	- -	15 00	9.4280525
To T. KZ	- -	9 28	9.2218842

And ſo by the ſame Analogy I find the Horizontal Diſtances of 10 and 2 o'Clock to be 19° 45', of 9 and 3 = 31° 54', of 8 and 4 = 47° 9', and of 7 of 5 = 66° 42', and 6 and 6 = 90°. Theſe being laid off by help of the Line of Chords on the Sector, will be the true Hour

Lines on a South Erect Direct Plane for the Latitude of 51° 32' North. And for the North Dial you may either continue the Hour Lines of 4 and 5 thro' the Center to ſerve for 7 and 8, and 7 and 8 ſhall be 4 and 5 on the North Plane, or elſe lay them off by help of the Chords, as above has been ſhewn. The Gnomon or Stile muſt hang at right Angles over the 12 o'Clock Hour Line, and becauſe the top Edge of all Dial Clocks repreſents the Earth's Axis, and conſequently muſt lye parallel to it; and in theſe Dials it muſt make an Angle with the Plane equal to the Complement of the Latitude of the Place: And in the South Dial it muſt point downward to the South Pole, and the Cock of the North Dial muſt point upwards to the North Pole. In all the Caſes of Dialling we ſuppoſe the Earth no more than a Point, in reſpect of the great Diſtance that it is from the Sun, for if it were reckoned otherways, the Stile of the Dial could not be parallel to the Earth's Axis, and at the ſame time point to the true Poles of the World.

When the Sun has North Declination, (which he has from the Ninth of *March* to the Twelfth of *September*), he comes upon the South Erect direct Plane in the Morning at the ſame time that he is due Eaſt, or upon the Prime Vertical, and goes off in the Afternoon when he is due Weſt.

III. *Of an Erect Declining Plane.*

THE firſt Thing to be done, is carefully to take the Plane's Declination, which is no more than an Arch of the Horizon comprehended between the Axis of the Plane and the Meridian of the Place; and that you may the better diſtinguiſh Planes, you are to conſider and underſtand three ſeveral Lines. The Firſt is the Horizontal Line; the Second, the Line perpendicular to the Plane, commonly called the Axis of the Plane. Laſtly, The Meridian Line. There are four ſeveral Ways to find the Declination of a Plane, as (1) by the Magnetic Needle,

which

which is of all the moſt uncertain by Reaſon of the Variation. (2) By two Obſervations, the one made in the Forenoon, the other in the Afternoon. *See my Mechanic Dialling.* (3) By applying a Dial made for the ſame Latitude to a Circle divided into 360° upon a Board placed Horizontally to the Plane, move the Dial to the true Hour of the Day and then the Twelve o'Clock Line on the Dial will point to the Degree of the Plane's Declination on the divided Circle. Laſtly, By the Sun's Azimuth and Horizontal Diſtance, which is thus, with a good *Aſtronomical Quadrant*, (ſuch as is now made by *John Barſton* and *Joſ. Turner* in *Hatton Garden, London)* take the Sun's Altitude, then in the Triangle A Z P, page 251, we have given A Z the Co-Altitude, Z P the Co-Latitude, and A P the Sun's Diſtance from the North Pole, or Co-Declination, to find the Angle A Z P, work as is taught in page 259, and you will have the Sun's Azimuth from the North, whoſe Complement to 180 is the Angle Æ Z A the Sun's Azimuth from the South.

At the ſame Moment of Time that you take the Sun's Altitude, let an Aſſiſtant fix a Semicircle divided into an 180 Degrees to the Horizontal Line of the Plane, ſo that the Limb, or graduated Edge be towards the Sun, hold up a Thread or Plummet near the Limb, ſo that the Shadow thereof may paſs through the Center of the Semicircle, and obſerve the Degrees cut by the Shadow of the Thread, and Number them from that Side of the Semicircle which is perpendicular to the Plane, for thoſe Degrees are the Horizontal Diſtance ſought. This being had with the Sun's Azimuth from the South, obſerve theſe two Rules

RULE I.

If the Shadow of the Thread fall between the South and that Side of the Quadrant, which is perpendicular to the Plane, then add the Sun's Azimuth from the South to the Horizontal Diſtance, and that Sum is the Declination; and it is to the ſame Coaſt with the Sun's Azimuth, that is, Eaſt, if the Obſervation be made in the Forenoon, but Weſtward in the Afternoon.

II. If the Shadow of the Thread fall not between the South and that Side of the Quadrant that is perpendicular to the Plane, then the Difference between the Sun's Azimuth from the South, and the Horizontal Diſtance is the Declination of the Plane; and if the Azimuth be the greater of the Two, then the Plane declines to the ſame Coaſt whereon the Sun is; but if the Horizontal Diſtance be the greater, then the Declination is to the contrary Coaſt.

EXAMPLE.

Anno 1725, *March* 17, at *London*, in the Afternoon I obſerved the Sun's Azimuth from the South 38° 52', and the Horizontal Diſtance fell not between the South and the Axis of the Plane, conſequently their difference 29° 8' is the Plane's Declination to the Eaſt. Theſe Things being found, there are four Things to be conſidered.

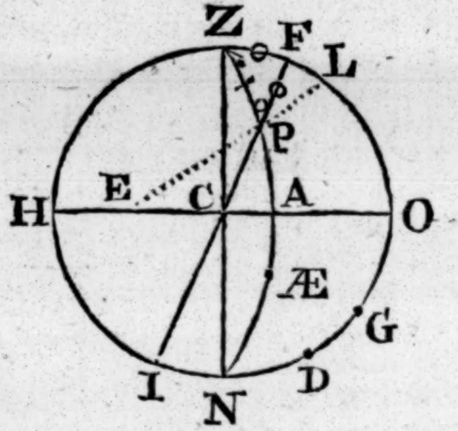
1. The Inclination of the Meridian of the Plane, with the Meridian of the Place.
2. The Height of the Pole or Stile above the Plane.
3. The Diſtance of the Subtile from the Meridian
4 The Diſtance of the Hour Lines from the Subtile.

All theſe I ſhall demonſtrate in the following Diagram.

Geometrical Conſtruction.

With the Chord of 60 draw the Primitive Circle which ſhall repreſent the Plane of the Dial, draw Z N and H O at right Angles, take the Chord of 29° 8' the Plane's Declination, and ſet it from N to D, lay a Ruler from Z to D and it will cut the Horizon in A, through which the Meridian of the Place muſt paſs; or ſet the Semi-tangent of the Declination from C will give A as before. Take the Co-Secant of the Plane's Declination = 60° 52' and draw the Meridian Z A N, it's Pole will be found at E; take the Latitude of the given Place 51° 32' from the Chord and ſet it from O to L, and from N to G, a Ruler laid from E to L cuts the Meridian in P the Pole of the World, and a Ruler laid from E to G cuts the Meridian in Æ, that is, where the Equinoctial cuts it. Laſtly, Lay a Ruler from P to C, and draw F I for the Subſtiler Line; now have we the Right angled Spherical Triangle Z F P, in which

Z F is the Subſtile Diſtance from the Meridian.
Z P is the Complement of the Latitude of *London.*
F P is the Height of the Pole above the Plane.



And the Angle { P F Z is right.
P Z F is the Complement Plane Declination.
F P Z is the Plane's diff. Longitude.

I. For the Angle F P Z the Inclination of Meridian's, or Plane's difference of Longitude.

In the Triangle ZFP, there are given PZF = 60° 52', the Complement of the Plane's Declination, and P Z the Complement of the Latitude of the Place = 38° 28' to find the Angle Z P F, the Inclination of Meridians.

Equation.

$$CS.\ PZ + R = CT < Z + CT < P.$$

Analogy.

As C T < Z Co-Declin.	60°	52'	9.7461320
To Radius -	90	00	10.0000000
So C S, P Z Co-Latitude	38	28	9.8937452
To CT < P Inclinat. Merid	35	27	10.1476132

Or by Tranſpoſition.

As Radius, to T. < Z: So C S. P Z, to C T. < P.

II. In the ſame Triangle there are the ſame Things given, to find Z F the Subſtile's Diſtance from the Meridian.

Equation.

$$CS < Z + R = CT.PZ + T.ZF.$$

Analogy.

As C T. P Z. Co-Lat.	38°	28′	10.0999135
To Radius . -	90	00	10.0000000
So C S. < Z. Co-Declinat.	60	52	9.6873895
To T. Z F. the Subſtile	21	9	9.5874760

Or by Tranſpoſition.

As Radius to T. P Z, So C S < Z to T. Z F.

III. There are the ſame Things given to find P F, the Height of the Pole Artic above the Plane equal to the Stile's Height.

Equation.

$$S.PF + R = S.PZ. + SZ.$$

Analogy.

As Radius - -	90°	00′	10.0000000
To S, PZ. Co-Latitude	38	28	9.7938317
So S < Z Co-Declinat.	60	52	9.9412575
To S. P F. Stile's Height	32	55	9.7350892

Theſe three Requiſites being found, the next Thing to be done is to make the Firſt and Second Columns of the following Table, which are done thus, the Plane's Difference of Longitude being 35° 27′, that is more than 30° = to two Hours in the Equinoctial in Time, ſhews that the Subſtile will fall between the Hours of Nine and Ten in the Forenoon, place this 35° 27′ right againſt the Hour of Twelve, and 21° 9′ againſt it in the next Column, becauſe

cauſe 3° 45′ anſwers to one Quarter of an Hour, and 15 to an Hour in the Equinoctial, therefore ſubtract 3° 45′ from 35° 27′ continually as long as it will bear, 'till I have only 1° 42′ remaining, which anſwers to three Quarters paſt 9 in the Morning; hence, becauſe this brings me to the Meridian, and Place of the Subſtile in the Dial, I ſubtract 1° 42′ from 3° 45′, and the Remainder is 2° 3′ the Equinoctial Degree anſwering $9\frac{1}{2}$, to which add 3° 45′ continually, 'till I come to 88° 18′ the Equinoctial Degrees anſwering $2\frac{3}{4}$ in the Morning, which Angles at the Pole, or Equinoctial Diſtances are more than we have Occaſion for in this Dial, as you will ſee when you come to lay down the Hour Lines upon the Plane by Help of the Line of Chords, which I leave for you to practice at your leiſure.

Hours		Equin. Diſt.		Diſt. à Subſtile		Hours		Equin. Diſt.		Diſt. à Subſtile.	
	2	77	3	67	4	10	0	5	27	2	58
	3	73	18	61	6		1	9	12	5	2
5	0	69	33	55	32		2	12	57	7	8
	1	65	48	50	24		3	16	42	9	16
	2	62	3	45	41	11	0	20	27	11	27
	3	58	18	41	21		1	24	12	13	43
6	0	54	33	37	21		2	27	57	16	5
	1	50	48	33	40		3	31	42	18	33
	2	47	3	30	17	12	0	35	27	21	9
	3	43	18	27	7		1	39	12	23	54
7	0	39	33	24	10		2	42	57	26	50
	1	35	48	21	24		3	46	42	29	58
	2	32	3	18	47	1	0	50	27	33	21
	3	28	18	16	19		1	54	12	37	0
8	0	24	33	33	56		2	57	57	40	57
	1	20	48	11	40		3	61	42	45	15
	2	17	3	9	28	2	0	65	27	49	57
	3	13	18	7	19		1	69	12	55	3
9	0	9	33	5	13		2	72	57	60	33
	1	5	48	3	10		3	76	42	66	58
	2	2	3	1	7	3	0	80	27	72	48
		Merid.		Subſtile			1	84	12	78	17
	3	1	42	0	55		2	87	57	86	14

Laſtly, To find the Diſtance of each Hour Line from the Subſtile, and the Analogy for this, is the very ſame as in the Horizontal Dial, *See the Triangle*, P12D, page 234, with this difference, that inſtead of the Elevation of the North Pole above the Horizon of the Place, commonly called the Latitude here you muſt make Uſe of the Height of the Pole or Stile above the Plane.

One Example will be ſufficient to illuſtrate the whole Proceſs to the Meaneſt Capacity, and that ſhall be to find the Diſtance of the Hour, or Quarter of an Hour before Ten in the Forenoon, the Angle at the Pole, or Equinoctial Diſtance, you ſee in the Second Column of the Table is 1° 42′.

Operation, By Tranſpoſition.

As Radius - -	90°	00′	10.0000000
To S. Stile's Height	32	55	9.7351345
To T < at the Pole	1	42	8.4724538
To T. Hour on the Plane	0	55	8.2075883

Now all the Work of this Dial is finiſhed, only transfering the Hour Lines to the Plane, and by help of your Line of Chords it is done the very ſame Ways as has been taught in the Horizontal Dial, therefore need no Repetition; only in fixing the Gnomon, obſerve to place it on the Subſtiler Line, at right Angles to the Plane, and to make an Angle therewith of 32° 55′, and it may be either a Plate of Braſs, Copper, *&c.* or an Iron Rod juſt the thickneſs of the Hour Lines.

A General Rule.

If the Plane decline Eaſt, the Subſtile falls among the Morning Hours, but if it declines Weſt, among the Afternoon Hours.

IV. *Of the Eaſt and Weſt Erect Direct Dials.*

THESE Dial Planes lying in the Plane of the Meridian, the Earth's Axis being parallel thereto, the Pole itſelf has no Elevation, and conſequently the Hour

Lines can have no Centers, but are parallel to each other, the Six o'Clock Hour Line being the Subſtile, the reſt of the Hour Lines are drawn by the Tangent of 15, 30, 45, 60, 75°, each Way as far as you have Occaſion for Hour Lines, and let them make an Angle with the Horizon equal to the Latitude of you Habitation: The Height of the Stile muſt be equal to the Tangent of 45°, or the Diſtance of the Nine and Three o'Clock Hour Line.

V. *Of an Equinoctial Dial.*

WHAT I call here an Equinoctial Dial, is not ſo in Fact, but an Horizontal Dial, or a Dial as is made under the Equinoctial Circle, for there the Pole has no Elevation; therefore the Hour Lines and Stile are all parallel to the Plane, and is made as has been juſt now ſhewn in the Eaſt and Weſt Dials, with this difference only, that this faces the Zenith, as the other do the Eaſt and Weſt Points of the Horizon.

VI. *Of a Polar Dial.*

THIS and the laſt mentioned Dial have different Names by different Writers; ſome call this an Equinoctial Dial, ſome a Polar Dial, but I ſhall not in this Place diſpute Words, for they are of no more Uſe than to convey the Idea of Things from one Perſon to another. Therefore if you would make a Dial for the Latitude of 90° either North or South, it is no more than to divide a Circle into 24 equal Parts, and from thoſe Parts draw Lines to the Center, and they ſhall be the true Hour Lines, and the Stile is no more than a Wire ſet perpendicular to the Plane in the Center of the Hour Lines.

VII. *To make an Equinoctial Dial.*

THIS Plane has two Faces, the Upper in North Latitudes ſerves all the Time the Sun is in Northern Signs, ♈, ♉, ♊, ♋, ♌, ♍: The under Face of the Plane ſerves the other Part of the Year, *viz.* while the Sun paſſes through ♎, ♏, ♐, ♑, ♒, ♓. and the making of it and it's Stile is all one and the ſame with the laſt mentioned Plane; and this Dial has an Advantage above many others, *viz.* it is Univerſal, for in what Latitude ſoever you be, place it ſo that the Plane may lye parallel to the true Equinoctial in the Heavens, and the Stile parallel to the Meridian of the Place, and then it will ſhew the true Hour of the Day.

VIII. *Of a South Reclining, or North Inclining Dial.*

THESE Planes muſt firſt be reduced to a new Latitude, in which they may become Horizontal Planes, and is thus performed: If the Reclination be leſs than the Complement of your Latitude, then ſubtract the Reclination from the Complement of the Latitude, and the Remainder is the new Latitude, in which it will become an Horizontal Plane.

2. But if the Reclination exceeds the Co-Latitude, ſubtract the Co-Latitude from the Reclination and this difference in the New Latitude, ſo that to theſe new Latitudes thus found, draw Hour Lines, as has been taught in the Horizontal Dial, and they will be the true Hour Lines upon the South Reclining Plane.

3. But if the Reclining be equal to the Complement of your Latitude, and Recline from the Zenith towards the North Pole, then doth ſuch a Plane lye directly in the Earth's Axis and have no Elevation; a Dial in ſuch a Plane

as this muſt be made in all Reſpects as has been taught in the Equinoctial Dial, in the Fifth hereof. That is, An Horizontal Dial under the Equinoctial Circle.

IX. *Of the Eaſt and Weſt Reclining and Inclining Dials.*

THESE Dials muſt alſo be refer'd to a new Latitude, that ſo they may become upright declining Planes, which is done thus: The new Latitude is always the ſame with the Complement of the Latitude in which the Plane Reclineth, and the Declination in that new Latitude will be always the ſame with the Complement of the Plane's Reclination: So that in the Latitude of 51° 32' if an Eaſt or Weſt Plane ſhould recline from the Zenith 35°, and you would know in what Latitude that Dial ſhall become an upright declining Plane. Then Note, That the new Latitude is 38° 28', the Complement of the given Latitude, and the Declination on the Plane is 55°, the Complement of the Reclination: So if you make a Vertical Dial (as has been taught in page 328), for the Latitude of 38° 28', and the Declining 55°, that Dial will be a true Dial, for an Eaſt or Weſt Plane Reclining from the Zenith 35° in the Latitude of 51° 32'.

X. *To make a Declining Reclining Dial.*

THESE Planes muſt be refer'd to a new Latitude, and to a new Declination before Hour Lines can be drawn upon them.

EXAMPLE.

In the Latitude of 51° 32', a Plane declines to the Eaſt or Weſt 34°, and Reclines 50° from the Zenith, what's the new Latitude and new Declination?

For the new Latitude always ſay,

As Radius - -	90°	0'	10.0000000
To C S. Declination	34	0	9.9185742
So C T Reclination	50	0	9.9238135
To T. of - -	34	55	9.8423877

Now obſerve theſe Rules.

I. This fourth Tangent muſt be compared with the old Latitude, and the Complement of the Difference is the new Latitude.

Operation.

Old Latitude -	51°	32'	
Tangent ſubtract	34	55	
Difference -	16	37	Complement 73° 23'

is the new Latitude.

II. *To find the new Declination.*

As Radius - -	90°	00'	10.0000000
To C S. Reclination	50	00	9.8080675
So S. Old Declination	34	00	9.7475616
To S. new Declination	21	4	9.5556291

Secondly, Suppoſe a Plane Recline juſt ſo far as to paſs thro' the Pole; then will the Hour Lines be all parallel to one another, as on a direct Eaſt or Weſt Equinoctial, Horizontal, *&c.* Then the Analogy for finding the Reclination is this.

As C S Declination, to Radius, ſo T. Latitude, To C T of the Reclination.

For the Declination.

As Radius, to C S Reclination found in the laſt, ſo is S. old Declination, to S new Declination.

Laſtly, Suppoſe a Plane Recline from the Zenith 20, and Declination 34 as before; *Quary* the new Latitude and new Declination? Say as in the firſt, and you will find the new Latitude 75° 14', and new Declination 31° 42'. And theſe are all the Varieties that can happen. I might here have ſpoke of Reflective, Refractive, Concave, and Convex Dials, with the Globe and Croſs Dials, but theſe I have already explained in my *Mechanic Dialling*, to which I refer my Reader for further Satisfaction.

CHAP X.

Of Surveying of Land.

SURVEYING of Land is a very uſeful Branch of the Mathematicks; and requires that the Practitioner be well ſkilled in Numbers, both Whole and Broken, with Extraction of Roots, Geometry, and in Plain Trigonometry, which this Treatiſe will make him a compleat Maſter of, without any other Book or Tutor. Therefore I would have the Young Student to perfect himſelf in one Chapter before he proceeds in another; and firſt you are to Note, that by a Statute made in the 33d Year of the Reign of King *Edward* I, *Anno* 1305, an Acre of Land was to contain 160 Square Perches, and the Perch 5½ Yards; 40 of which Poles or Perches makes one Furlong, and 8 Furlongs makes one Mile, as is better and more at large expreſſed in this Table.

Inches.	Feet.	Yard.	Pole.	Furl.	Miles.
12	1				
36	3	1			
198	16½	5½	1		
7920	660	220	40	1	
63360	5280	1760	320	8	1

All Land is now meaſured by Mr *Gunter*'s Chain, which contains ſour Poles, each Pole is 16½ Feet, the Whole Chain being 22 Yards, or 100 Links, but more largely and better expreſſed in this Table.

Inches.	Link	Yard.	Pole.	Chain.	Mile.
7.92	1				
36	4.55 *ferè.*	1			
198	25	5½	1		
792	100	22	4	1	
63360	8000	1760	320	80	1

And for the Learner's better Information I ſhall here preſent him with a new Table of Square Meaſure.

A Table of Square Meaſure, which is to be read thus, viz. *in* 1 *Chain there are* 16 *Square Perches,* 174.24 *Square Paces,* 484 *Square Yards,* 4356 *Square-Feet,* 10000 *Square Links, and* 627264 *Square Inches,* &c.

Inches.	Links.	Feet.	Yards.	Paces.	Perches.	Chains.	Acres.	Mile.
1								
62.7264	1							
144	2.295	1						
1296	20.75	9	1					
3600	57.38	25	27	1				
39264	625	272.25	30.25	10.89	1			
627264	10000	4356	484	174.24	16	1		
6272640	100000	43560	4840	1742.4	160	10	1	
4014489600	64000000	27878400	3097600	1115136	102400	6400	640	1

A Table ſhewing how Square Links are made in any Piece of Land.

Square Links.	Per-ches.	Square Links.	Acr.	Rods	Per-ches.
625	1	14375			23
1250	2	15000			24
1875	3	15625			25
2500	4	16250			26
3125	5	16875			27
3750	6	17500			28
4375	7	18125			29
5000	8	18750			30
5625	9	19375			31
6250	10	20000			32
6875	11	20925			33
7500	12	21250			34
8125	13	21875			35
8750	14	22500			36
9375	15	23125			37
10000	16	23750			38
10625	17	24375			39
11250	18	25000		1	
11875	19	50000		2	
12500	20	75000		3	
13125	21	100000	1		
13750	22				

By the foregoing Table you may caſt up the Content of any Piece of Land by Inſpection, for when you have multiply'd your Dimenſions in Chains and Links together, that Product is ſquare Links, as ſuppoſe you had 35127 Square Links, what is the Content?

Look for the neareſt leſs in the Table, (if you cannot find the exact Number) which is here 25000 anſwering to one Rood of Land, and the overpluſs 10127, anſwers to 16 Perches, and 127 Square Links over, which being leſs than a Perch is not regarded.

Now becauſe *Gunter*'s Chain is the only Chain now made Uſe of in Surveying of Land, you are to obſerve that 10 Chains long, and 1 Chain broad doth make a Statute Acre of 160 Square Perches, as by this Table is more fully ſhewn.

Length in Chains.	Breadth in Chains, Links, and Parts of a Link.
1	10.00
2	5.00
3	3.33 Parts 33
4	2.50
5	2.00
6	1.66 Parts 66
7	1.42 Parts 85
8	1.25
9	1.11 Parts 11.

Likewiſe if you would ſet out an Acre in Yards, you may ſee by the Table, page 340, that the Chain is 22 Yards long, which multiply'd by 10 makes 220 Yards long, and 22 Yards broad makes one Acre. Becauſe 220 × 22 = 4840 Square Yards; and if you would ſet out an Acre by Perches you muſt know that 40 Perches long, and 4 broad, is an Acre = 160 Square Perches.

II. *Of the Inſtruments that the Surveyor muſt be furniſhed with.*

1 A *Gunter*'s Chain, as above deſcribed.
2 A Caſe of Inſtruments.
3. A Parallel Ruler.
4 A Plain Table.
5. A Theodolite.
6. A Plotting Scale, which has on it two Diagonal Scales, commonly one of 10, and the other of 20 in an Inch on one Side, and on the other Scales of 11, 12, 16, 20, 24,

20, 24, and 30 in an Inch, *&c.* As the Protractor and Line of Chords are uſed in laying down Angles, ſo are theſe Plotting Scales uſed in laying down of ſtreight Lines, *i. e.* the Sides of Incloſures, the Diagonals, Perpendiculars and Off-ſets which were actually taken in the Field.

Now a Scale of any Number of equal Parts whatſoever, is no more than to divide the Inch into as many equal Parts as you deſign the Scale ſhall contain Parts. As if I would have a Scale of 10 equal Parts in an Inch, (which Scale will make the greateſt Plot of all), divide the Inch into 10 equal Parts and 'tis done. If you would have a Scale of 16 in an Inch, divide the Inch into 16 equal Parts, then 10 of thoſe Parts will be a Scale of 16 in an Inch, *&c.*

N. B. I here recommend the Uſe of the Theodolite before that of the Plain Table, becauſe the Paper upon the Table in the Field will be ſubject to contract, or expand according as the Air is dry or moiſt, and conſequently will make your Plot too much, or too little, and err from the Truth conſiderably. Theſe Inſtruments, with all others for Practical Mathematics, are made and ſold by Mr *John Fowler*, at the *Globe* in *Swithin*'s-*Alley*, by the *Royal Exchange*, *London*, fitted up at the beſt Manner, and at the loweſt Prices.

In Meaſuring with the Chain, be careful to get the ſhorteſt Diſtance between any two Objects, otherwiſe you will make more of the Land than it really is. In Order hereunto you muſt provide Ten ſmall Sticks like Arrows, ſharp at the End to ſtick in the Ground; having ſet up White's at the Angles of the Field, and at other Places where you ſee Occaſion, let him, (your Aſſiſtant) that leads the Chain, carry the Arrows in his Left Hand, all, ſave one in the Right, with the Chain, and always when he ſticks down one of his Arrows, be ſure that it be in a right Line with the Eye of him that follows the Chain, who is to gather up the Sticks, go on thus to the End of the Diſtance you are to meaſure, whether it be a Diagonal, Perpendicular, or the Boundary, Off-ſet, *&c.* by this Method he that follows the Chain and gathers up the Sticks has a ſure Method of counting the Number of Chains, without any Trouble, or fear of Error: As ſuppoſe I find a Diſtance to be 8 Chains and 30 Links, you muſt ſet it down in your Field Book, ruled in proper Columns for that purpoſe, and it will ſtand thus 8.30, and 4 Chains 7 Links will ſtand thus, 4.07, always minding to ſet a a Cypher in the Place of Primes in the Links, if they be under Ten. Now the Reader being well acquainted with Decimal Arithmetic, and with the Menſuration of Superficies, as taught in this Treatiſe, he is qualified to go into the Field, in Order to meaſure the ſame.

III. *How to take the Plot of a Field, by the Chain only, and to caſt up the Content thereof.*

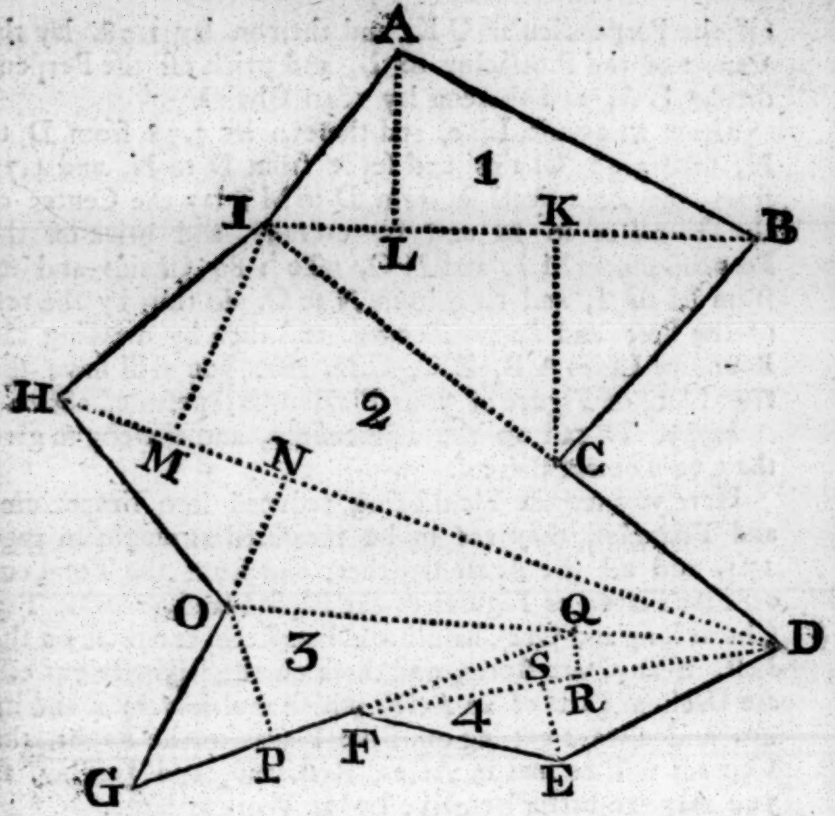
IN Order hereunto, you muſt provide yourſelf with a ſmall Inſtrument call'd a Croſs, which you may make of Wood thus, provide a Piece of Box Wood, and let it be turn'd round, and about two or three Inches Diameter, with a fine Saw cut two Slits croſs ways, that ſo they may be exactly at right Angles with each other, and on the under Side of it let there be a Hole for Conveniency of placing it upon a Staff about your own Height in Time of Practice. Being thus prepared, go into a Field, and upon a Piece of Paper draw the rough Draught of it, as near the Shape as poſſible you can, while you are doing this, let your Aſſiſtant place Pieces of white Paper at A, B, C, D, E, F, G, O, H and I. Then your Aſſiſtant taking the Chain and Arrow, ſtanding at B, direct him to I, and as you go along, you muſt by help of your Croſs take the Perpendicular K C, that is done by looking through one of the Slits in the Croſs to I, then looking thro' the other Slit at C and find the Point K, put down the Chains and Links B K 150, leave a Mark at K, and meaſure on to L; here by help of your Croſs looking at B and A, you will find out the Point L, that is where the Perpendicular A L doth fall, ſet down the Number of Chains either from B to L, or from K to L, it matters not which, leave a Mark at L, and meaſure on to I, and ſet down the Whole Diagonal B I 3.75. Go back to L and meaſure the Perpendicular L A 1.40, and C K 1.78, which ſet down upon your rough Draught. And thus is the Trapezium ABCI finiſhed, which I Note with the Figure 1.

Secondly, For the Trapezium I D O H, which I mark with the Figure 2. Then going to D, direct the Aſſiſtant towards H, and having meaſured 3.93 Chains, I obſerve the Point N, and the Point M fall at 4.77, where leave Marks, and meaſuring on to H, I find the Whole Baſe D H to be 5.78 Chains, which I inſert in my rough Draught as before, then meaſuring the Perpendicular M I, I find it to be 1.80, and N O 1.09.

Thirdly, I go into the Triangle G O Q, which I mark with the Figure 3, and beginning at G, I meaſure towards Q, and as I go along, I obſerve with my Croſs that the Perpendicular O P falls at P. 1.11 from G at P, I leave a Mark, and meaſure to Q, which I find to be 3.54 Chains. Then going back to P, I meaſure P O and find it to be 1.08 Chains.

Laſtly, Here ſtill remains the Trapezium F Q D E unmeaſured, which I Note with the Figure 4. Then coming to F, I meaſure toward D, and I find the Perpendicular S E to fall at 1.43 Chains from F, which I Note down in my rough Draught as before; meaſuring on I find, the Perpendicular R Q to fall at 1.72 Chains from F, which I alſo Note down: And meaſuring to D, I find F D to be 3.23 Chains, which I alſo Note down Then coming back to the Marks which I left at R and S, I meaſure S E and find it 0.60 Link, and Q R 0.35 Link, which I ſet down in my Field Draught, and ſo is the meaſuring all finiſhed. And for the Reader's better Information, I ſhall here inſert each particular Meaſurement in Chains and Links.

N. B. You muſt remember, when in the Field, to meaſure I D and D O, they being of Uſe in the plotting, but not in caſting up of the Dimenſions.



B K	1.50	N O	1.09	I D	4.94
K L	1.22	G P	1.11	DO	4.15
B I	3.75	G Q	3.54		
A L	1.40	P O	1.08		
C K	1.78	F S	1.43		
DN	3.93	F R	1.72		
DM	4.77	F D	3.23		
DH	5.78	SE	0.60		
M I	1.80	QR	0.35		

Secondly, Shewing how to Plot the ſame.

When you have finiſhed the Work in the Field, you may plot and caſt it up at your leaſure thus.

Take a clean Sheet of Paper, and with a Black Lead Pencil draw a Line at Pleaſure, take one Chain 50 Links from your Scale and lay it from B to K, take 1.22 and lay it from K to L, and 3.75 Chains from B to I. This done, take your Protractor and lay the Center to K, prick off the Perpendicular C K, and thereon lay 1.78, lay the Center of the Protractor to L, and prick off the Perpendicular L A, and thereon lay 1.40 Chains.

Draw an occult Line, and thereon let 5.78 from D to H, take 3.93 Chains and ſet it from D to N, and 4.77 from the ſame Scale ſet from D to M; lay the Center of the Protractor to M and N ſeverally, and prick off the Perpendiculars M I, and N O, take 1.80 Chains and ſet from M to I, and 1.09 from N to O, do thus by the reſt of the Baſes and Perpendiculars, and then by drawing the Boundary Lines A B, B C, C D, *&c.* you will have the true Plot, or Figure of your Fields as is repeſented above.

Laſtly, To caſt up the Dimenſions, and thereby to give the true Content thereof.

Here you ſee the Field being reduced into Trapeziums and Triangles, they are to be meaſured as taught in page 174, add all the Areas together, and from the Total cut off Five Places, or Figures to the Right Hand, (becauſe Ten Chains long and One Chain broad is an Acre) and thoſe on the Left, if any, are Acres, and thoſe on the Right ſo cut off, are Decimal Parts of an Acre, which multiply by 4 and by 40, and always cutting off Five Places to the Right, the Content will be had in Acres, Roodland, and Perches, as you may the better perceive by the Work at large.

A L = 1.40
C K = 1.78
3.18
Half = 1.59
1 B = 3.75
795
1113
477
1 = .59625
2 = .83521
3 = .19116
4 = .15181
Acres. 1.77443
4
Roods = 2.99772
40
Poles = 39.90880

I M = 1.80
N O = 1.09
Sum - - 2.89
Half DH = 2.89
2601
2312
578
.83521

G Q = 3.54
Half O P = .54
1416
1770
.19116

F D = 3 . 23
Half S E & Q R = . 47
2261
1292
.15181

S E = 0 .60
Q R = .35
Sum = .95
Half = .47

II. *How to take an Angle in the Field either with the Plain Table or Theodolite.*

SUPPOSE in the Field above, I would take the Angle at A, ſet up two Marks at B and I, place your Inſtrument at A, bring the Index to 360 on the Limb, and turn the Inſtrument about till through the Sights you ſee the white at B, here fix the Inſtrument, and move the Index till you ſee the Mark at I, the Degrees cut by Index on the Limb are the Quantity of the Angle B A I, and after the ſame manner may any Angle be taken: And for Proof of your Work, when you have taken all the Field, before you begin to Plot, if you would know whether it will Cloſe, multiply 180° by a Number always leſs by 2 than the Number of Angles within the Field, and if that Product be equal to the Sum of all the Angles, then is the Work right, other ways not.

III. *How to take the Plot of a Field by going round the ſame, or any large Champion Ground, as* Saliſbury-Plain, Hounſlow-Heath, *Forreſt of* Dalameer, *&c.*

IN the 312 Page of the Abridgments of Acts of Parliament, we are told that *Hounſlow-Heath* contains 4293 Acres, and one Rood Land of Ground. Now let us ſuppoſe the Figure in page 347 to repreſent one of the above-mentioned Champion Grounds.

With your Theodolite, Chain and Aſſiſtant, beginning at A, take that Angle, and at every turning of its Bounds as at B, C, D. *&c.* take the Angle as has been taught above, and as you go along meaſure the Diſtances A B, B C, C D, *&c.* with your Chain, and find them as is here ſet down.

Sides			Angles			
A	B	3.10	A	98		
B	C	2.31	B	76		
C	D	2.15	C	93	30	external.
D	E	1.80	D	68	00	
E	F	1.64	E	140	00	
F	G	1.73	F	145	00	external.
G	O	1.53	G	45	00	
O	H	2.00	O	114	30	external.
H	I	2.04	H	91	00	
I	A	1.70	I	168	00	external.

The Sum of the internal Angles is 1440 which is equal to 180 multiplied by 8, that is leſs by 2 the Number of Angles, which is a Proof of the Work.

IV. *How to take the Horizontal Line of a Hill.*

IT is beſt to Meaſure Hilly Ground by the Chain, by ſetting up Beacons upon ſeveral of the higheſt Hills, from which you can ſee the Angles in the Hedges, and direct your Sights from Station to Station, for the Chain being drawn over the Hills and Dales, muſt neceſſarily make a larger Plot, than it would do if you went all round the Hedges, all the way upon level or even Ground; now becauſe you cannot make a Convex Superficies upon the Paper, you muſt only Plot the Horizontal Line or Baſe thereof.

Place your Theodolite at one ſide of the Hill, and cauſe a Mark to be ſet on the Top of the Hill, juſt as high as your Inſtrument is from the Ground, and take the Angle of Altitude, and meaſure with your Chain from the Inſtrument to the Mark on the Top: Then by Trignonometry ſay, As Radius, to the Diſtance from the Inſtrument to the Top of the Hill; So is C S, Altitude, to the part of the Baſe from the Inſtrument to the Perpendicular Line. Go to the oppoſite Side of the Hill and make the like Obſervations, find by the above Analogy the Diſtance between the Inſtrument and the Perpendicular Line, which added to the other juſt now found, and the Sum is the Horizontal Line ſought.

Or it may be found at one Operation thus, As the Sine of the Angle at the Inſtrument is to the oppoſite Side of the Hill, ſo is the Sine of the Angle at the Top of the Hill, to the Length of the Baſe.

V. *How to take the Plot of a Field at one Station, when all the Angles are ſeen.*

PLANT your Inſtrument any where in the Field where all the Angles can be ſeen, ſet up Marks, take all the Angles and note them down in your Field-Book, then meaſure the Diſtance with your Chain from your Inſtrument to every Angle, and ſet them down in your Field-Book, and the Work is done.

N. B. If it happens that you cannot ſee all the Angles at one Station, you muſt make two or three as you ſee moſt convenient for your purpoſe, meaſuring your Stationary Diſtances.

VI. *To take the Plot of a Field, or the Map of a Country.*

MAKE choice of two Stations where you can ſee all the Angles of the Field, place your Theodolite at one of the Stations, and take the Angle of the Field, meaſure to your ſecond Station the Diſtance in Chains and Links, Plant your Theodolite at your ſecond Station, and take all the Angles of the Field, and Note them down in your Field-Book: Now theſe Angles and Stationary Diſtance being Protracted upon Paper, the Interſection of the Lines from each Station will determin the juſt Figure of the Field, or the true Situation of a Country, and make a perfect Map of places in your View, which Diſtances are gain'd either by Trigonometry, or by meaſuring them by the ſame Scale by which the Stationary Diſtance was laid down.

VII. *How to Survey Woods, or great Pools of Water.*

THIS muſt be done by going round the ſame on the out Side, and at every Bending take the Angle and meaſure the Diſtance from one Bending to another, Note all down in your Field-Book, and when you come round to the Place where you began, if your Plot cloſe, you may be aſſur'd your Work is right, other ways there is ſome Error. Then when you have plotted your Work, take off the Baſes and Perpendiculars from the ſame Scale you laid down the Plot, and from thence caſt up the Contents in Acres as has been taught in page 348.

VIII. *An Expeditious Method, to caſt up the Superficial Content of Land in Acres.*

WHAT we have already ſaid on this Head, is to reduce the Field into Trapezias and Triangles, and thoſe Baſes and Perpendiculars meaſured by the Scale, may poſſibly produce ſome Error if great care is not taken in drawing the Lines fine; therefore in Geometry we have ſhewn how to reduce a Figure of many Sides to a Triangle, *&c.* by which means many Operations are ſaved in caſting it up, and conſequently liable to leſs Error.

IX. *How to take the Plot of a Gentlemans Eſtate, Mannor or Lordſhip.*

IT will be beſt firſt of all to take a View of the Land, and conſider where it is beſt to begin; then provide a Station Staff about the ſame Height with your Inſtrument, and ſlit it at the Top to put in a quarter of a Sheet of white Paper, beginning at any convenient place; fix your Inſtrument and take the Bearing or Situation of your firſt Station, and ſend your Station Staff by an Aſſiſtant as far as you can ſee, meaſure to every Off ſet as you go along, and the Off-ſet themſelves, entering all in your Field-Book, remembering where you begin to make that your firſt Meridian, and then as you go on in the Work, take Notice how ſuch or ſuch a remarkable Place bears from your firſt Meridian, by this means the Mannor will be kept in its true Poſition; proceeding thus, through all the Work, always being careful to take Notice of all Things that are remarkable, and incert them in your Field-Book, that ſo when you come Home to Plot your Days work at Night, you may readier diſcern any Error if any ſuch Thing ſhould be. When the Map is made, you muſt incert the Name of every Incloſure, with its Content in Acres, and diſtinguiſh each with their proper Colours, the Gentleman's Seat, Coat of Arms, and Points of the Compaſs, truly repreſenting the Situation of the Eſtate; and alſo a Scale of Miles adapted to the Map. The North-ſide of the Map is always placed upwards, and the Eaſt on the right, as the Compaſs more plainly expreſſes it. If the Lordſhip is large it will be beſt to draw Upright, or North and South, and Eaſt and Weſt Lines, denoted by Letters at the top and bottom, and alſo on the ſides; to be refer'd to by the Table of References, for the ready finding any Field or Parcel of Land therein contain'd.

X. *How to find the Diameter of a Circle that ſhall contain any Number of Acres.*

RULE.

As 11 is to 14, ſo is the Number of the Acres given, to the Square of the Diameter of the Circle required.

EXAMPLE

What's the Diameter of a Circle whoſe Area is one Acre?

Turn the given Acre into ſquare Links thus 1.00000, then,

As 11 : 14 : : 100000 : 12.727270(3.57 □√ *ferè*.

XI. *To Set out an Acre, three Rood Land, half an Acre, one Rood, or any Number of Perches under* 40 *Perches, or a Rood Land.*

Say, As the Length, or Breadth,

	Chains			
Is to	10.00	So is	1 00	Acre.
	7.50		1.00	three Rood.
	5, 0		1.00	half Acre.
	2.50		1.00	Rood.
	.625		.01	one Perch.
	.625		.02	two Perch.
	.625		.07	three Perch.

EXAMPLE

Suppoſe, I had 1.32 Chains in length, how much muſt I have in Breadth to make 10 Perches.

As 1.32 : .625 : : 10 : .48 Links, *ferè*.

XII. *How to reduce a large Plot of Land or Map into a leſſer Compaſs, according to any given Proportion, or on the contrary how to enlarge it.*

THE beſt way to do this, if it be a Field or two, *&c.* is to Plot it over again by a greater or leſſer Scale as need requireth, but if it be large as the Map of a Country or Mannor, *&c.* the only way is to Circumſcribe it with a Geometric Square, and Divide that great Square into ſeveral other leſſer Squares, and by this means, every Cloſe, Houſe, Mill, Tree, *&c.* in one will fall in the ſame Square in the other.

XIII. *To find how many Inches makes a Mile, half a Mile, or a quarter of a Mile, by any Scale whatſoever.*

FIRST you are to conſider that the leaſt Number of parts any Scale is Divided into are 10, and this Scale makes the largeſt Plot of any other. Secondly, You ſee by the Table in page 340, that 80 Chains of all Scales whatſoever make one Mile, ſo that by the Scale 10 in an Inch, 80 Inches in one Mile, but to find how many Inches is a Mile by any other Scale, obſerve theſe inverted Proportions.

	Scales		
As 10 : 80 : :	11 : 72.72		Inches in a Mile.
	12 : 66.66		
	16 : 50.		
	20 : 40.		
	24 : 33.33		
	25 : 32.		
	30 : 26.26		
	40 : 20		
	50 : 16		

XIV. *To reduce Statute Meaſure to Cuſtomary, and the Contrary.*

THAT which is called Statute Meaſure, has 5 ½ yards to the Pole or Perch, = 16 ½ feet, and the Acre hath 160 of thoſe Square Perches: But in *Lancaſhire* they have 8 yards to the Pole = 24 Feet, which they call a Rood, and 160 Square Poles to the Acre. In *Cornwal* (I am informed) their Perch is 3 yards, and the Acre 640 Poles or Perches. The *Iriſh* Perch is 7 yards = 21 Feet, and 160 Square Perches to the Acre. Now how many Acres Statute Meaſure will 20 Acres in *Lancaſhire* make? See the Work.

As 30.25 : 20 : : 64 : 42.3 Statute.

Again, Suppoſe 42.3 Statute Acre, how many Acres will they make Cuſtomary in *Lancaſhire*?

As 64 : 42.3 : : 30.25 : 20 Acres.

That is, As the Square of the Cuſtomary Perch,
Is to the given Number of Acres,
So is the Square of the Statute Perch,
To the Number of Acres ſought.

FINIS.

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